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Parity-violating decays of charmed baryons in a quark model

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We study parity-violating nonleptonic decays of charmed baryons as arising through single-quark and two-quark transitions. We notice that single-quark transition acquires null contribution from the Glashow-Iliopoulos-Maiani (GIM) weak Hamiltonian ($\underline{20}''+\underline{84}$). $\underline{20}''$ dominance in the GIM model forbids $B(3)\to B(8)+P(3^*)$ and $B(3)\to B(6)+P(8)$ decays in two-quark transitions. We also include $\underline{45}$, $\underline{45}^*$ components in the weak Hamiltonian, which may occur through unconventional currents. Only π^+-emitting decays then occur through single-quark transitions. Thus the weak decays of charmed baryons arise predominantly through two-quark transitions.

I. INTRODUCTION

The weak hadronic decays of charmed baryons have been discussed in higher-symmetry schemes.[1] The general current $\otimes$ current weak Hamiltonian belongs to the representations present in the direct product $15\otimes 15$. The conventional Glashow-Iliopoulos-Maiani (GIM) model,[2] where the Hamiltonian belongs to $\underline{20}''+\underline{84}$ representations, has several unsatisfactory features[3] for the weak hadronic decays of both the charmed and the ordinary (uncharmed) hadrons. Even at the SU(3) level, $\underline{6}^*$ dominance is not able to explain the observed charmed-D-meson decays.[4,6] For the ordinary baryons, one may consider an admixture of the adjoint representation but for the charmed sector, particularly for the Cabibbo-enhanced mode, other higher representations like $\underline{45}$, $\underline{45}^*$, and $\underline{84}$, seem to contribute significantly.[5] If all the representations are included, it becomes hard to draw any conclusion about these decays based on SU(4) symmetry alone. Various additional symmetries, such as u-s quark symmetry,[6] generalized charged symmetry,[7] spin–unitary-spin symmetry,[8] etc., have been considered.

In the present paper, we employ a quark model to study the parity-violating (PV) weak decays of charmed baryons. In quark models, the weak decays of ordinary baryons have earlier been studied by many authors.[9,10] Nakagawa and Trofimenkoff[10] were able to obtain for the ordinary hadrons current-algebra results in a quark model, considering single-quark and two-quark transitions. In a similar model we discuss the PV weak decays of the charmed hyperons in the Cabibbo-favored ($\Delta C=\Delta S$) mode. Using the usual quark-model assumption,[10] we show that single-quark transitions for all the weak decay modes of baryons do not get any contribution from the GIM weak Hamiltonian. We consider then the two-quark transitions, where $B(3)\to B(8)+P(3^*)$ and $B(3)\to B(6)+P(8)$ decay modes are forbidden in the $\underline{20}''$-dominant GIM weak Hamiltonian. Therefore, $B(3)$ charmed ($C=2$) multiplet is allowed to decay to $B(3^*)$ baryons only. The $\underline{20}''$-forbidden channels are allowed to occur through the $\underline{84}$ component of the GIM Hamiltonian. In our study, we include other representations $\underline{45}$, $\underline{45}^*$ for the sake of completeness. These representations may appear in the weak interaction through SU(4) breaking and/or unconventional currents, like second-class currents,[11] right-handed current,[12] etc. In the presence of $\underline{45}+\underline{45}^*$ weak Hamiltonian,[12] single-quark transition Hamiltonian allows only those decays of charmed baryons in which π^+ is emitted. So $B(3)\to B(8)+P(3^*)$ decay mode is totally forbidden in single-quark transition. Therefore weak hadronic decays of charmed baryons seem to occur predominantly through two-quark transitions. We include two-quark transitions from the $(\underline{45}+\underline{45}^*)$ piece and compare the decay-amplitude sum rules with those in the GIM model.

II. PRELIMINARIES

We make the following assumptions:

(i) The weak Hamiltonian H_W (a) is CP invariant, (b) is of the current $\otimes$ current form, i.e., $H_W=\frac{1}{2}(JJ^\dagger+J^\dagger J)$, and (c) belongs in general to all the representations present in the direct products

$$15\otimes 15=1+15_S+15_A+20_S+45_A+45_A^*+84_S.$$

In the GIM model, H_W belongs only to $\underline{20}''$ and $\underline{84}$.[13] Other representations can appear through SU(4) breaking, or additional currents.[11,12] We discuss the $\Delta C=\Delta S$ mode only, where $\underline{15}$ and singlet representation do not contribute.

