Hyperfine splitting of the ground state of baryonium using a harmonic-oscillator potential

C. S. Kalman
Concordia University–Université du Québec Elementary Particle Physics Group, 1455 de Maisonneuve Boulevard West, Montreal, P.Q., Canada H3G 1M8

Richard L. Hall
Concordia University–Université du Québec Elementary Particle Physics Group and Department of Mathematics, Concordia University, 1455 de Maisonneuve Boulevard West, Montreal, P.Q., Canada H3G 1M8

Sushil K. Misra
Concordia University–Université du Québec Elementary Particle Physics Group and Department of Physics, Concordia University, 1455 de Maisonneuve Boulevard West, Montreal, P.Q., Canada H3G 1M8
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The hyperfine splitting of the ground state of baryonium is calculated using parameters derived solely from baryon data. The model used employs a harmonic-oscillator potential suitably modified to describe an attractive potential at short range (Coulomb-type piece derived from quantum chromodynamics) and deviations from the harmonic-oscillator form at large distances. Comparisons with other theoretical calculations and experimental results are made. The hyperfine levels computed in this paper are in close agreement with those derived using a linear plus Coulomb potential.

I. INTRODUCTION

Recently an ever increasing number of narrow-width mesons with large coupling to baryon-antibaryon channels has been observed.[1] The favored explanation[2] is that these states, now frequently referred to as baryonium,[3] are the $qq\bar{q}\bar{q}$ states whose existence is required by dual models.[4] Such four-quark states have also been anticipated in potential models[5] and several quark models, such as the MIT bag model[6] and dynamical group models.[7] Several researchers[8-11] have described baryonium as a color-magnetic coupling of a diquark with an antidiquark. Of the various possible theoretical states only the 3-$\bar{3}$ and 6-$\bar{6}$ are physically acceptable, since the composite state must be a color singlet. (This can be seen as follows. Each quark is a triplet in color SU(3) [$\mathrm{SU}(3)_c$] and $3\otimes 3=\bar{3}\oplus 6$. Hence the diquark can be either in a $\bar{3}$ or 6 representation of $\mathrm{SU}(3)_c$.) Chan and Hogassen[12] note that for high values of angular momentum the 3-$\bar{3}$ composite state which they call true baryonium (T) has normal hadronic width into baryon-antibaryons. The 6-$\bar{6}$ state, which they refer to as mock baryonium (M) is not a geniune $B\bar{B}$ state at all and decays into $B\bar{B}$ pairs only by default of the meson modes.

The color hypothesis was first introduced[13] to ensure that the baryon wave functions are antisymmetric under the exchange of quark indices. Up to now this hypothesis has had only three experimental tests: the decay $\pi^0\to 2\gamma$, the ratio of the decays of the τ^- lepton to leptons and to hadrons, and the ratio $R=\sigma(e^+e^-\to \text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$. Since M baryonium states exist only by virtue of the hidden color, it is of great interest to establish the mass of these states so as to provide an additional experimental test of the color hypothesis. Very few calculations of the ground-state system of baryonium exist at present. Hendry and Hinchliffe[8] and Lichtenberg and Johnson[10] have considered only the T baryonium system; the treatments of Jaffe,[6] and Barbour and Ponting[11] have included the M baryonium system.

It is the purpose of this paper to describe splitting of the baryonium ground state due to the hyperfine interaction using parameters derived exclusively from baryon data. The model used is the Isgur-Karl quark shell model,[14] which has been successfully employed to predict the mixing angles of the p-wave baryons and violations of SU(6) selection rules.

II. GENERAL PROBLEM

In the ground state there are no spin-orbit or tensor hyperfine interactions. The Isgur-Karl Hamiltonian then has the form

$$H=\sum_i m_i+H_0+H_{\mathrm{hyp}}\,, \tag{2.1}$$

where $m_i=m$ is the common constituent quark mass

$$H_0=\sum_i p_i^{\,2}/2m+\sum_{i<j}V_{\mathrm{conf}}^{ij}-\Big(\sum_i p_i\Big)^2\Big/\Big(2\sum_i m_i\Big)\,, \tag{2.2}$$

$$H_{\mathrm{hyp}}=-8\sqrt{2}A\pi\sum_\alpha\sum_{i>j}(\vec{S}_i\cdot\vec{S}_j)\Lambda_i^\alpha\Lambda_j^\alpha\delta^{(3)}(\vec{r}_{ij})\,, \tag{2.3}$$

where the $\vec{r}_{ij}$ are the interparticle distances and $\vec{s}_i$ and Λ_i^α are the spins and color vectors of the quarks and antiquarks. The coefficient of Eq. (2.3) has been chosen for ease of evaluation by comparison to the baryon states described by Isgur and Karl.[14]

Numerical studies have shown[14] that the low-lying states of many potentials (for example, linear plus Coulomb) can be approximated by harmonic-oscillator wave functions. Thus,

$$V_{\rm conf}^{ij} = \vec{\Lambda}_i \cdot \vec{\Lambda}_j [\tfrac{1}{2} K r_{ij}^2 + U(r_{ij})] . \tag{2.4}$$

$U(r_{ij})$ is some unknown potential which incorporates an attractive potential at short range [a Coulomb-type piece derived from quantum-chromodynamics (QCD)] and deviations from the harmonic-oscillator form at large distances. U and $H_{\rm hyp}$ can be treated by first-order perturbation theory using the harmonic-oscillator wave function.

