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Rare radiative decays of tensor mesons

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The relativistically invariant coupling structures realized in a quark model with broken $SU_6 \times O_3$ group symmetry are applied here to the calculation of the rare radiative decays of tensor mesons. In particular, we examine the decay modes $(f, A_2) \rightarrow \gamma\gamma$, $\rho\gamma$, and $\omega\gamma$, based on the additional requirement of vector-meson dominance of the electromagnetic interactions of hadrons. Calculated partial decay rates are compared with previous estimates.

It has been amply recognized during the past several years that the vast plethora of experimental data on strong and electromagnetic decays of hadrons and their resonances can be successfully accounted for in terms of a simple quark model with phenomenological coupling structures. The version of the quark model developed largely by Mitra and collaborators[1] has proved to be particularly useful in this respect. This model embodies the essential requirement of relativistic invariance and the expressions of the relevant hadron couplings deduced in this scheme from phenomenological considerations are so designed as to be compatible with the provisions of the broken $SU_6 \times O_3$ group symmetry. The $SU_6 \times O_3$ group is known to have the distinct advantage over the SU_6 group that the former (unlike the latter) provides an underlying theoretical basis for a description of the various hadronic states in a unified framework. In our earlier work[2] we examined the performance of this model by considering its applications to the calculation of the decay rates of a vast variety of strong and electromagnetic modes of pseudoscalar and vector mesons, including some rare three- and four-body decays.[3] The results of these calculations were found to be in excellent accord with the available experimental data.

A natural extension of this scheme is to incorporate the description of tensor-meson decays. It need be emphasized at this stage that the study of the decay patterns of tensor mesons deserves serious consideration, especially in view of the possibility of their being detected in the future. Motivated by the recent observation of the rare modes such as $A_2 \rightarrow \omega\pi\pi$ and $f \rightarrow 4\pi$ we calculated the corresponding decay rates along with several other modes, namely, $A_2 \rightarrow \rho\pi, \pi\gamma$, and $f \rightarrow K\bar{K}\pi, \eta\eta$, in the context of the present model.[4] In the present paper we continue this program to study the hitherto elusive one- and two-photon radiative decay processes of tensor mesons. In particular, we calculate the decay rates of the rare modes $(f, A_2) \rightarrow \gamma\gamma$, $\rho\gamma$, and $\omega\gamma$ using the model prescriptions in conjunction with the idea of vector-meson dominance of the electromagnetic interactions of hadrons.

The appropriate coupling structures entering the calculation of the decay rates of the $V\gamma$ and $\gamma\gamma$ ($V = \rho, \omega$) modes of f and A_2 mesons are of the tensor-vector-vector type. While a fairly detailed account of the derivation of the TVV couplings within the general formulation of the broken $SU_6 \times O_3$ quark model is given elsewhere,[5] we would only rereproduce here the relevant vertices.

For the construction of TVV couplings it is necessary that one of the vector mesons (V) be regarded as a quantum of radiation. The essential details of construction of these couplings follow exactly the same pattern as encountered in the case of couplings formulated previously[1-5] where the pseudoscalar meson (P) acts as the radiation quantum. However, the interaction of the V meson now results in additional varieties of couplings, viz., magnetic and charge type, which when formulated at the quark level are governed by the respective forms $\bar{q}\sigma_{\mu\nu}\partial_\mu V_\nu q$ and $\bar{q}i\gamma_\mu V_\mu q$, where q stands for the quark field (of mass M_q and momentum P_q) corresponding to the primitive vertex $\bar{q}Vq$. In analogy with the P- meson case, the low-momentum limits of these couplings are given by $q^\dagger V_0 q$ (charge) and $q^\dagger\vec{\sigma}\cdot\hat{\epsilon} \times \vec{k}q$ (magnetic), where V_0 is the scalar component of the radiation quantum V with m_V, k_μ $(\vec{k}, i\omega_k)$, and $\hat{\epsilon}$ as its mass, four-momentum, and unit polarization, respectively. The P-meson case for the magnetic type is recovered by the replacement $\vec{k} \rightarrow \hat{\epsilon} \times \vec{k}$, with the recoil effect entering through $\vec{k} \rightarrow \vec{k} - M_q^{-1}\omega_k\vec{P}_q$. The charge coupling for which there exists no P-meson analog is rather straightforward and free from any recoil effects. This must, of course, be multiplied by m_V so as to reconcile its dimensions with the corresponding magnetic coupling.

Consequent to the interchange in the role of the radiation quantum from the P to the V meson, there exist three different types of couplings, namely, M_sVM_t (magnetic only), M_sVM_s (charge only), and M_tVM_t (charge and magnetic) with the subscripts t, s signifying the quark spins for the meson states (t = quark spin triplet, $S=1$; s = quark spin singlet, $S=0$). For the purposes of the present calculation we need only the $M_t V M_t$ type of couplings, the magnetic and charge structures for which at the SU_6 level (i.e., prior to switching on the L excitation) have the form

$$\begin{aligned} &\text{magnetic } (D){:}\quad \vec{k}\cdot\hat{u}\hat{\omega}\cdot\hat{\epsilon} - \vec{k}\cdot\hat{\omega}\hat{u}\cdot\hat{\epsilon}\ ,\\ &\text{magnetic } (R){:}\quad \hat{u}_i\hat{\omega}\cdot\hat{\epsilon} - \hat{\omega}_i\hat{u}\cdot\hat{\epsilon}\ , \\ &\text{charge:}\quad m_V V_0 \hat{u}\cdot\hat{\omega}\ , \end{aligned} \tag{1}$$

where $\hat{\omega}$ and $\hat{u}$ represent the initial and final polarization vectors. The notations D and R correspond to the direct and recoil contributions, respectively.