(ii) The baryons are nonrelativistic bound states of three quarks in the s state and are described by the spin–unitary-spin wave functions[14] belonging to the $\underline{120}$ representation of SU(8).

(iii) The $\underline{\text{PV}}$ weak decays of the type $B(\frac{1}{2}^+)\to B'(\frac{1}{2}^+)+P(0^-)$ occur through the emission of a pseudoscalar meson accompanied by single-quark transition[10] and two-quark transitions, in such a manner that these transitions have no dynamical influence on the spectator quarks. The transition amplitude for the decay $B \to B' + P$ is then written as

$$\langle BP | H_W | B \rangle = \left\langle B'P \left| \sum_{i=1}^{3} H_1^{(i)} + \sum_{i \neq j=1}^{3} H_2^{(i,j)} \right| B \right\rangle ,$$

where $H_1^{(i)}$ and $H_2^{(i,j)}$ are weak Hamiltonians for meson emission accompanied by a ith single-quark and (i,j)th two-quark transitions, respectively.

(iv) The recoil energy momentum of the quark taking part in the meson emission is negligible. Effects of recoil are expected to be of the order of m_P/m_B and are expected to be small when an uncharmed meson is emitted. However, for charmed-meson emission, neglect of recoil may introduce an appreciable error.

(v) The current and constituent quarks are identical. This assumption is justified as there is no orbital angular momentum involved in the processes considered. Moreover, it has been shown by Sebastian[15] that in the nonrelativistic quark model this assumption is valid.

III. DECAY-AMPLITUDE SUM RULES

A. GIM model

1. Single-quark transition

The weak Hamiltonian H_1 for the single-quark transition can be written as

$$H_1 = a^n (\bar{q}^a F q_b) \phi_B + \text{H.c.}$$

ϕ_B represents pseudoscalar-meson wave function, n denotes the dimensionality of the representation to which H_W belongs, and F is a function of momenta of quarks a and b. One can also include the momentum of meson emitted. But, that can be expressed in terms of momenta of quarks due to energy-momentum conservation. Taking quarks to be free inside the baryons, the momenta can then be converted into masses by using free wave equation for quarks, thus making F a scalar constant.

The GIM weak Hamiltonian has the following structure:

$$H_W^{20''} = a^{20''} (\bar{q}^a F q_b) P_d^c H_{[a,c]}^{[b,d]} ,$$

$$H_W^{84} = a^{84} (\bar{q}^a F q_b) P_d^c H_{(a,c)}^{(b,d)} ,$$

where the tensors $H_{[a,c]}^{[b,d]}$ and $H_{(a,c)}^{(b,d)}$ represent the transformation properties of weak Hamiltonian $\underline{20}''$ and $\underline{84}$ representations, respectively. CP invariance leads to

$$a^{20''} = a^{84} = 0 .$$

The GIM weak Hamiltonian, therefore, gives no contribution to the weak nonleptonic decays through single-quark transition.

2. Two-quark transitions

We treat the two-quark transition in general symmetry arguments, without going into the detailed nature of the quark-pair interaction. As the quarks have been assumed to be in the s state, the weak Hamiltonian causing the two-quark transitions is written as

$$H_2 = [b^n (\bar{q}^a q_b)(\bar{q}^c q_d) + c^n (\bar{q}^a \vec{\sigma} q_b) \cdot (\bar{q}^c \vec{\sigma} q_d)] \phi_P + \text{H.c.}$$