It should be noted that in the bag model[6] $\langle H_0\rangle$ is the same for both T and M baryonium. Chan and Høgassen[12] also made this assumption for their calculation of these systems. Although our calculations (see Secs. IV and V) show $\langle H_0\rangle_T \simeq \langle H_0\rangle_M$, there is a definite (though small) difference in energy which affects the diagonalization of the total Hamiltonian. Furthermore, in the bag model, $C = 8\sqrt{2}A\pi\langle\delta^{(3)}(\vec{r}_{12})\rangle$ and $C' = 8\sqrt{2}A\pi\langle\delta(\vec{r}_{13})\rangle$ are equal. Again Chan and Høgassen make the assumption that "$C' \simeq C$ where $L=0$, but depends on the angular momentum L in general." Our results (see Sec. VI) show that even for $L=0$, there is a large difference between the value of C' and C.

In the following sections we separately examine the various factors entering into the total Hamiltonian. The final results for the masses of all the states appear in Sec. VI and a discussion of the results is given in Sec. VII.

III. COLOR FACTORS

In this section the computation of potentials for the T and M baryoniums will be considered. The term in Eq. (2.2) to be reduced is

$$H_c = \sum_{i<j} V_{\rm conf}^{ij} = \sum_{i<j} V_{ij} \vec{\Lambda}_i \cdot \vec{\Lambda}_j , \tag{3.1}$$

where $V_{ij} = V(r_{ij})$; r_{ij} is the distance between i and j particles (quark or antiquark). We first note that the symmetry of the baryonium system dictates that the spatial expectation values $\langle V_{ij}\rangle$ are such that

$$\langle V_{12}\rangle = \langle V_{34}\rangle (= V_a)$$

and

$$\langle V_{13}\rangle = \langle V_{14}\rangle = \langle V_{23}\rangle = \langle V_{24}\rangle (= V_b) .$$

Thus

$$\begin{aligned}\langle H_c\rangle &= V_a(\vec{\Lambda}_1\cdot\vec{\Lambda}_2 + \vec{\Lambda}_3\cdot\vec{\Lambda}_4) + V_b(\vec{\Lambda}_1\cdot\vec{\Lambda}_3 + \vec{\Lambda}_1\cdot\vec{\Lambda}_4 + \vec{\Lambda}_2\cdot\vec{\Lambda}_3 + \vec{\Lambda}_2\cdot\vec{\Lambda}_4)\\ &= \tfrac{1}{2}(V_a - V_b)[(\vec{\Lambda}_1+\vec{\Lambda}_2)^2 - (\vec{\Lambda}_1{}^2 + \vec{\Lambda}_2{}^2) + (\vec{\Lambda}_3+\vec{\Lambda}_4)^2 - (\vec{\Lambda}_3{}^2+\vec{\Lambda}_4{}^2)]\\ &\quad + \tfrac{1}{2}V_b[(\vec{\Lambda}_1+\vec{\Lambda}_2+\vec{\Lambda}_3+\vec{\Lambda}_4)^2 - (\Lambda_1{}^2+\Lambda_2{}^2+\Lambda_3{}^2+\Lambda_4{}^2)] .\end{aligned} \tag{3.2}$$

We next note the following. Since the composite state must be a color singlet, (i) $\vec{\Lambda}_1+\vec{\Lambda}_2+\vec{\Lambda}_3+\vec{\Lambda}_4 = 0$, (ii) $\Lambda_i{}^2 = \frac{4}{3}$ $(i=1,2,3,4)$, (iii) $(\vec{\Lambda}_1+\vec{\Lambda}_2)^2 = (\vec{\Lambda}_3+\vec{\Lambda}_4)^2 = \frac{4}{3}$ for T baryonium,[6] and (iv) $(\vec{\Lambda}_1+\vec{\Lambda}_2)^2 = (\vec{\Lambda}_3+\vec{\Lambda}_4)^2 = \frac{10}{3}$ for M baryonium,[6] one obtains from Eq. (3.2)

$$(\vec{\Lambda}_1\cdot\vec{\Lambda}_2)_T = -\tfrac{2}{3}, \quad (\vec{\Lambda}_1\cdot\vec{\Lambda}_2)_M = \tfrac{1}{3} ,$$
$$(\vec{\Lambda}_1\cdot\vec{\Lambda}_3)_T = -\tfrac{1}{3} , \tag{3.3}$$

and

$$(\vec{\Lambda}_1\cdot\vec{\Lambda}_3)_M = -\tfrac{5}{6} .$$

These results are identical with those obtained by Anderson and Joshi.[15] The spatial expectation values $\langle H_c\rangle$ are as follows:

$$T \text{ baryonium:}\ \langle H_c\rangle_T = -\tfrac{4}{3}V_a - \tfrac{4}{3}V_b , \tag{3.4}$$

$$M \text{ baryonium:}\ \langle H_c\rangle_M = \tfrac{2}{3}V_a - \tfrac{10}{3}V_b . \tag{3.5}$$

The values given by Eqs. (3.4) and (3.5) will now be used to compute the harmonic-oscillator energies for T and M baryonium states in Sec. IV below. These resulting values will, in turn, be used to compute the spectra for the various states in Sec. VI.

