Analysis of $\Delta S=1$ nonleptonic weak decays and the $\Delta I=1/2$ rule

John F. Donoghue
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Eugene Golowich and William A. Ponce
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003

Barry R. Holstein
Physics Division, National Science Foundation, Washington D. C. 20550

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Current-algebra and quark-model methods are used to investigate hyperon and kaon transition amplitudes for various contributions to the weak nonleptonic Hamiltonian of the Weinberg-Salam model. Substantial enhancements are found for matrix elements of certain operators which are induced by quantum-chromodynamic radiative corrections and contain right-handed currents. These operators improve the agreement with experiment in most cases, although problems remain in P-wave hyperon amplitudes. In this approach, the $\Delta I = 1/2$ rule is understandable in both kaons and baryons under plausible assumptions.

I. INTRODUCTION

Several years ago, the advent of non-Abelian gauge theories prompted considerable theoretical activity on the subject of the $\Delta S=1$ nonleptonic interactions. These calculations, as well as others in weak-interaction physics, were typically based upon four concepts which can be considered as likely ingredients in any ultimately successful theory. They are the following:

(i) Current algebra and PCAC (partial conservation of axial-vector current), which relate a process involving a pion to one with the pion removed. The chiral structure of the nonleptonic weak Hamiltonian is the dominant consideration here. Successful applications of this technique place meaningful constraints on models of the weak Hamiltonian.[1]

(ii) Quantum-chromodynamic (QCD) radiative corrections [see Figs. 1(a) and 1(b)] to the interactions of the known left-handed weak currents. Renormalization-group summation of these corrections reveal enhancement of the $\Delta I=\frac{1}{2}$, SU(3) octet operators and suppression of the $\Delta I=\frac{3}{2}$ SU(3) 27-plet operator.[2]

(iii) The existence of hadrons as singlets under the dynamical gauge group of color SU(3). A direct consequence of this is the Pati-Woo theorem which states that baryon-to-baryon matrix elements of the $\Delta I=\frac{3}{2}$ part of a weak Hamiltonian constructed from left-handed currents must vanish.[3]

(iv) Utilization of quark-model wave functions to describe hadronic structure. Presumably, the ability to produce realistic wave functions will improve with time. An especially useful method which yields reasonable results for a variety of hadronic properties is the MIT bag model.[4] In addition, quark diagrams can provide useful insights as to which dynamical mechanisms can contribute to a given transition.[5]

At first it appeared that taken together, the above concepts might lead to a successful explanation of the $\Delta I=\frac{1}{2}$ rule. Unfortunately, subsequent calculations indicated such optimism to be premature.[6] However, the work[7] of Shifman, Vainshtein, and Zakharov (SVZ), which emphasizes the role of mass scales defined by the set of light *and* heavy quarks, has renewed hopes among workers in the field.[8]

Perhaps the main implication of the SVZ analysis is that the effective $\Delta S=1$ nonleptonic operators contain right-handed currents as well as their traditional left-handed counterparts. Two kinds of current-current operators thus appear, "left-left" and "left-right." It is contended in Ref. 7 that the latter have substantially enhanced hadronic matrix elements relative to the former, and as a result these new "left-right" operators play a dominant role in the hyperon and kaon decays. In light of its importance, the claim deserves careful study.

Our primary task in this paper is to present a comprehensive evaluation of nonleptonic operator matrix elements. We shall generally employ the soft-pion method in order to reduce the matrix elements to those between single-particle states. These are then simple enough to permit analysis of many properties and also allow for straightforward numerical evaluation where necessary in terms of the relatively trustworthy bag model.

Employment of the operator-product expansion

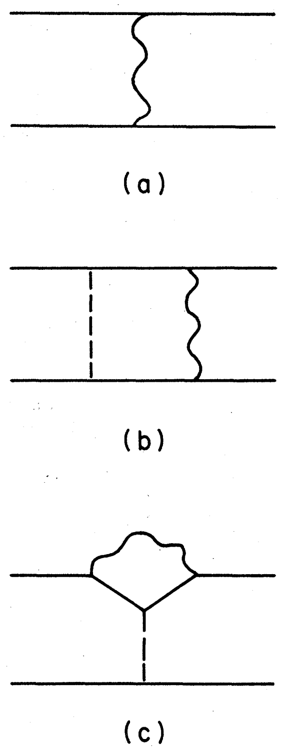

FIG. 1. $\Delta S=1$ transitions. The solid, wavy, and dashed lines represent quarks, W bosons, and gluons, respectively.

and the renormalization-group techniques allows the effective weak Hamiltonian to be expressed in terms of local operators involving quark fields (and in some models, gluons as well). In particular, the SVZ $\Delta S=1$ nonleptonic Hamiltonian can be written

$$\mathcal{H}_w=\frac{G_F}{2\sqrt{2}}\cos\theta_C\sin\theta_C\sum_{i=1}^{6}c_i\mathcal{O}_i+\text{H.c.}\,, \tag{1}$$

where [all the operators appearing in Eqs. (2)–(4) are understood to be normal-ordered and color indices are suppressed][9]

$$\begin{aligned}
\mathcal{O}_1&=\mathcal{H}_A-\mathcal{H}_B\,,\\
\mathcal{O}_2&=\mathcal{H}_A+\mathcal{H}_B+2\mathcal{H}_C+2\mathcal{H}_D\,,\\
\mathcal{O}_3&=\mathcal{H}_A+\mathcal{H}_B+2\mathcal{H}_C-3\mathcal{H}_D\,,\\
\mathcal{O}_4&=\mathcal{H}_A+\mathcal{H}_B-\mathcal{H}_C\,,
\end{aligned} \tag{2}$$

with

$$\begin{aligned}
\mathcal{H}_A&=\bar{d}\Gamma_L^{\mu}u\bar{u}\Gamma_{L\mu}s\,,\\
\mathcal{H}_B&=\bar{u}\Gamma_L^{\mu}u\bar{d}\Gamma_{L\mu}s\,,\\
\mathcal{H}_C&=\bar{d}\Gamma_L^{\mu}d\bar{d}\Gamma_{L\mu}s\,,\\
\mathcal{H}_D&=\bar{s}\Gamma_L^{\mu}s\bar{d}\Gamma_{L\mu}s\,,
\end{aligned} \tag{3}$$

and

$$\begin{aligned}
\mathcal{O}_5&=\bar{d}\Gamma_L^{\mu}t^As\bar{Q}\Gamma_{R\mu}t^AQ\,,\\
\mathcal{O}_6&=\bar{d}\Gamma_L^{\mu}s\bar{Q}\Gamma_{R\mu}Q\,.
\end{aligned} \tag{4}$$