The GIM weak Hamiltonian $H_W^{(20''+84)}$ has the following components:

$$\begin{aligned} H_W^{20''} = \{ & [b_1^{20''} (\bar{q}^f q_c)(\bar{q}^a q_f) P_d^b H_{[a,b]}^{[c,d]} + b_2^{20''} (\bar{q}^f q_c)(\bar{q}^a q_d) P_f^b H_{[a,b]}^{[c,d]} + b_3^{20''} (\bar{q}^a q_f)(\bar{q}^b q_c) P_d^f H_{[a,b]}^{[c,d]}] \\ & + [c_1^{20''} (\bar{q}^f \vec{\sigma} q_c) \cdot (\bar{q}^a \vec{\sigma} q_f) P_d^b H_{[a,b]}^{[c,d]} + c_2^{20''} (\bar{q}^f \vec{\sigma} q_c) \cdot (\bar{q}^a \vec{\sigma} q_d) P_f^b H_{[a,b]}^{[c,d]} + c_3^{20''} (\bar{q}^a \vec{\sigma} q_f) \cdot (\bar{q}^b \vec{\sigma} q_c) P_d^f H_{[a,b]}^{[c,d]}] \} , \end{aligned}$$

$$\begin{aligned} H_W^{84} = \{ & [b_1^{84} (\bar{q}^f q_c)(\bar{q}^a q_f) P_d^b H_{(a,b)}^{(c,d)} + b_2^{84} (\bar{q}^f q_c)(\bar{q}^a q_d) P_f^b H_{(a,b)}^{(c,d)} + b_3^{84} (\bar{q}^a q_f)(\bar{q}^b q_c) P_d^f H_{(a,b)}^{(c,d)}] \\ & + [c_1^{84} (\bar{q}^f \vec{\sigma} q_c) \cdot (\bar{q}^a \vec{\sigma} q_f) P_d^b H_{(a,b)}^{(c,d)} + c_2^{84} (\bar{q}^f \vec{\sigma} q_c) \cdot (\bar{q}^a \vec{\sigma} q_d) P_f^b H_{(a,b)}^{(c,d)} + c_3^{84} (\bar{q}^a \vec{\sigma} q_f) \cdot (\bar{q}^b \vec{\sigma} q_c) P_d^f H_{(a,b)}^{(c,d)}] \} . \end{aligned}$$

CP invariance leads to the following relations:

$$b_1^{20''} = b_1^{84} = c_1^{20''} = c_1^{84} = 0 ,$$

$$b_2^{20''} = -b_3^{20''} , \quad c_2^{20''} = -c_3^{20''} ,$$

$$b_2^{84} = -b_3^{84} , \quad c_2^{84} = -c_3^{84} .$$

In the light of the present mass spectroscopy[16] of hadrons, all the states except Ω_1^0 of $B(6)$ multiplet $(\Sigma_1^{++,+,0}, \Xi_1^{+,0}, \Omega_1^0)$ can decay to $B(3^*)$ multiplet $(\Lambda_1'^{+}, \Xi_1'^{+,0})$ through strong, electromagnetic interactions.[17] In the following, we discuss the weak decays of $B(3^*)$, $B(3)$ multiplet $(\Xi_2^{++,+}, \Omega_2^+)$ and of Ω_1^0 baryons in $\Delta C = \Delta S = -1$ mode. We follow the notations of Hendry and Lichtenberg[16] for the charmed baryon states.

(a) $\underline{20}''$ dominance. The above conditions with the $20''$ dominance of the GIM weak Hamiltonian give the relations

(i) $B(3^*) \to B(8) + P(8)$:

$$0 = \langle \Lambda \pi^+ | \Lambda_1'^{+} \rangle , \tag{3.1}$$

$$(\tfrac{2}{3})^{1/2} \langle P \bar{K}^0 | \Lambda_1'^{+} \rangle = \frac{1}{\sqrt{2}} \langle \Sigma^+ \eta' | \Lambda_1'^{+} \rangle , \tag{3.2}$$

$$-\frac{2}{\sqrt{3}}\langle \Xi^-\pi^+|\Xi_1'^0\rangle=\langle \Xi^0\eta'|\Xi_1'^0\rangle, \tag{3.3}$$

$$-\langle\Sigma^+\eta'|\Lambda_1'^+\rangle=(\tfrac{2}{3})^{1/2}\langle\Sigma^+\pi^0|\Lambda_1'^+\rangle=\frac{1}{\sqrt{3}}\langle\Sigma^0\pi^+|\Lambda_1'^+\rangle$$
$$=(\tfrac{2}{3})^{1/2}\langle\Sigma^+\bar{K}^0|\Xi_1'^+\rangle, \tag{3.4}$$