IV. THE HARMONIC-OSCILLATOR STATES

If we label the four particles ($qq\bar{q}\bar{q}$), each of mass m, by (1234), respectively, and let r_{ij}

$= |\vec{r}_i - \vec{r}_j|$, then the harmonic-oscillator part H_{HO} of the Hamiltonian H may be written

$$H_{HO} = \frac{1}{2m}\sum_{i=i}^{4}\vec{p}_i^{\,2} - \frac{1}{8m}\left(\sum_{i=i}^{4}\vec{p}_i\right)^2 + \tfrac{1}{2}f_1K(r_{12}^2+r_{34}^2) + \tfrac{1}{2}f_2K(r_{13}^2+r_{14}^2+r_{23}^2+r_{24}^2)\,, \tag{4.1}$$

where the color factors (f_1,f_2) have the following values [see Eq. (3.3). The factors and those of Eq. (3.3) differ by an overall constant which has been absorbed into K of Eq. (4.1). This has been done for ease of comparison with the baryon-data calculations of Isgur and Karl[14]]:

$$T \text{ baryonium: } (f_1,f_2) = (1,\tfrac{1}{2})\,, \qquad M \text{ baryonium: } (f_1,f_2) = (-\tfrac{1}{2},\tfrac{5}{4})\,. \tag{4.2}$$

The Hamiltonian H_{HO} separates in terms of a natural set of orthogonal relative coordinates given by

$$\begin{bmatrix}\vec{\rho}_2\\ \vec{\rho}_3\\ \vec{\rho}_4\end{bmatrix} = \begin{bmatrix}1/\sqrt{2} & -1/\sqrt{2} & 0 & 0\\ 0 & 0 & 1/\sqrt{2} & -1\sqrt{2}\\ \tfrac{1}{2} & \tfrac{1}{2} & -\tfrac{1}{2} & -\tfrac{1}{2}\end{bmatrix}\begin{bmatrix}\vec{r}_1\\ \vec{r}_2\\ \vec{r}_3\\ \vec{r}_4\end{bmatrix} \tag{4.3}$$

and the corresponding conjugate momenta

$$\vec{\pi}_j = -i\hbar\nabla_{\vec{\rho}_j}\,,\quad j = 2,3,4\,,$$

for we have (Hall[16])

$$H_{HO} = (1/2m)(\vec{\pi}_2^{\,2}+\vec{\pi}_3^{\,2}+\vec{\pi}_4^{\,2}) + \tfrac{1}{2}K[2(f_1+f_2)(\rho_2^2+\rho_3^2)+4f_2\rho_4^2]\,. \tag{4.4}$$

Since H_{HO} is now diagonal, we may immediately write down the energies and corresponding eigenstates. For convenience we relate all the parameters arising in the present four-body problem to the parameters α and ω used by Isgur and Karl[14] for the baryons. Thus we define

$$\alpha^2 = (3mK/\hbar^2)^{1/2}\,,\quad \omega = (3K/m)^{1/2}\,, \tag{4.5}$$

$$\alpha_1^2 = \alpha^2 s_1\,,\quad \omega_1 = \omega s_1\,, \tag{4.6}$$

and

$$\alpha_2^2 = \alpha^2 s_2\,,\quad \omega_2 = \omega s_2\,, \tag{4.7}$$

where

$$s_1 = [2(f_1+f_2)/3]^{1/2} \text{ and } s_2 = (4f_2/3)^{1/2}\,. \tag{4.8}$$

The ground state ψ_0 and the ground-state energy E_{HO} of H_{HO} are consequently given by

$$\psi_0(\vec{\rho}_2,\vec{\rho}_3,\vec{\rho}_4) = \frac{\alpha_1^{\,3}\alpha_2^{\,3/2}}{\pi^{9/4}}\exp[-\tfrac{1}{2}(\alpha_1^2\rho_2^2+\alpha_1^2\rho_3^2+\alpha_2^2\rho_4^2)] \tag{4.9}$$

and

$$E_{HO} = (3\hbar\omega_1+\tfrac{3}{2}\hbar\omega_2) = \tfrac{3}{2}\hbar\omega(2s_1+s_2)\,. \tag{4.10}$$