In Eqs. (3) and (4) we have $\Gamma_L^{\mu}\equiv\gamma^{\mu}(1+\gamma_5)$, $\Gamma_R^{\mu}\equiv\gamma^{\mu}(1-\gamma_5)$, and $\mathrm{Tr}(t^At^B)=2\delta^{AB}$. The operator Q in Eq. (4) is summed over the three light-quark flavors u, d, s. $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_5$, and $\mathcal{O}_6$ all transform like SU(3) octets and carry isospin of one half. $\mathcal{O}_3$ and $\mathcal{O}_4$ are 27-plets under SU(3) and $I=\frac{1}{2}, \frac{3}{2}$, respectively.

The coefficients c_i are calculated by studying the QCD renormalization behavior, as will be discussed in more detail in Sec. II. A representative set of coefficients suggested by SVZ is $c_1=2.5$, $c_2=0.08$, $c_3=0.08$, $c_4=0.4$, $c_5=(-0.06)\to(-0.14)$, and $c_6=(-0.01)\to(-0.05)$. Of these, $c_1\to c_4$ appear reasonably secure. However, doubts can be raised about the values of c_5 and c_6. For example, they were calculated using free-field propagators for the quarks and gluons in a diagram where the whole effect arises at momentum below the charm mass. Bound-state effects could easily alter the numbers. For example, a calculation[6] within the MIT bag model of the diagram which generates $\mathcal{O}_5$ yields larger effects than would be predicted with c_5 and c_6 of SVZ. However, the form of $\mathcal{O}_5$ and $\mathcal{O}_6$ is more general. Wise and Witten[7] have recently shown that the only effects which are generated in perturbation theory have the form of the set of local operators considered by SVZ. Thus is what follows we shall discuss matrix elements of the operators $\mathcal{O}_1, \ldots \mathcal{O}_6$, and will not feel constrained to use the SVZ evaluation of c_5 and c_6. One of the consequences of our calculations will be to provide estimates for the kind of c_i which are needed phenomenologically.

In Sec. II we present a qualitative discussion of the "box" and "penguin" QCD radiative corrections which will hopefully clarify the issues being raised. Section III includes analyses of S-wave hyperon decay while Sec. IV treats the P waves. This is followed in Sec. V by a treatment of kaon nonleptonic decays. Our results are summarized in Sec. VI and some details regarding wave-function overlap integrals are presented in the Appendix.

II. BACKGROUND

Before presenting details of our calculation, we wish to review aspects of the application of renormalization-group methods to the calculation of effective nonleptonic Hamiltonians as well as outline the specific proposal of SVZ. The intent here is in part pedagogical. It has become clear to us in discussions with nonexperts, who are nonetheless interested in keeping up with recent progress in the field, that the physical nature of renormalization-group-induced effects is still arcane to many.

Although the operator-product expansion generally provides the most convenient framework for discussing QCD corrections to nonleptonic operators, for simplicity let us start by considering the W-boson-mediated graph of Fig. 1(a). One type of gluon radiative correction to this process is the box diagram of Fig. 1(b) (there are three related graphs as well). To see how significant such corrections are as a function of some scale mass μ, we must evaluate the associated dynamical integral (quark masses are assumed negligible):

$$I(\mu)=\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4(k^2-\mu_W^2)}$$
$$=\frac{-i}{8\pi^2\mu_W^2}\ln\left[\frac{\kappa}{(\kappa^2+\mu_W^2)^{1/2}}\right]\Bigg|_{\mu}^{\infty}. \quad (5)$$

The mass M_W provides a cutoff; that is, for $\mu>M_W$ we find $I(\mu)\cong 0$ whereas for $\mu<M_W$, $I(\mu)$ is proportional to the logarithmic factor $\ln(M_W/\mu)$. For energies which lie above quark masses, this should be the only class of radiative correction which contributes significantly. In other words, if we associate Fig. 1(a) with the nonleptonic operator which in the operator-product expansion reduces in lowest order to $\mathcal{H}_A$ [see Eq. (3)], then associated with Fig. 1(b) is a radiative correction to $\mathcal{H}_A$ which begins to contribute for $\mu\cong M_W$ and then grows for decreasing μ as $\ln(M_W/\mu)$.

Another noteworthy feature of these corrections is that a *new* nonleptonic operator is thereby generated. In particular, the box process of Fig. 1(b) (and related processes) induces the modification

$$\mathcal{H}_A\to\mathcal{H}_A-\frac{g^2}{16\pi^2}\ln(M_W^2/\mu^2)(3\mathcal{H}_A-\mathcal{H}_B), \quad (6)$$

where g is the quark-gluon coupling constant and we assume the number of quark colors to be three throughout this paper. Thus the effect of renormalization on $\mathcal{H}_B$ must be considered as well. As explained in Ref. 2, it turns out that the linear combinations

$$\mathcal{O}_\pm\equiv\tfrac{1}{2}(\mathcal{H}_A\pm\mathcal{H}_B) \quad (7)$$

remain form-invariant under renormalization of the type under discussion. To second order in g, the coefficient functions which accompany the operators $\mathcal{O}_\pm$ are given by

$$c_\pm=1+d_\pm\frac{g^2}{16\pi^2}\ln(M_W^2/\mu^2), \quad (8)$$

where $d_+=-2$ and $d_-=4$. The quantities $d_\pm$, essentially the anomalous dimensions of $\mathcal{O}_\pm$, arise in the diagonalization procedure implied by Eq. (7). The $\Delta I=\frac{1}{2}$ operator $c_-\mathcal{O}_-$ is thus enhanced whereas the operator $c_+\mathcal{O}_+$, which causes both $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ transitions, is suppressed. Observe how sensitive the quantities $c_\pm$ are to the value of the coupling $g^2/16\pi^2$. For example, if for definiteness we choose $M_W/\mu\cong 80$, and employ the not totally unreasonable value $g^2/4\pi\cong 0.72$, then $c_+=0$ and we obtain the long sought-after suppression of $\Delta I=\frac{3}{2}$ nonleptonic transitions. Of course, this line of reasoning is specious. If the second-order correction is large enough to totally cancel the zeroth-order term, then what must be the effect of even higher orders?