Meson couplings for any arbitrary J ($J = L+S$) clearly involve the L-excitation which must now be incorporated for a general treatment, based on the assumption of the dominance of the $(L\pm 1)$ wave transitions by the D and R contributions, respectively. The vertices are now of the type $A^J VV$, where A^J is a normalized tensor of rank J. The tensor mesons, e.g., f and A_2, whose decay modes are studied in this investigation, are generated by the tensor A^{L+1}. The appropriate $A^{L+1}VV$ structures are then worked out explicitly in the context of the present model giving rise to the magnetic and charge couplings in the form

$$\begin{aligned} &\text{magnetic } (D){:}\quad A^{L+1}_{\mu\mu_1\cdots\mu_L} k_{\mu_1}\cdots k_{\mu_L}(\epsilon_\mu k_\nu - k_\mu \epsilon_\nu)u_\nu\ ,\\ &\text{magnetic } (R){:}\quad A^{L+1}_{\mu\nu\mu_2\cdots\mu_L} k_{\mu_2}\cdots k_{\mu_L} u_\mu \epsilon_\nu\ , \\ &\text{charge:}\quad m_V A^{L+1}_{\mu_0\mu_1\cdots\mu_L} k_{\mu_1}\cdots k_{\mu_L} u_{\mu_0}\epsilon_\mu\ . \end{aligned} \tag{2}$$

These expressions are supplemented with appropriate form factors and normalizations. For the magnetic (D) coupling the normalization is $(2M_L)$, whereas for the charge type the corresponding factor is $(p_\mu + P_\mu)$ with M_L and P_μ denoting the mass and four-momentum of the decaying meson (the tensor mesons in our case) and p_μ the four-momentum of the vector meson (other than the radiation quantum). The magnetic (R) coupling, however, has to be examined in relation to the Gell-Mann–Oakes–Renner (GOR) effect which manifests itself in the overall normalization $(M_L{}^2 - m_V{}^2)$ with this term. Combining the magnetic D and R terms, the final forms of the magnetic and charge type structures for the $A^{L+1}VV$ vertices are expressible as

magnetic:

$$\begin{aligned} &f_L^{(+)}(2M_L)A^{L+1}_{\mu\mu_1\cdots\mu_L}k_{\mu_1}\cdots k_{\mu_L}(\epsilon_\mu k_\nu - k_\mu\epsilon_\nu)u_\nu \\ &\quad + f_L^{(-)}(M_L{}^2 - m_V{}^2)A^{L+1}_{\mu\nu\mu_2\cdots\mu_L}k_{\mu_2}\cdots k_{\mu_L}u_\mu\epsilon_\nu\ , \end{aligned} \tag{3}$$

charge: $f_L^{(+)} m_V (p_\mu + P_\mu) A^{L+1}_{\mu_0\mu_1\cdots\mu_L} k_{\mu_1}\cdots k_{\mu_L} u_{\mu_0}\epsilon_\mu$,

where $f_L^{(+,-)}$ is indeed the meson form factor for the $(L\pm 1)$ wave supermultiplet transitions.

Among the most successful constructions of the meson form factor, the one which gives the best performance and which is specially tailored to suit the requirements of the off-shell extension so as to be applicable to electroproduction and photoproduction processes has the structure[6]

$$f_L^{(+,-)} = g_L\left(\frac{2M_L s_F}{M_L{}^2 \pm m^2 - m_V{}^2}\right)^{L+1,L}\ , \tag{4}$$

where s_F (= 1.16) is a scale factor and g_L is the reduced coupling constant (dimensionless) governing the entire supermultiplet transition. Here m is the mass of the vector meson other than the radiation quantum. Since there exist no guiding Ansätze with respect to the radiation convention, the lighter vector meson is chosen as the radiation quantum on the basis of purely phenomenological arguments, and to this extent it can at best be regarded as arbitrary. In the calculation of the $V\gamma$ and $\gamma\gamma$ modes of the tensor mesons f and A_2, the two vector mesons involved in the intermediate state are ρ and ω, as evidenced by the fact that the f meson couples to the $\rho\rho$, $\omega\omega$ states, whereas the A_2 meson couples only to the $\rho\omega$ state, viz., $f\to\rho\rho$, $\omega\omega$, and $A_2\to\rho\omega$. Since the ρ and ω mesons differ only slightly in their masses we take them essentially at the same mass in the calculation of the decay rates, in which case the masses m and m_V in expression (4) become identically equal. We note that the above form factor is reminiscent of the GOR effect and possesses the desired symmetry breaking.