From (4.2) and (4.8) we find

$$T:\ (s_1,s_2) = (1,(\tfrac{2}{3})^{1/2})\,, \tag{4.11}$$

and

$$M:\ (s_1,s_2) = ((\tfrac{1}{2})^{1/2},(\tfrac{5}{3})^{1/2})\,, \tag{4.12}$$

and, by using the value $\omega = 250$ MeV from the baryon fit of Isgur and Karl,[14] we have from (4.10)

$$E_0^T = 1056 \text{ MeV and } E_0^M = 1014 \text{ MeV}. \tag{4.13}$$

V. THE CONTRIBUTION OF $\hat{U}$ IN FIRST-ORDER PERTURBATION

The Hamiltonian, excluding the hyperfine interactions, is given by $(H_{HO}+\hat{U})$, where

$$\hat{U} = f_1[U(r_{12})+U(r_{34})]+f_2[U(r_{13})+U(r_{14})+U(r_{23}) + U(r_{24})]\,, \tag{5.1}$$

and (f_1,f_2) are the color factors (4.2). In first-order perturbation the contribution of $\hat{U}$ to the energy is just $\langle\hat{U}\rangle = (\psi_0,\hat{U}\psi_0)$, where ψ_0 is the ground state (4.9) of H_{HO}. The central potential $U(r)$ itself is unknown, but from the baryon model of Isgur and Karl[14] we do have values for the integrals $a(1)$, $b(1)$, and $c(1)$ of $U(r)$, where

$$a(s) = (3\alpha^3 s^{3/2}/\pi^{3/2})\int d^3\rho\, U(\sqrt{2}\rho)\exp(-s\alpha^2\rho^2)\,, \tag{5.2}$$

$$b(s) = (3\alpha^5 s^{5/2}/\pi^{3/2})\int d^3\rho\, U(\sqrt{2}\rho)\rho^2\exp(-s\alpha^2\rho^2)\,, \tag{5.3}$$

and

$$c(s) = (3\alpha^7 s^{7/2}/\pi^{3/2}) \times \int d^3\rho\, U(\sqrt{2}\rho)\rho^4\exp(-s\alpha^2\rho^2)\,. \tag{5.4}$$

Thus, for the baryons, Isgur and Karl[14] obtain

$$L=0,\quad E(S) = 3m+3\omega+a(1)\,, \tag{5.5}$$

$$L=1,\quad E(P) = 3m+4\omega+a(1)/2+b(1)/3\,, \tag{5.6}$$

$$L=2,\quad E(S') = 3m+5\omega+5a(1)/4-b(1)+c(1)/3\,, \tag{5.7}$$

and, using $\omega = 250$ MeV and $m = m_u = m_d = 350$ MeV (Isgur and Karl[14]), one obtains

$$a(1) = -650 \text{ MeV},\quad b(1) = -405 \text{ MeV}\,,$$

and

$$c(1) = -908 \text{ MeV}\,. \tag{5.8}$$

By constructing quadratic approximations about

$s=1$ for $a(s)$, $b(s)$, and $c(s)$ we find from (5.5), (5.6), (5.7), and (5.8) that

$$a(s) \simeq A+Bs+Cs^2\,, \tag{5.9}$$

$$b(s) \simeq (3A+Bs-Cs^2)/2\,, \tag{5.10}$$

and

$$c(s) \simeq (15A+3Bs-Cs^2)/4\,, \tag{5.11}$$

where

$$A=-170,\quad B=-390,\quad C=-90 \text{ MeV}\,. \tag{5.12}$$

We are now able to evaluate $\langle \hat{U} \rangle$ for the four-body problem.

From the permutation symmetry of the ground state ψ_0 we have from (5.1)

$$\langle \hat{U} \rangle = 2f_1\langle\langle U(r_{12})\rangle\rangle + 4f_2\langle\langle U(r_{13})\rangle\rangle \tag{5.13}$$

and from (5.2) we have

$$\langle\langle U(r_{12})\rangle\rangle = a(s_1)/3\,. \tag{5.14}$$

In order to evaluate $\langle\langle U(r_{13})\rangle\rangle$ we choose a new set of relative coordinates $(\vec{\sigma}_2, \vec{\sigma}_3, \vec{\sigma}_4)$ and then approximate $\psi_0{}^2$ in such a way that $\langle\langle U(r_{13})\rangle\rangle$ can be related to the integral $a(s)$ for some s. Thus,

$$\begin{pmatrix} \vec{\rho}_2 \\ \vec{\rho}_3 \\ \vec{\rho}_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 1/\sqrt{2} \\ -\frac{1}{2} & \frac{1}{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}, \tag{5.15}$$

$$r_{13} = \sqrt{2}\,\sigma_2,$$

and

$$\psi_0{}^2 = \frac{\alpha_1{}^6\alpha_2{}^3}{\pi^{9/2}} \exp[-\tfrac{1}{2}(\alpha_1{}^2+\alpha_2{}^2)(\sigma_2{}^2+\sigma_3{}^2) - \alpha_1{}^2\sigma_4{}^2 - (\alpha_2{}^2-\alpha_1{}^2)(\vec{\sigma}_2\cdot\vec{\sigma}_3)]\,. \tag{5.16}$$