The renormalization-group (RG) analysis provides an answer to this question by taking into account the sum of all leading-logarithm terms $[g^2\ln(M_W/\mu)]^n$. The dependence of the coefficient functions is modified from that of Eq. (8) to

$$c_\pm=[K(g,M_W,\mu)]^{\gamma_\pm} \quad (9)$$

with

$$K(g,M,\mu)=1+\frac{g^2}{16\pi^2}b\ln(M^2/\mu^2) \quad (10)$$

for arbitrary mass $M>\mu$. The exponent in Eq. (9) is given by $\gamma_\pm=d_\pm/b$ where $b=(33-2n)/3$ for three quark colors, n being the number of quark flavors. Before discussing the physical significance of $K(g,\mu)$, it is worthwhile to estimate the magnitudes of $c_\pm$ as computed first to second order as in Eq. (8) and then to all leading-logarithm orders as in Eq. (9). A reasonable set of parameters to employ is $n=3$ (i.e., quark flavors u, d, s), $M_W/\mu=80$, and $g^2/4\pi\cong 1$ at 1 GeV. We then find

	second order	RG
c_-	3.8	2.42
c_+	−0.39	0.64

Thus numerically the effect of the renormalization group is to moderate the excesses of the second-order contributions (which for our seemingly innocent choice of parameters has driven c_+ down through zero and into negative values).

At present, it is hard to assign precise values to coefficient functions like $c_\pm$, depending as they do on both the number of quark flavors n and the coupling constant $g^2(\mu)/4\pi$ at mass scales appropriate to decays of light hadrons. However, all estimates that we are aware of for these quantities give substantial enhancements ($c_-\cong 2$ to 4) and suppressions ($c_+\cong 0.7$ to 0.5) relative to the free-field limit $c_-=c_+=1$. This is in marked contrast to the familiar correction in electrodynamics wherein the magnetic moment of a point charge is shifted upward by about only 0.1%. What makes the QCD radiative corrections to the nonleptonic operators so large? Evidently, there are two dominant considerations: (i) $\ln(M_W/\mu)$ represents the fact that two vastly different mass scales are present, and (ii) $g^2/4\pi(\mu)$ inevitably gets large at the mass scales considered here if prevailing ideas regarding quark confinement are correct; thus, instead of $e^2/4\pi \cong 1/137$, we encounter a rather larger value for $g^2/4\pi$. Another question related to the large size of the QCD radiative corrections is whether we can really neglect terms which lie outside the renormalization-group summation. Our tendency is to answer this in the affirmative because at least on an individual basis, terms such as $[g^4 \ln(M_W{}^2/\mu^2)]^n$, etc., really are small compared to those taken into account.

Up to this point, we have considered only two mass scales, M_W and μ, which enter the physics of nonleptonic transitions. However, others are expected to occur as well. Witten has shown that masses of heavy quarks also provide important scales, and that effective local operators can be generated to account for the effects of heavy quarks.[10] The papers of SVZ explore the role of heavy-quark masses in the theory, particularly emphasizing the effect of the process in Fig. 1(c) on the renormalization of the nonleptonic operators.

Before addressing Fig. 1(c), we first describe the effect of heavy-quark masses on the "box" coefficient functions $c_\pm$ considered so far. For simplicity we assume like SVZ that the *only* effect of the heavy-quark masses is to produce a decoupling of a given quark flavor from the quantity $b(n)$ as μ is lowered through the associated mass threshold.[11] Thus in a six-quark model (u, d, s, c, b, t with $m_c = 1.55$ GeV, $m_b = 4.6$ GeV, and $m_t = 13.8$ GeV) the value of $b(n)$ increases as we descend through the t-quark, b-quark, and c-quark thresholds down to $\mu \cong 1$ GeV. In between any two heavy-quark mass thresholds, $c_\pm$ vary as in Eqs. (9) and (10). The quantity $K(g, M, \mu)$ in these equations physically represents the variation in coupling strength over the energy range $M \geq E \geq \mu$,

$$K(g, M, \mu) = g^2(\mu)/g^2(M) \tag{11}$$

which can be directly inferred from

$$\int_{g(M)}^{g(m)} dx\, \beta^{-1}(x) = \ln(M/m),$$

where β is the function denoting the coupling-constant dependence in the renormalization-group equation. As a specific example, in the six-quark model mentioned above, the choice of scale mass $\mu \cong 1$ GeV implies for the coefficients $c_\pm$ the form (we suppress the notation for coupling constant in the argument of K)

$$\begin{aligned} c_-(\mu) &= K(M_W, m_t)^{0.57} K(m_t, m_b)^{0.52} \\ &\quad \times K(m_b, m_c)^{0.48} K(m_c, \mu)^{0.44}, \\ c_+(\mu) &= [c_-(\mu)]^{-1/2}, \end{aligned} \tag{12}$$

where we have employed the appropriate values for $b(n)$ in the exponents. It is sufficient to examine $c_-(\mu)$ numerically. Assuming $g^2(\mu)/4\pi = 1$ at $\mu = 1$ GeV, we have $c_-(\mu = 1 \text{ GeV}) = 2.50$ overall, with the specific contributions being

$$K(M_W, m_t)^{0.57} = 1.239, \quad K(m_b, m_c)^{0.48} = 1.356,$$
$$K(m_t, m_b)^{0.52} = 1.207, \quad K(m_c, \mu)^{0.49} = 1.233.$$

Thus, as we proceed from energy M_W down to energy $\mu \cong 1$ GeV, the coefficient $c_-(\mu)$ derives its largest growth from the interval $m_b \geq E \geq m_c$, where evidently $g^2/4\pi$ has grown to a sufficiently large value which together with the length of the interval provides the numerical dominance. Incidentally, the sensitivity to changing $g^2(\mu)/4\pi$ is not extreme [e.g., for $g^2/4\pi = \frac{1}{3}$ at $\mu = 1$ GeV we have $c_-(\mu) = 2.38$] so that our qualitative findings may be considered stable.