$$(\tfrac{2}{3})^{1/2}\langle\Xi^0\pi^+|\Xi_1'^+\rangle=\tfrac{2}{3}\langle\Lambda\bar{K}^0|\Xi_1'^0\rangle=\frac{2}{\sqrt{3}}\langle\Sigma^0\bar{K}^0|\Xi_1'^0\rangle$$
$$=-\langle\Xi^0\eta|\Xi_1'^0\rangle=\frac{1}{\sqrt{3}}\langle\Xi^0\pi^0|\Xi_1'^0\rangle, \tag{3.5}$$

$$\langle\Sigma^+\pi^0|\Lambda_1'^+\rangle=-\langle\Xi^0\pi^+|\Xi_1'^+\rangle, \tag{3.6}$$

$$\langle P\bar{K}^0|\Lambda_1'^+\rangle=-\langle\Xi^-\pi^+|\Xi_1'^0\rangle, \tag{3.7}$$

$$0=\langle\Sigma^+K^-|\Xi_1'^0\rangle=\langle\Xi^0K^+|\Lambda_1'^+\rangle. \tag{3.8}$$

(ii) $B(3)\to B(3^*)+P(8)$:

$$-\langle\Xi_1'^+\bar{K}^0|\Omega_2^+\rangle=\langle\Xi_1'^0\pi^+|\Xi_2^+\rangle=\frac{\sqrt{3}}{2}\langle\Xi_1'^+\eta'|\Xi_2^+\rangle, \tag{3.9}$$

$$-\frac{1}{\sqrt{2}}\langle\Xi_1'^+\pi^0|\Xi_2^+\rangle=(\tfrac{3}{2})^{1/2}\langle\Xi_1'^+|\Xi_2^+\rangle=\langle\Lambda_1'^+\bar{K}^0|\Xi_2^+\rangle$$
$$=\langle\Xi_1'^+\pi^+|\Xi_2^{++}\rangle. \tag{3.10}$$

(iii) $B(3)\to B(8)+P(3^*)$:

$$0=\langle\Sigma^+D^0|\Xi_2^+\rangle=\langle\Xi^0F^+|\Xi_2^+\rangle=\langle\Lambda D^+|\Xi_2^+\rangle$$
$$=\langle\Sigma^0D^+|\Xi_2^+\rangle=\langle\Sigma^+D^+|\Xi_2^{++}\rangle=\langle\Xi^0D^+|\Omega_2^+\rangle. \tag{3.11}$$

(iv) $B(3)\to B(6)+P(8)$:

$$0=\langle\Omega_1^0\pi^+|\Omega_2^+\rangle=\langle\Sigma_1^{++}\bar{K}^0|\Xi_2^{++}\rangle=\langle\Sigma_1^{++}K^-|\Xi_2^+\rangle$$
$$=\langle\Sigma_1^+\bar{K}^0|\Xi_2^+\rangle=\langle\Xi_1^+\pi^0|\Xi_2^+\rangle=\langle\Xi_1^+\bar{K}^0|\Omega_2^+\rangle$$
$$=\langle\Xi_1^0\pi^+|\Xi_2^+\rangle=\langle\Omega_1^0K^+|\Omega_2^+\rangle=\langle\Xi_1^+\eta|\Xi_2^+\rangle$$
$$=\langle\Xi_1^+\eta'|\Xi_2^+\rangle=\langle\Xi_1^+\pi^+|\Xi_2^{++}\rangle. \tag{3.12}$$

The decay channels $B(3)\to B(8)+P(3^*)$ and $B(3)\to B(6)+P(8)$ are forbidden. This result has earlier been obtained in the SU(8) symmetry scheme.[8] Decay channels $B(3^*)\to B(8)+P(8)$ and $B(3)\to B(3^*)+P(8)$ are related through

$$2\langle P\bar{K}^0|\Lambda_1'^+\rangle=\langle\Xi_1'^+\bar{K}^0|\Omega_2^+\rangle, \tag{3.13}$$
$$2\langle\Sigma^+\pi^0|\Lambda_1'^+\rangle=\langle\Xi_1'^+\pi^+|\Xi_2^{++}\rangle. \tag{3.14}$$