In the present scheme, the tensor mesons f and A_2 belong to the $L=1$ supermultiplet while the vector mesons ρ and ω are assigned to the $L=0$ supermultiplet. Therefore, the reduced coupling constant for the $(L=1)\to(L=0)$ transitions is g_1, which not only governs the tensor and vector mesons but also accounts for all members in these respective supermultiplets [including, for instance, the A mesons ($L=1$) and the P mesons ($L=0$)]. The coupling constant g_1 as estimated from the decay $A_1\to\rho\pi$ in our earlier work[1,3-5] has the value given by $g_1{}^2/4\pi = 0.08$. As for the SU_6 coefficients for the $A_2\rho\omega$, $f\rho\rho$, and $f\omega\omega$ vertices we have a factor of 2 for each.

The effective couplings appropriate for the evaluation of the radiative decays of f and A_2 involving the $TV\gamma$ and $T\gamma\gamma$ vertices are obtained from the TVV vertex through the use of the vector-meson-dominance hypothesis by successive replacements of the vector-meson fields V_μ (= ρ_μ or ω_μ) by their electromagnetic equivalents in accordance with the transformations

$$\rho_\mu \to \frac{e}{g_{\rho\pi\pi}} A_\mu \text{ and } \omega_\mu \to \tfrac{1}{3}\frac{e}{g_{\rho\pi\pi}} A_\mu , \qquad (5)$$

where A_μ is the photon field and $g_{\rho\pi\pi}$ the $\rho\pi\pi$ coupling constant evaluated in terms of the reduced self-coupling $[(L=0)\to(L=0)]$ g_0 ($g_0{}^2/4\pi = 0.03$). The decay rates for one- and two-photon decay modes of f and A_2 are then readily computed with the aid of the above prescription. The predictions of the model are displayed in Table I.

We stress at this point that the results obtained here can hardly be established on firm footing unless confronted with the experimental data. Unfortunately, no experimental detection of these rare modes has been reported so far. We, therefore, take recourse to other theoretical models for the sake of comparison. There have been several attempts in the past to estimate these decays within the framework of a variety of models. For instance, an earlier model due to Renner[7] based on the tensor-meson dominance of the energy-momentum tensor predicted the partial decay rates of the f meson as $f\to(\gamma\gamma, \rho\gamma, \omega\gamma) = (8.0\text{ keV}, 2.0\text{ MeV}, 0.2\text{ MeV})$. A similar calculation by Kleinert *et al.*[8] gave the values $f\to(\gamma\gamma,\rho\gamma) = (7.0\text{ keV}, 1.3\text{ MeV})$. Using finite-energy sum rules, Bramon and Greco[9] examined the $\gamma\gamma$ mode of f and A_2 where the corresponding rates turn out to be $(f, A_2)\to\gamma\gamma = (0.8, 0.3)$ keV, while a dispersion-theoretic technique employed by Schierholtz and Sundermeyer[10] yields for the $f\to\gamma\gamma$ decay rate a value as high as 11.3 keV. More recent predictions of these radiative transitions have been reported in a comprehensive dual amplitude analysis carried out by Levy, Singer, and Toaff.[11] We list in Table I these results along with those of a quark-model calculation of Berger and Feld[12] in order to facilitate comparison of our results.

An examination of the results in Table I indicates that the predictions of the present model are in general agreement with previous calculations, especially those of Levy, Singer, and Toaff. It may be pointed out that our $V\gamma$ results are close to those of Berger and Feld, while the $\gamma\gamma$ modes, on the other hand, are a bit too large, which may presumably be attributed to the type of symmetry-breaking mechanism in the model we have used. We, however, believe that marginal differences do not merit serious consideration at this stage, especially in view of the paucity of data on these decay modes. As mentioned earlier, the only rare modes of tensor mesons detected thus far are $A_2\to\omega\pi\pi$ and $f\to 4\pi$ for which a theoretical description in terms of the present model found good agreement with the observed rates. We, therefore, feel encouraged that this model is likely to survive the tests of future experiments when more reliable data on these one- and two-photon decay modes of f and A_2 become available. As such, the present situation prevents firm conclusions from being drawn.

TABLE I. The calculated decay rates for one- and two-photon decay modes of tensor mesons $f(1270)$ and $A_2(1310)$. The corresponding rates calculated by Levy, Singer, and Toaff (LST, Ref. 11) as well as Berger and Feld (BF, Ref. 12) are also listed for the sake of comparison. The $V\gamma$ and $\gamma\gamma$ modes are expressed in MeV and keV, respectively.

Decay mode	Present calculation	Ref. 11 (LST)	Ref. 12 (BF)
$f\to\gamma\gamma$	4.31	5.07	1.2 or 2.3
$f\to\rho\gamma$	0.98	0.58	0.84 or 2.2
$f\to\omega\gamma$	0.11	0.06	0.096 or 0.24
$A_2\to\gamma\gamma$	1.16	2.05	0.46 or 0.81
$A_2\to\rho\gamma$	0.085	0.08	0.11 or 0.29
$A_2\to\omega\gamma$	0.73	0.68	0.84 or 2.5

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