We now delete the relatively small cross term in $\vec{\sigma}_2\cdot\vec{\sigma}_3$, renormalize, and obtain the approximation

$$\psi_0{}^2 \simeq \phi_0{}^2 = \frac{\alpha_3{}^6\alpha_1{}^3}{\pi^{9/2}} \exp[-\alpha_3{}^2(\sigma_2{}^2+\sigma_3{}^2) - \alpha_1{}^2\sigma_4{}^2]\,, \tag{5.17}$$

where

$$\alpha_3{}^2 = \tfrac{1}{2}(\alpha_1{}^2+\alpha_2{}^2) = \tfrac{1}{2}\alpha^2(s_1+s_2)\,. \tag{5.18}$$

Consequently we have

$$\langle \hat{U} \rangle = \tfrac{2}{3}f_1 a(s_1) + \tfrac{4}{3}f_2 a[\tfrac{1}{2}(s_1+s_2)]\,, \tag{5.19}$$

and by substituting the numerical values [Eq. (4.2), (4.11), and (4.12)] in (5.2), we find

$$\langle \hat{U} \rangle_T = -832 \text{ MeV} \tag{5.20}$$

and

$$\langle \hat{U} \rangle_M = -919 \text{ MeV}\,. \tag{5.21}$$

VI. THE HYPERFINE INTERACTION

The two quarks and two antiquarks also interact via gluon exchange giving rise to a color-magnetic force between diquark and antidiquark. For the $L=0$ states, there is no spin-orbit or tensor term and the only contribution is the Fermi contact term which has the form[6]

$$H_{\text{hyp}} = -8\sqrt{2}A\pi \sum_{\alpha} \sum_{i>j} \vec{S}_i\cdot\vec{S}_j \Lambda_i^\alpha \Lambda_j^\alpha \delta^3(r_{ij})\,, \tag{6.1}$$

where $\vec{S}_i$ and Λ_i^α are the spin and color vectors for the ith quark and r_{ij} is the distance between the ith and jth quark. The coefficients have been chosen for ease of evaluation by comparison to the baryon states described by Isgur and Karl.[14]

The spatial integrals required for the contact terms are of the form $\langle\langle\delta(\vec{r}_{ij})\rangle\rangle$, and there are just two distinct integrals since the symmetry of the wave function implies

$$\langle\langle\delta(\vec{r}_{12})\rangle\rangle = \langle\langle\delta(\vec{r}_{34})\rangle\rangle\,, \tag{6.2}$$

and

$$\langle\langle\delta(\vec{r}_{13})\rangle\rangle = \langle\langle\delta(\vec{r}_{14})\rangle\rangle = \langle\langle\delta(\vec{r}_{23})\rangle\rangle = \langle\langle\delta(\vec{r}_{24})\rangle\rangle\,. \tag{6.3}$$

Hence Eq. (6.1) takes the form

$$\begin{aligned} H_{\text{hyp}} = &-4A\pi[2\sqrt{2}\langle\langle\delta(r_{13})\rangle\rangle]\sum_{\alpha}\sum_{i>j}\vec{S}_i\cdot\vec{S}_j\Lambda_i^\alpha\Lambda_j^\alpha \\ &-8\sqrt{2}A\pi[\langle\langle\delta(\vec{r}_{12})\rangle\rangle - \langle\langle\delta(\vec{r}_{13})\rangle\rangle] \\ &\times(\vec{S}_1\cdot\vec{S}_2\vec{\Lambda}_1\cdot\vec{\Lambda}_2 + \vec{S}_3\cdot\vec{S}_4\vec{\Lambda}_3\cdot\vec{\Lambda}_4)\,. \end{aligned} \tag{6.4}$$

Now $\delta(\vec{r}_{12}) = 2^{-3/2}\delta(\vec{\rho}_2)$, where ρ_2 is defined by (4.3), and consequently we have from (4.9)

$$\langle\langle\delta(\vec{r}_{12})\rangle\rangle = s_1{}^{3/2}\alpha^3/(2\pi)^{3/2}\,. \tag{6.5}$$

Meanwhile $\delta(\vec{r}_{13}) = 2^{-3/2}\delta(\vec{\sigma}_2)$, where σ_2 is defined by (4.3) and (5.15), and therefore from (4.9) we find

$$\langle\langle\delta(\vec{r}_{13})\rangle\rangle = \left(\frac{2s_1s_2}{s_1+s_2}\right)^{3/2}\alpha^3/(2\pi)^{3/2}\,. \tag{6.6}$$