Ω_1^0 decay is related to $B(3^*)\to B(8)+P(8)$ channel as follows:

$$\langle\Xi^0\bar{K}^0|\Omega_1^0\rangle=\sqrt{6}\,\langle\Sigma^+\pi^0|\Lambda_1'^+\rangle. \tag{3.15}$$

(b) Total GIM weak Hamiltonian $H_W^{(20''+84)}$. If we further include $\underline{84}$ part of the weak Hamiltonian we obtain the sum rules

(i) $B(3^*)\to B(8)+P(8)$: Relations (3.1) to (3.5) remain valid and in addition we get

$$\langle\Sigma^+K^-|\Xi_1'^0\rangle=\langle\Xi^0K^+|\Lambda_1'^+\rangle. \tag{3.16}$$

(ii) $B(3)\to B(3^*)+P(8)$: $\underline{84}$ part of the GIM weak Hamiltonian vanishes in this channel, therefore relations (3.9) and (3.10) remain valid. Decays forbidden in $H_W^{20''}$ and arising through $\underline{84}$ component obey the relations listed in (iii).

(iii) $B(3)\to B(8)+P(3^*)$:

$$0=\langle\Sigma^+D^0|\Xi_2^+\rangle=\langle\Xi^0F^+|\Xi_2^+\rangle, \tag{3.17}$$

$$\langle\Sigma^+D^+|\Xi_2^{++}\rangle=-(\tfrac{2}{3})^{1/2}\langle\Lambda D^+|\Xi_2^+\rangle=-\langle\Xi^0D^+|\Omega_2^+\rangle$$
$$=\sqrt{2}\,\langle\Sigma^0D^+|\Xi_2^+\rangle. \tag{3.18}$$

(iv) $B(3^*)\to B(6)+P(8)$:

$$0=\langle\Omega_1^0\pi^+|\Omega_2^+\rangle=\langle\Sigma_1^{++}\bar{K}^0|\Xi_2^{++}\rangle, \tag{3.19}$$

$$\sqrt{2}\,\langle\Sigma_1^+\bar{K}^0|\Xi_2^+\rangle=\langle\Sigma_1^{++}K^-|\Xi_2^+\rangle, \tag{3.20}$$

$$\langle\Xi_1^+\eta|\Xi_2^+\rangle=-\sqrt{2}\,\langle\Xi_1^+\eta'|\Xi_2^+\rangle, \tag{3.21}$$

$$-\sqrt{2}\,\langle\Xi_1^{++}\bar{K}^0|\Xi_2^{++}\rangle=\langle\Sigma_1^{++}K^-|\Xi_2^+\rangle=-\sqrt{3}\,\langle\Xi_1^+\eta|\Xi_2^+\rangle$$
$$=\langle\Xi_1^+\pi^0|\Xi_2^+\rangle=\sqrt{2}\,\langle\Xi_1^0\pi^+|\Xi_2^+\rangle$$
$$=\langle\Omega_1^0K^+|\Xi_2^+\rangle=-\sqrt{2}\,\langle\Xi_1^+\bar{K}^0|\Omega_2^+\rangle. \tag{3.22}$$

Different channels in the total GIM weak Hamiltonian $H_W^{(20''+84)}$ are related through the relations

$$\frac{1}{\sqrt{6}}(\langle P\bar{K}^0|\Lambda_1'^+\rangle+\langle\Xi^-\pi^+|\Xi_1'^0\rangle)$$
$$=\langle\Xi_1^+\pi^+|\Xi_2^{++}\rangle=\frac{1}{\sqrt{3}}\langle\Lambda D^+|\Xi_2^+\rangle=-\frac{1}{\sqrt{3}}\langle\Xi^0K^+|\Lambda_1'^+\rangle, \tag{3.23}$$

$$\langle\Lambda\bar{K}^0|\Lambda_1'^+\rangle+\frac{\sqrt{3}}{2}\langle\Xi^0\eta'|\Xi_1'^0\rangle=-\langle\Xi_1'^0\pi^+|\Xi_2^+\rangle, \tag{3.24}$$

$$\sqrt{2}\,\langle\Sigma^+\pi^0|\Lambda_1'^+\rangle+\frac{2}{\sqrt{3}}\langle\Lambda\bar{K}^0|\Xi_1'^0\rangle=-\langle\Xi_1'^+\pi^0|\Xi_2^+\rangle. \tag{3.25}$$

Ω_1^0 decay is related to the channel $B(3^*)\to B(8)+P(8)$ as in relation (3.15).