A related issue is the problem of which value of μ is most appropriate for evaluating matrix elements. The matrix elements of the operators also depend on the mass scale μ, and changing μ would move part of the matrix elements to the coefficients or vice versa.[10] However, we only know how to evaluate the matrix elements at a particular phenomenologically determined value of μ. The scale in the coefficients must be chosen to match this. Unfortunately, we have no clear way for determining the best μ, although we feel that in the bag model most scales are set by $(1 \to 2)/R \sim 0.2 \to 0.4$ GeV. The coefficients c_1, c_2, c_3, and c_4 do not depend strongly on the choice of μ. However, c_5 and c_6 do, and this increases the uncertainty in these coefficients.

Our final comments regard the impact of the so-called "penguin diagram," Fig. 1(c), on the calculation of $\mathcal{H}_W$ in the context of a four-quark model (u, d, s, c). The latter restriction keeps us within the SVZ model and minimizes the number of unknown parameters (i.e., mixing angles, thought to be small,[12] associated with the heavier b, t quarks). The main effect of this diagram is to introduce the "left-right" operators $\mathcal{O}_5$, $\mathcal{O}_6$ which to order g^2 are proportional to $\ln(m_c{}^2/\mu^2)$, the factor of M_W previously in the logarithm being removed by the Glasow-Iliopoulos-Maiani cancellation[13] between intermediate u quarks and c quarks. It is shown in Ref. 7 how operators $\mathcal{O}_5$ and $\mathcal{O}_6$ mix with $\mathcal{O}_1$ and $\mathcal{O}_2$ to produce a 4×4 renormalization matrix, and ultimately the form of $\mathcal{H}_W$ as given in

Eq. (1). As stated previously we shall calculate in this paper the matrix elements of the $\mathcal{O}_i$ because of their importance to the SVZ conclusion and also because there linger some questions as to the values of the coefficient functions.

III. HYPERON DECAY: S-WAVE AMPLITUDES

We shall consider only the parity-violating weak Hamiltonian $\mathcal{H}_w^{\rm PV}$ in this section. The S-wave hyperon decay amplitudes are obtained by means of the familiar current algebra formula

$$\lim_{q_\lambda \to 0} \langle B'\pi^0(q)|\mathcal{H}_w^{\rm PV}|B\rangle = \frac{-i}{F_\pi}\langle B'|[F_3^5, \mathcal{H}_w^{\rm PV}]|B\rangle \tag{13a}$$

$$= \frac{i}{2F_\pi}\langle B'|\mathcal{H}_w^{\rm PC}|B\rangle . \tag{13b}$$

Equation (13b) is correct only if

$$[F_3^5, \mathcal{H}_w^{\rm PV}] = [F_3, \mathcal{H}_w^{\rm PC}], \tag{14}$$

a point we shall return to shortly. If so, then an estimate of the S-wave amplitudes is obtainable in terms of the single-baryon matrix element of $\mathcal{H}_w^{\rm PC}$ as in Eq. (13b). We have neglected possible baryon pole diagrams which involve single-baryon matrix elements $\mathcal{H}_w$ of $\mathcal{H}_w^{\rm PV}$. This is done since SU(3) predicts that $\langle B'|\mathcal{H}_w^{\rm PV}|B\rangle$ vanishes.[14]

The experimental S-wave amplitudes for the decays $B \to B'\pi$ are given by the following dimensionless quantities[15]:

$$\begin{aligned} A(\Lambda n) &= (237 \pm 4)\times 10^{-9}, \\ A(\Sigma^+ p) &= (-328 \pm 11)\times 10^{-9}, \\ A(\Xi^0 \Lambda) &= (-343 \pm 7)\times 10^{-9}. \end{aligned} \tag{15}$$

Although these numbers are of individual interest in any calculation which takes symmetry breaking into account, it is also worthwhile to fit them in terms of SU(3) octet parameters f and d (Ref. 16):

$$\begin{aligned} A(\Lambda n) &= d + 3f, \\ A(\Sigma^+ p) &= \sqrt{6}(d - f), \\ A(\Xi^0 \Lambda) &= d - 3f. \end{aligned} \tag{16}$$

With $f = -2d$, we predict $A(\Lambda n) = 5f/2$, $A(\Sigma^+ p) = -3\sqrt{6}f/2$, and $A(\Xi^0\Lambda) = -7f/2$ with $f \simeq 96 \times 10^{-9}$. The overall fit is quite reasonable.

The relation between commutators in Eq. (14) is obeyed by the "left-left" operators $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$, and $\mathcal{O}_4$ of Eq. (2). However, because of normal-ordering and the presence of right-handed currents this is not true of the operators $\mathcal{O}_5$ and $\mathcal{O}_6$. For example, explicit calculation yields

$$[F_3^5, \mathcal{O}_6] = [F_3, \mathcal{O}_6] + \mathcal{O}_6^{(c)}, \tag{17}$$

where $\mathcal{O}_6^{(c)}$ can be cast in the form of a normal-ordered two-quark operator

$$\mathcal{O}_6^{(c)} = \tfrac{2}{3}\langle 0|\bar{d}d|0\rangle \colon \bar{d}(1+\gamma_5)s\colon . \tag{18}$$

Similarly, we find

$$[F_3^5, \mathcal{O}_5] = [F_3, \mathcal{O}_5] + \mathcal{O}_5^{(c)}, \tag{19}$$

where $\mathcal{O}_5^{(c)} = \frac{16}{3}\mathcal{O}_6^{(c)}$, the familiar numerical factor of $\frac{16}{3}$ arising from color matrices. Let us postpone numerical evaluation of the quantities $\mathcal{O}_{5,6}^{(c)}$ until after we study the structure of the single-baryon matrix elements of the operators $\mathcal{O}_i$, $i = 1, \ldots, 6$.