Now Isgur and Karl[14] show that $4A\alpha^3\pi^{-1/2} = \Delta - N \simeq 300$ MeV. Hence using Eqs. (4.11) and (4.12) in Eqs. (6.5) and (6.6), we find

$$2\sqrt{2}A\pi\langle\delta_{13}\rangle_T = 63.93 \text{ MeV}\,, \tag{6.7}$$

$$2\sqrt{2}A\pi\langle\delta_{13}\rangle_M = 65.51 \text{ MeV}\,, \tag{6.8}$$

$$2\sqrt{2}A\pi[\langle\langle\delta(\vec{r}_{12})\rangle\rangle_T - \langle\langle\delta(\vec{r}_{13})\rangle\rangle_T] = 11.07 \text{ MeV}\,, \tag{6.9}$$

$$2\sqrt{2}A\pi[\langle\langle\delta(\vec{r}_{12})\rangle\rangle_M - \langle\langle\delta(\vec{r}_{13})\rangle\rangle_M] = -20.91 \text{ MeV}\,. \tag{6.10}$$

Furthermore,

$$(\vec{\Lambda}_1\cdot\vec{\Lambda}_2)_T = (\vec{\Lambda}_3\cdot\vec{\Lambda}_4)_T = -\tfrac{2}{3}\,, \tag{6.11}$$

$$(\vec{\Lambda}_1\cdot\vec{\Lambda}_2)_M = (\vec{\Lambda}_3\cdot\vec{\Lambda}_4)_M = \tfrac{1}{3}\,, \tag{6.12}$$

and

$$\vec{S}_1 \cdot \vec{S}_2 = \tfrac{1}{4} \quad \text{for } S_Q = 1$$
$$= -\tfrac{3}{4} \quad \text{for } S_Q = 0\,, \qquad (6.13)$$
$$\vec{S}_3 \cdot \vec{S}_4 = \tfrac{1}{4} \quad \text{for } S_{\bar{Q}} = 1$$
$$= \tfrac{3}{4} \quad \text{for } S_{\bar{Q}} = 0\,. \qquad (6.14)$$

The labels $Q, \bar{Q}$ refer, respectively, to the diquark made up of quarks 1 and 2 and the antidiquark made up of antiquarks 3 and 4. Finally, following Jaffe[6] we note that the products $S^k\Lambda^\alpha$ are among the generators of $SU(6)_{cs}$. Specifically the generators of $SU(6)_{cs}$ can be defined as follows:

$$\{\Omega\} = \begin{cases} (\tfrac{2}{3})^{1/2} S^k, & k = 1,2,3 \\ \Lambda^\alpha, & \alpha = 1,2,\ldots,8 \\ S^k\Lambda^\alpha\,. \end{cases} \qquad (6.15)$$

The Casimir operators of $SU(6)_{cs}$, $SU(3)_c$, and $SU(2)_s$ are then given by

$$C_6 = \sum_{\mu=1}^{35} \left(\sum_i \Omega_i^\mu\right)^2, \qquad (6.16)$$

$$C_3 = \sum_{\alpha=1}^{8} \left(\sum_i \Lambda_i^\alpha\right)^2, \qquad (6.17)$$

$$S(S+1) = \sum_{k=1}^{3} \left(\sum_i S_i^k\right)^2. \qquad (6.18)$$

Hence,

$$-4\sum_\alpha \sum_{i>j} \vec{S}_i \cdot \vec{S}_j \Lambda_i^\alpha \Lambda_j^\alpha = 2N + 2C_6(\text{tot}) - S_{\text{tot}}(S_{\text{tot}}+1)/3 + C_3(Q) + 2S_Q(S_Q+1)/3 - 4C_6(Q) + C_3(\bar{Q}) + 2S_{\bar{Q}}(S_{\bar{Q}}+1)/3 - 4C_6(\bar{Q})\,. \qquad (6.19)$$

The label tot refers to the entire baryonium system.

From (6.19), we see that to find the eigenvalues of H_{hyp}, we must construct wave functions which are in definite $SU(6)_{cs}$ representations. Our construction is essentially that of Jaffe.[6] Quarks are [6] in $SU(6)_{cs}$, thus the representations for a diquark are given by

$$[6]\otimes[6] = [15]\oplus[21] \qquad (6.20)$$
$$= [(6,1)\oplus(\bar{3},3)]\oplus[(6,3)\oplus(\bar{3},1)]\,. \qquad (6.21)$$

The second line gives the decomposition of the $SU(6)_{cs}$ representation $[n_{cs}]$ into $SU(3)_c \oplus SU(2)_s$ representations (n_c, n_s):

$$[n_{cs}] = [(n_c^1, n_s^1)\oplus(n_c^2, n_s^2)]\,, \qquad (6.22)$$

where n_{cs}, n_c, and n_s are, respectively, the dimension of the $SU(6)_{cs}$, $SU(3)_c$, and $SU(2)_s$ representations. Antidiquarks are in conjugate representations. The ground state of the diquark and antidiquark must be antisymmetric in color-spin isospin. Since the u and d quarks form an isospin $\tfrac{1}{2}$ doublet, the diquark can be in either an isospin 0 or 1 state. For both $SU(6)_{cs}$ and $SU(2)_I$, the smaller value of n_{cs} and I represents a symmetric state. The appropriate anti-symmetrized states are then $[n_{cs}, I] = [15,1], [21,0]$. To form the baryonium system we must then look for color singlets contained in the four possible combinations $[n_{cs}]\otimes[\bar{n}'_{cs}]$, $n_{cs} = 15, 21$, $\bar{n}_{cs} = \overline{15}, \overline{21}$.