3. Most general weak Hamiltonian

We have stated before that in addition to the GIM contribution ($\underline{20}''+\underline{84}$), other representations like $\underline{45}$, $\underline{45}^*$ can appear in weak interaction through $\underline{\mathrm{SU}(4)}$ breaking or due to the presence of unconventional currents. For the single-quark transition, the weak Hamiltonian transforming as $\underline{45}+\underline{45}^*$ has the form

$$H_W^{45}=a^{45}(\bar{q}^aFq_b)P_d^cH_{(a,c)}^{[b,d]},$$
$$H_W^{45^*}=a^{45^*}(\bar{q}^aFq_b)P_d^cH_{[a,c]}^{(b,d)},$$

where the tensors $H_{(a,c)}^{[b,d]}$ and $H_{[a,c]}^{(b,d)}$ represent the transformation properties of the weak Hamiltonian in the $\underline{45}$ and $\underline{45}^*$ representation of SU(4), respectively. CP invariance implies

$$a^{45}=a^{45^*}.$$

We notice that through single-quark transition, all the decays involving pseudoscalar mesons other than π^+ do not occur. The decay channel $B(3) \to B(8)+P(3^*)$ is completely forbidden. Thus the weak decays of charmed baryons occur predominantly through two-quark transitions. With the most general weak Hamiltonian, for the two-quark transitions, we notice that the decay amplitude sum rules (3.16), (3.17), (3.18), (3.20), and (3.21) remain valid.

V. SUMMARY AND CONCLUSION

The weak nonleptonic decays are not well understood even at the phenomenological level. The simple generalization of the $\Delta I = \frac{1}{2}$ enhancement, i.e., $\underline{20}''$ dominance in the GIM model leads to several unsatisfactory features[3] for both charmed and uncharmed hadrons. Even at the SU(3) level, $\underline{6}^*$ dominance of the charm-changing weak Hamiltonian is not a good assumption.[4] This might imply the possibility of the presence of other representations, such as $\underline{45}$, $\underline{45}^*$ and $\underline{84}$, in the weak Hamiltonian. These higher representations could be generated through large SU(4) breaking or due to addition of unconventional currents, such as the second-class current[11] and the right-handed current,[12] for which experimental evidence is not yet available. But the introduction of those unconventional currents appears to help in the understanding of the weak decays.[12] In a recent paper, two of us,[18] using simple dynamic assumptions such as the nonexoticity of intermediate states, etc., have shown that the GIM contribution $(\underline{20}''+\underline{84})$ to charmed hadronic decays is small and the dominant contribution would come from $\underline{45}$, $\underline{45}^*$ representations; we have here considered all the representations in order to make our study most general. In order to obtain constraints on the weak Hamiltonian, we have used a quark model to study the PV weak decays of charmed baryons. Firstly we have discussed the single-quark transition and two-quark transitions to nonleptonic decays in GIM model. The single-quark transition, however, gives null contribution for the GIM weak Hamiltonian. And through two-quark transitions $B(3) \to B(8)+P(3^*)$ and $B(3) \to B(6)+P(8)$, decay modes are forbidden in $\underline{20}''$-dominant GIM weak Hamiltonian. Therefore, in the GIM model, $B(3)$ multiplet is allowed to decay only to $B(3^*)$ baryons. Then we included $\underline{45}$, $\underline{45}^*$ components of weak Hamiltonian and found that single-quark transition allows only π^+-emitting decays of charmed baryons and through the two-quark transition $\underline{20}''$-forbidden decay channels are allowed to occur.

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