Hereafter we assume that baryons are color singlets of three quarks whose dynamics is governed by the SU(3) color gauge symmetry. Within this framework, some of our results will be model dependent whereas others are exact consequences of the approach described thus far. We shall consider the latter first. To begin with there is the relation

$$\langle B'|\mathcal{H}_A + \mathcal{H}_B|B\rangle = 0 \tag{20}$$

which has been proved elsewhere.[3] Two additional null statements are also derivable,

$$\langle B'|\mathcal{H}_C|B\rangle = 0, \tag{21a}$$

$$\langle B'|\mathcal{H}_D|B\rangle = 0. \tag{21b}$$

To prove Eq. (21b), we note that after performing a Fierz transformation and realizing that the spin labels on the field operators appearing in $\mathcal{H}_D$ [see Eq. (37)] are summed over and thus are dummy indices, we can write the normal-ordered creation and annihilation operators in $\mathcal{H}_D$ as

$$\tfrac{1}{2}b_i^\dagger(d,\lambda_d)b_j^\dagger(s,\lambda_1) \times [b_j(s,\lambda_2)b_i(s,\lambda_3) + b_i(s,\lambda_2)b_j(s,\lambda_3)]. \tag{22}$$

The initial baryon state is antisymmetric in color indices whereas the bracketed quantity is symmetric, and so Eq. (21b) follows immediately. Equation (21a) is a consequence of similar reasoning. We can write $\mathcal{H}_C$ as

$$\tfrac{1}{2}[b_i^\dagger(d,\lambda_1)b_j^\dagger(d,\lambda_2) + b_j^\dagger(d,\lambda_1)b_i^\dagger(d_1\lambda_2)] \times b_j(d,\lambda_3)b_i(s,\lambda_s), \tag{23}$$

and applying this operator to the final state B', the ensuing contraction of symmetric and antisymmetric quantities gives zero. The results (20) and (21) when cast in terms of the operators of Eq. (2) imply

$$\langle B'|\mathcal{O}_2|B\rangle = \langle B'|\mathcal{O}_3|B\rangle = \langle B'|\mathcal{O}_4|B\rangle = 0. \tag{24}$$

Finally, we claim for baryons in the valence model considered here that

$$\langle B'|\mathcal{O}_5|B\rangle = -\tfrac{8}{3}\langle B'|\mathcal{O}_6|B\rangle . \tag{25}$$

To prove this we use the SU(3) identity

$$t_{ij}^A t_{kl}^A = 2\delta_{il}\delta_{kj} - \tfrac{2}{3}\delta_{ij}\delta_{kl}$$

to write

$$\mathcal{O}_5 = -\tfrac{2}{3}\mathcal{O}_6 + 2\mathcal{O}_5' , \quad (26a)$$

where

$$\mathcal{O}_5' = :\ \bar{d}_i \Gamma_L^\mu s_j \bar{Q}_j \Gamma_{R\mu} Q_i : . \quad (26b)$$

But from arguments like those used for the null theorems just proved, we have $\langle B'|\mathcal{O}_5'|B\rangle = -\langle B'|\mathcal{O}_6|B\rangle$, from which Eq. (25) directly follows.

Taken together, the results of the preceding paragraphs demonstrate that for baryons we need calculate only matrix elements of the operators $\mathcal{O}_1$ and $\mathcal{O}_5$ (or $\mathcal{O}_6$). The importance of this is that matrix elements of the $\Delta I = \frac{3}{2}$ operator $\mathcal{O}_4$ vanish. *Thus the interaction transforms like an SU(3) octet, and has $\Delta I = \frac{1}{2}$ and yields the desired $\Delta I = \frac{1}{2}$ rule in the hyperons.*

In the course of forming matrix elements, we employ wave functions for quarks confined within baryons as given by the bag-model fit of Ref. 4. Our procedure is to compute the matrix elements

$$(2F_\pi)^{-1}\langle B'|\mathcal{O}_i|B\rangle \quad (27)$$

for each relevant set of baryons B, B' thereby inferring the dimensionless parameters f_i, d_i ($i = 1,5$). For comparison the reader should keep in mind the phenomenological values $f \simeq -2d \simeq 96\times 10^{-9}$ discussed earlier. We will first present an analysis based on exact SU(3), giving all quarks a common mass, and later consider the individual decay modes in the more realistic situation of unequal quark masses.

It is already known[6] that for matrix elements $\langle B'|\mathcal{O}_1|B\rangle$ one finds $d_1 = -f_1$ with

$$f_1 = \frac{\sqrt{3}}{4}\cos\theta_C \sin\theta_C \frac{G_F R^{-3}}{F_\pi}\frac{N^4}{4\pi}[\bar{A}(mR) + \bar{B}(mR)] , \quad (28)$$

where m is the common quark mass, R is the bag radius, N is the normalization factor for the quark field, and $\bar{A}+\bar{B}$ is a dynamical integral. All of these are discussed in the Appendix. The dependence of $\bar{A}$ and $\bar{B}$ on quark mass m is given in Fig. 2. For m varying from 0 to 280 MeV, we find f_1 going from 24×10^{-9} to 30×10^{-9}. Remember that before it can be compared with the phenomenological parameter f, f_1 must be multiplied by the coefficient function c_1. With $d = -f$ we have $A(\Lambda n) = 2f$, $A(\Sigma^+ p) = 2\sqrt{6}f$, and $A(\Xi^0\Lambda) = -4f$. This is clearly unsatisfactory in that it predicts $|A(\Sigma^+ p)/A(\Xi^0\Lambda)| = 1.23$ and $|A(\Xi^0\Lambda)/A(\Lambda n)| = 2.0$, each of which disagrees with experiment [Eq. (15)]. In order to improve the fit f must be increased relative to d.