The resulting states are given in Table I. Note that I and K and also J and L are identical as far as H_{hyp} and H_{tot} are concerned. States with identical spin values and originating from the same $[n_c']\otimes[n'_{cs}]$ combination such as A and B are not eigenvalues of H_{hyp} and are therefore mixed by the total Hamiltonian. The eigenstates of H_{tot} are as follows

$$|A'\rangle = 0.8404[(6,3)\otimes(\bar{6},3)] + 0.5419[(\bar{3},1)\otimes(3,1)]\,, \qquad (6.23)$$

$$|B'\rangle = 0.5330[(6,3)\otimes(\bar{6},3)] - 0.8462[(\bar{3},1)\otimes(3,1)]\,, \qquad (6.24)$$

$$|C\rangle = (6,3)\otimes(\bar{6},3)\,, \qquad (6.25)$$

$$|D\rangle = (6,3)\otimes(\bar{6},3)\,, \qquad (6.26)$$

$$|E'\rangle = 0.7282[(\bar{3},3)\otimes(3,3)] + 0.6854[(6,1)\otimes(\bar{6},1)]\,, \qquad (6.27)$$

$$|F'\rangle = 0.6944[(\bar{3},3)\otimes(3,3)] - 0.7197[(6,1)\otimes(\bar{6},1)]\,, \qquad (6.28)$$

$$|G\rangle = (\bar{3},3)\otimes(3,3)\,, \qquad (6.29)$$

$$|H\rangle = (\bar{3},3)\otimes(3,3)\,, \qquad (6.30)$$

$$|I'\rangle = 0.7119[(\bar{3},1)\otimes(3,3)] - 0.7023[(\bar{6},3)\otimes(6,1)]\,, \qquad (6.31)$$

$$|J'\rangle = 0.7105[(\bar{3},1)\otimes(3,3)] + 0.7037[(\bar{6},3)\otimes(6.1)]\,, \qquad (6.32)$$

$$|K'\rangle = 0.7119[(\bar{3},3)\otimes(3,1)] - 0.7023[(\bar{6},1)\otimes(6,3)]\,, \qquad (6.33)$$

$$|L'\rangle = 0.7105[(\bar{3},3)\otimes(3,1)] + 0.7037[(\bar{6},1)\otimes(6,3)]\,. \qquad (6.34)$$

Corresponding energy eigenvalues of the total Hamiltonian are given in Table II.

VII. DISCUSSION

The masses coresponding to the hyperfine splitting of baryonium as computed using the mo-

TABLE I. Diquark-antidiquark combinations formed into color-spin $SU(6)_{cs}$ representations. n_{cs}, n_c, and n_s are the dimensions of the $SU(6)_{cs}$, $SU(3)_c$, and $SU(2)_s$ representations.

State	Origin $[n_{cs}]\otimes[\overline{n'_{cs}}]$	S	I	$[n_{cs}]$	Origin as a combination of $(n_c,n_s)\otimes(\overline{n}_c,n_s)$ factors
A	$[21]\otimes[\overline{21}]$	0	0	[1]	$(\frac{6}{7})^{1/2}[(6,3)\otimes(\overline{6},3)]+(\frac{1}{7})^{1/2}[(\overline{3},1)\otimes(3,1)]$
B	$[21]\otimes[\overline{21}]$	0	0	[405]	$(\frac{1}{7})^{1/2}[(6,3)\otimes(\overline{6},3)]-(\frac{6}{7})^{1/2}[(\overline{3},1)\otimes(3,1)]$
C	$[21]\otimes[\overline{21}]$	1	0	[35]	$(6,3)\otimes(\overline{6},3)$
D	$[21]\otimes[\overline{21}]$	2	0	[405]	$(6,3)\otimes(\overline{6},3)$
E	$[15]\otimes[15]$	0	0,1,2	[1]	$(\frac{3}{5})^{1/2}[(\overline{3},3)\otimes(3,3)]+(\frac{2}{5})^{1/2}[(6,1)\otimes(\overline{6},1)]$
F	$[15]\otimes[15]$	0	0,1,2	[189]	$(\frac{2}{5})^{1/2}[(\overline{3},3)\otimes(3,3)]-(\frac{3}{5})^{1/2}[(6,1)\otimes(\overline{6},1)]$
G	$[15]\otimes[15]$	1	0,1,2	[35]	$(\overline{3},3)\otimes(3,3)$
H	$[15]\otimes[15]$	2	0,1,2	[189]	$(\overline{3},3)\otimes(3,3)$
I	$[21]\otimes[\overline{15}]$	1	1	[35]	$(\frac{1}{3})^{1/2}[(\overline{3},1)\otimes(3,3)]-(\frac{2}{3})^{1/2}[(\overline{6},3)\otimes(6,1)]$
J	$[21]\otimes[\overline{15}]$	1	1	[280]	$(\frac{2}{3})^{1/2}[(\overline{3},1)\otimes(3,3)]+(\frac{1}{3})^{1/2}[(\overline{6},3)\otimes(6,1)]$
K	$[15]\otimes[\overline{21}]$	1	1	[35]	$(\frac{1}{3})^{1/2}[(\overline{3},3)\otimes(3,1)]-(\frac{2}{3})^{1/2}[(\overline{6},1)\otimes(6,3)]$
L	$[15]\otimes[\overline{21}]$	1	1	$[\overline{280}]$	$(\frac{2}{3})^{1/2}[(\overline{3},3)\otimes(3,1)]+(\frac{1}{3})^{1/2}[(\overline{6},1)\otimes(6,3)]$