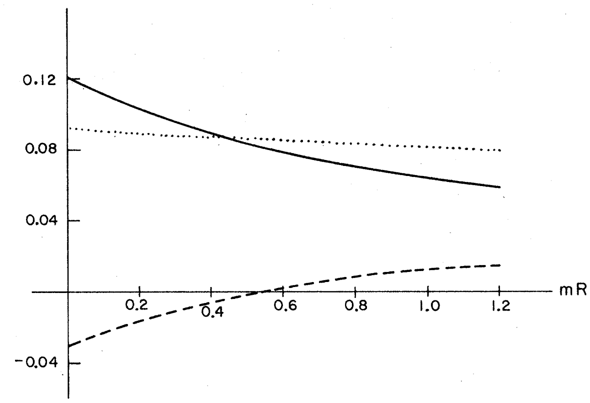

FIG. 2. Dynamical integrals in the SU(3) limit. The solid, dotted, and dashed curves represent respectively the integrals $\bar{A}+\bar{B}$, $\bar{C}$, and $\bar{A}-\bar{B}$ defined in the text.

Matrix elements of $\mathcal{O}_5$ (and of $\mathcal{O}_6$) have a slightly more complicated structure in that both f_5 and d_5 are nonzero and their ratio is a function of the dynamical integrals $\bar{A},\bar{B}$. We find for matrix elements of $\mathcal{O}_5$,

$$d_5 = \frac{\sqrt{3}}{9}\cos\theta_C \sin\theta_C \frac{G_F R^{-3}}{F_\pi}\frac{N^4}{4\pi}[3\bar{A}(mR) - \bar{B}(mR)] \quad (29a)$$

and

$$f_5 = \frac{\sqrt{3}}{27}\cos\theta_C \sin\theta_C \frac{G_F R^{-3}}{F_\pi}\frac{N^4}{4\pi}[3\bar{A}(mR) + 7\bar{B}(mR)] \quad (29b)$$

so that

$$\frac{d_5}{f_5} = 3\,\frac{[3\bar{A}(mR) - \bar{B}(mR)]}{3\bar{A}(mR) + 7\bar{B}(mR)} . \quad (30)$$

The f_5, d_5 parameters are roughly comparable in magnitude to f_1 and are of the same phase. For example, for quark mass between 0 and 280 MeV, f_5/f_1 goes from 0.82 to 0.65. Furthermore, the matrix elements of $\mathcal{O}_5$ tend to be predominantly f type for small quark mass but less so as the mass is increased. That is, for quark mass in the range 0–280 MeV, we find d_5/f_5 varying between 0.265 and 1.10. To the extent that this new contribution is f type, it can be viewed as having the potential to assist in bringing the model into agreement with experiment. However, there is an even more important left-right contribution to which we now turn our attention.

Perhaps the most striking of our numerical results involves the new terms arising from the commutation relations [see Eqs. (16)–(19)]. These commutator-induced operators are proportional to $:d(1+\gamma_5)s:$. As such they are pure $\Delta I = \frac{1}{2}$ and indeed are pure f type.[18] For the dominant contribution, which arises from $\mathcal{O}_5$, we find in the limiting case of SU(3) invariance that

$$f_5^{(c)}=\frac{8}{9\sqrt{3}}\frac{G_F\cos\theta_C\sin\theta_C}{F_\pi}\langle 0|\bar{d}d|0\rangle N^2\bar{C}\,, \quad (31a)$$

where the superscript in $f_5^{(c)}$ stands for "commutator" and $\bar{C}$ is a scalar density amplitude, defined in the Appendix. From estimates of the "current-algebra" d-quark mass,[19] one can infer that

$$\langle 0|\bar{d}d|0\rangle=\frac{F_\pi^2}{2m_d^{(c)}}[m^2(K^-)-m^2(K^0)-m^2(\pi^+)]$$
$$=-0.007\,\text{GeV}^3\,. \quad (31b)$$

The computation of this number is sufficiently subtle to warrant an explanation. In Ref. 18 a "renormalized" quark mass m^* is related to current-algebra quark mass $m^{(c)}$ by a renormalization constant $m^*=Zm^{(c)}$. From SU(3)-symmetry breaking it is estimated that $m_s^*\simeq 150$ MeV, and so $m_d^*=m_s^*/20\simeq 7.5$ MeV from a current-algebra relation. The quantity Z which is defined by the expectation value $\langle\bar{q}_f q_k\rangle=ZN_{hk}$ in hadron h, where N_{hk} is the number of k-flavored quarks in hadron h, can be directly computed in the bag model[20]:

$$Z=\frac{\omega+2mR(\omega-1)}{2\omega(\omega-1)+mR}\,, \quad (32)$$

where m is the bag-model quark mass. For the d quark, m_d is small, so that we will use $m=0$ throughout our work. Thus we estimate $Z=0.48$ from which our value for $\langle 0|\bar{d}_i d_i|0\rangle$ follows. (The reader should also be careful of the sign convention in Ref. 19.) This together with analytic evaluation of the integral $\bar{C}$ implies that for a common quark mass in the range $0\le m\le 280$ MeV, the quantity $f_5^{(c)}$ goes from -45×10^{-9} to -62×10^{-5}. That is, $f_5^{(c)}$ is about a factor of 2 larger than either f_1 or $f_5^{(1)}$ and carries a phase opposite to both of them.

The origin of the terms involving $O_{5,6}^{(c)}$ is not as mysterious when examined from another point of view. They are, in fact, the "vacuum-intermediate-state" or "factorization" contributions originally calculated by SVZ, as we will now show.[21] Requiring that only connected pieces of, e.g., O_5 contribute to weak processes, we have

$$\mathcal{O}_5=\text{“}\mathcal{O}_5\text{”}+\tfrac{32}{9}\langle 0|\bar{d}d|0\rangle\bar{d}(1+\gamma_5)s$$
$$+\tfrac{32}{9}\langle 0|\bar{s}s|0\rangle\bar{d}(1-\gamma_5)s\,, \quad (33)$$

where "$\mathcal{O}_5$" is not normal-ordered,

$$\text{“}\mathcal{O}_5\text{”}=\bar{d}\gamma^\mu(1+\gamma_5)t^A s\bar{Q}\gamma_\mu(1-\gamma_5)t^A Q\,, \quad (34)$$

and as before, Q is summed over flavors u,d,s. If we make a Fierz transformation of the first piece of this operator into a form $\mathcal{O}_{SP}$ involving scalar, pseudoscalar densities then

$$\mathcal{O}_5=\mathcal{O}_{SP}+\tfrac{32}{9}\langle 0|\bar{d}d|0\rangle\bar{d}(1+\gamma_5)s$$
$$+\tfrac{32}{9}\langle 0|\bar{s}s|0\rangle\bar{d}(1-\gamma_5)s \quad (35)$$

with

$$\mathcal{O}_{SP}=-4\bar{Q}^i(1+\gamma_5)u^j\bar{u}^k(1-\gamma_5)$$
$$\times Q^k(\delta_{ij}\delta_{kl}-\tfrac{1}{3}\delta_{il}\delta_{kj})\,. \quad (36)$$

We can now pick out the factorization contribution to hyperon decay, wherein $Q=u,d$ and the pion is annihilated by the appropriate pseudoscalar density, $\bar{u}\gamma_5 d$ or $\bar{d}\gamma_5 d$, while the associated scalar density, $\bar{s}u$ or $\bar{s}d$, connects the two baryons. Using the divergence relation

$$-iZ\partial^\mu\frac{(\bar{q}_2\gamma_\mu\gamma_5 q_1)}{m_1^*+m_2^*}=\bar{q}_2\gamma_5 q_1 \quad (37)$$

we find, e.g.,

$$\langle B'\pi^0|\mathcal{O}_5|B\rangle_{\text{fact}}=i\frac{32Z}{9}\frac{F_\pi m_\pi^2}{2m_d^*}\langle B'|\bar{d}s|B\rangle\,. \quad (38)$$

Aside from the factor of Z, this is the method employed by SVZ. We wish to compare this with the soft-pion limit

$$\langle B'\pi_q^0|\mathcal{O}_5|B\rangle\underset{q\to0}{\longrightarrow}\frac{-i}{F_\pi}\langle B'|[F_3^5,\mathcal{O}_5]|B\rangle\,. \quad (39)$$

Using the commutation relation

$$[F_3^5,\mathcal{O}_5]=-\tfrac{1}{2}\mathcal{O}_5+\tfrac{32}{9}\langle 0|\bar{d}d|0\rangle\bar{d}(1+\gamma_5)s\,, \quad (40)$$

we find

$$\langle B'\pi_q^0|\mathcal{O}_5|B\rangle\underset{q\to0}{\longrightarrow}\frac{i}{2F_\pi}[\langle B'|\mathcal{O}_{SP}|B\rangle_{\text{connected}}$$
$$-\tfrac{64}{9}\langle 0|\bar{d}d|0\rangle\langle B'|\bar{d}s|B\rangle]\,. \quad (41)$$

Finally, assuming the Weinberg mass relations[19] we have

$$\langle 0|\bar{d}d|0\rangle=-Z\frac{F_\pi^2 m_\pi^2}{m_u^*+m_d^*}\,. \quad (42)$$

Thus

$$\langle B'\pi_8^0|\mathcal{O}_5|B\rangle\underset{q\to0}{\longrightarrow}\frac{i}{2F_\pi}\langle B'|\mathcal{O}_{SP}|B\rangle_{\text{connected}}$$
$$+i\frac{32}{9}Z\frac{F_\pi^2 m_\pi^2}{m_u^*+m_d^*}\langle B'|\bar{d}s|B\rangle\,, \quad (43)$$

so we see that the factorization terms are associated with the anomalous terms in the PCAC commutator while the effects of the nonfactorization pieces are taken into account by use of the *connected* matrix elements of $\mathcal{O}_{SP}$. These can be calculated, for example, in the MIT bag model as done above.

The evaluation of $f_5^{(c)}$, of course, also shares

TABLE I. Structure of single-particle matrix elements. The factors X and Y are defined in Eqs. (45) and (46). The dynamical integrals A, B, A^*, B^*, C are given in the Appendix.

	$\mathcal{O}_1$	$\mathcal{O}_5$	$\mathcal{O}_5^{(c)}$
Λn	$2\sqrt{6}X(A+B)$	$\frac{8}{3}\sqrt{6}X(A+B)$	$\frac{8YC}{3\sqrt{3}F_\pi}$
$\Sigma^+ p$	$-12X(A+B)$	$\frac{16}{3}X(A-5B/3)$	$-\frac{16YC}{9\sqrt{2}F_\pi}$
$\Xi^0\Lambda$	$-4\sqrt{6}X(A+B)$	$-\frac{8}{3}\sqrt{6}X(A^*+B^*/3-A+B)$	$-\frac{8YC}{3\sqrt{3}F_\pi}$

with SVZ the dependence on the values of the current-algebra light-quark masses. These may not be well determined as can be seen from the recent controversy over whether the up-quark mass vanishes.[22] This dependence appears to be unavoidable. Our calculation differs from SVZ by a more careful treatment of the mass with the inclusion of the renormalization constant Z. Our matrix elements are smaller than SVZ by a factor of Z^2, which equals $(0.48)^2=0.23$ when Z is evaluated in the bag model. Adding all the contributions together (for $m_q \to 0$) we find

$$f=(24c_1-25c_5-c_6)\times10^{-9}\,, \quad d=-(24c_1+5c_5-2c_6)\times10^{-9}\,. \tag{44}$$

We see that the effect of the new operator $\mathcal{O}_5$ is roughly comparable to that of $\mathcal{O}_1$. Neither $\mathcal{O}_1$ nor $\mathcal{O}_5$ by itself could adequately describe the SU(3) structure. However, adding the $\mathcal{O}_5$ contribution to that of $\mathcal{O}_1$ does help the results (recall that for SVZ $c_5<0$). Both the magnitude of f and the f/d ratio are thereby improved.