del proposed in this paper are listed in Table II. For comparison, this table also contains the masses computed theoretically by Jaffe,[6] Barbour *et al.*,[11] Hendry *et al.*,[8] and Lichtenberg *et al.*[10] The computations of Refs. 8 and 10 were confined only to T baryonium states, whereas those of this paper, as well as of Refs. 6 and 11 take into account both the T and M baryonium states. A meaningful comparison of the various calculations is thus possible only for the state G which appears as a pure T baryonium state in all cases. It is seen that the mass of the G state as computed in this paper is clearly consistent with that predicted by Barbour *et al.*[11] It is also consistent with the result of Lichtenberg *et al.*[10] (taking into account the error quoted by them). Our model, as well as those of Refs. 10 and 11, use baryon data for the evaluation of parameters used in the calculation, while in the calculation of Hendry *et al.*[8] all input parameters are based on experimental data, except the 1120-MeV state (corresponding roughly to the states A', B' of Table II). As far as those states which are different from G are concerned, the values given by Barbour *et al.*[11] are consistent with those predicted by the model used in this paper, whereas the values given by Jaffe[6] are invariably lower.

A comparison of the values of this paper with the experimental values will now be made. The state A' given by the present calculation is close

TABLE II. Mass of $l=0$ baryonium states. The quantum numbers l, J, P, and I refer to orbital angular momentum, total angular momentum, parity, and isospin of the whole system.

				Mass (MeV)				
State	J^P	I	Flavor [$(U(3)$ rep]	This calculation	Jaffe	Barbour and Ponting	Hendry and Hinchliffe	Lichtenberg and Johnson
A'	0^+	0	9	830	650	930	1120	1260
B'	0^+	0	9*	1530	1450	1510		
E'	0^+	0,1,2	36	1250	1150	1340	~1670	
F'	0^+	0,1,2	36*	1890	1800	1870		
C	1^+	0	9	1250	1200			
G	1^+	0,1,2	36	1640	1450	1690	~1670	1570
I',K'	1^+	1	18	1341	1250	1390	~1395	1450
J',L'	1^+	1	18*	1707	1650	1700		
D	2^+	0	9	1680	1650			
H	2^+	0,1,2	36	1810	1650	1840	~1670	1720

in energy to the $S^*(980)$ state.[17] If such identification is made, it would be consistent with the widely held view derived from the charmonium system that for the scalar mesons the four-quark ground state lies lower in mass than the two-quark ground state.[18] Further possible identification with experiment can be made as follows. Pavapoulos *et al.*[19] see three discrete lines in the photon spectrum $\bar{p}p \rightarrow X\gamma$ corresponding to masses 1395, 1646, and 1684 MeV. These states can be identified with I', G, and D states, respectively, of Table II.

The main feature of the approach adopted in this paper is the solution of the explicit four-quark problem with a pair potential derived solely from baryon data rather than from baryonium data. This enhances the reliability of the predictions of this paper as the baryon data are rich and relatively accurate, whereas the baryonium data are suspect.[8] It is hoped that the present work will lead to the establishment of a quark-quark pair potential which would enable one to predict the spectra of multiquark systems.[20]

The close agreement of the results of this paper with those derived by Barbour *et al.*,[11] with the use of a linear plus Coulomb potential builds our confidence in the use of Isgur-Karl model for the calculation of hyperfine splitting. Work is currently in progress towards the computation of excited state baryonium spectra, and will be published in due course.

Note added. After having completed the calculations reported in this paper, our attention was drawn to the calculation of Isgur *et al.*[21] which indicates that there are internal color transitions between the T and M baryoniums, where for the $T(M)$ system the diquark and antiquark are both in the $l=0$ states and for the $M(T)$ system the diquark and antidiquark are both in the $l=1$ states. The next step towards a realistic calculation appropriate to the baryonium system should take this into account. It is, nevertheless, hoped that the present work will serve as a useful basis for further research.

Note added in proof. In view of the recent results from the crystal-ball detector experiment [E. D. Bloom *et al.*, Report No. SLAC-PUB-2425, 1979 (unpublished)] it is no longer justifiable to assume that for the scalar mesons the four-quark ground state lies lower in mass than the two-quark ground state.

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