The SU(3) analysis carried out in the previous paragraphs gives us a quick but approximate overview of the hyperon decays. To get a more precise picture of the individual transitions as well as to take SU(3)-symmetry breaking into account, we summarize in Table I formulas relevant to the hyperon decays and in Table II the associated numerical values. In Table I, each hyperon matrix element of the four-quark operators $\mathcal{O}_1$ and $\mathcal{O}_5$ is given in units of the dimensionless factor

$$X=\frac{\cos\theta_C\sin\theta_C}{2\sqrt{2}}\,\frac{G_F R^{-3}}{2F_\pi}\,\frac{N^3N'}{4\pi}\,, \tag{45}$$

while each matrix element of the two-quark, commutator-induced operator $\mathcal{O}_5^{(c)}$ is given in units of the constant Y,

$$Y=0.007\ (\mathrm{GeV}^3)G_F\cos\theta_C\sin\theta_C NN'\,, \tag{46}$$

where the prime on N signifies strange-quark kinematics. In the course of our calculations, we studied the effects of SU(3) several different ways and found the results to be in qualitative accord with each other. The particular method displayed in Table II involves keeping the nonstrange-quark mass equal to zero as in Ref. 4 and

TABLE II. Numerical study of matrix elements. Nonstrange-quark mass is kept at zero. We use $R=5.0\ \mathrm{GeV}^{-1}$. Entries are in units of 10^{-9}. Each entry includes contributions from $G_F\sin\theta_C\cos\theta_C/2\sqrt{2}$ as in Eq. (1).

	m_sR	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
Λn	$\mathcal{O}_1$	48	48	48	49	49	49	49	49
	$\mathcal{O}_5$	64	64	64	65	65	65	65	65
	$\mathcal{O}_5^{(c)}$	−134	−138	−143	−146	−150	−154	−157	−160
$\Sigma^+ p$	$\mathcal{O}_1$	−117	−118	−118	−119	−119	−120	−120	−120
	$\mathcal{O}_5$	−35	−34	−32	−31	−30	−28	−27	−26
	$\mathcal{O}_5^{(c)}$	−109	−113	−116	−120	−122	−125	−128	−131
$\Xi^0\Lambda$	$\mathcal{O}_1$	−96	−96	−97	−97	−97	−98	−98	−98
	$\mathcal{O}_5$	−53	−54	−55	−56	−57	−58	−59	−61
	$\mathcal{O}_5^{(c)}$	−134	−138	−143	−146	−150	−154	−157	−160

considering the strange-quark mass over the range $0 \leq m_s \leq 280$ MeV. A scan of the numbers in Table II provides several immediate insights. Effects of nondegenerate quark mass are seen to be typically less than twenty percent. Table II serves to reinforce the qualitative conclusions arrived at in our study of the SU(3)-invariance limit. By and large, the matrix elements of $\mathcal{O}_1$ and of $\mathcal{O}_5$ are comparable. In particular, for the $\Sigma^+ p$ and $\Xi^0\Lambda$ transitions, $\mathcal{O}_1$ has a larger effect than does $\mathcal{O}_5$ while the situation reverses for Λn. However, for all of the hyperon transitions the matrix elements of $\mathcal{O}_5^{(c)}$ are the largest.

To get a feel for the effect of these operators, we display the three transitions in Table III for a fixed value of c_1 ($c_1 = 2.5$) and for three values of c_5 ($c_5 = 0, -.5, -1.0$).[23] It is easy to see that including $\mathcal{O}_5$ does improve the SU(3) structure of hyperon decays, although one needs values of c_5 substantially larger than SVZ suggest. With $c_5 \approx -0.5 \to -1$ one obtains a quite satisfactory pattern of S-wave amplitudes.

IV. HYPERON DECAY: *P*-WAVE AMPLITUDES

The analysis of the hyperon P-wave amplitudes (parity conserving) is not as secure as that of the S waves. We discuss first several contributions to the parity-conserving matrix elements, and then borrow results from other sections in order to give a numerical evaluation. We shall see that the ingredients which we include are not sufficient to give a satisfactory description of the P-wave amplitudes. The decay $\Sigma^+ \to n\pi^+$ serves as a particularly good example.

In the soft-pion limit we encounter a baryon-to-baryon matrix element analogous to Eq. (13), but involving the parity-*violating* Hamiltonian

$$\frac{-i}{F_\pi}\langle B'(p')|[F_3^5,\mathcal{H}_w^{\rm PC}]|B(p)\rangle = \frac{i}{2F_\pi}\langle B'(p')|\mathcal{H}_w^{\rm PV}|B(p)\rangle \,. \tag{47}$$

SU(3) symmetry forces this matrix element to vanish (at least when $p = p'$) for a Hamiltonian with the SU(3) properties of Eq. (1).[14] Quark models are not yet sophisticated enough to see if in this context SU(3) is a strong constraint on the amplitude. However, with no evidence to the contrary,

TABLE III. Hyperon decay modes with $c_1 = 2.5$, and a variety of values of c_5. All entries are in units of 10^{-8}.

Mode	$c_5=0$	$c_5=-0.5$	$c_5=-1$	Experiment
Λn	12	17	22	23.7 ± 0.4
$\Sigma^+ p$	-30	-35	-40	-32.8 ± 1.1
$\Xi^0\Lambda$	24	-30	-34	-34.3 ± 0.7

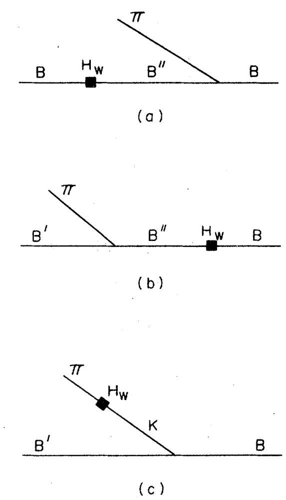

FIG. 3. Pole contributions to the parity-conserving amplitude in $B \to B'\pi$. Baryon poles are shown in (a) and (b), and the kaon pole is represented in (c).

we shall assume that Eq. (47) does vanish and so not include it in our analysis.

The standard treatment of the P waves[24] is to utilize baryon poles. As illustrated in Figs. 3(a) and 3(b), these consist of a $\Delta S = 1$ baryon-to-baryon transition accompanied by strong pion emission. The baryon matrix elements are those considered in the last section. With the notation

$$M_{AB} = \langle B|\mathcal{H}_w^{\rm PC}|A\rangle\,, \tag{48}$$

the baryon pole contributions are

$$B(\Lambda_0^0) = g(m_p + m_\Lambda)\left[\frac{M_{\Lambda n}}{(m_\Lambda - m_p)2m_p} + \left(\frac{2}{3}\right)^{1/2}\frac{dM_{\Sigma^+p}}{(m_\Sigma - m_p)(m_\Sigma + m_\Lambda)}\right],$$

$$B(\Sigma_0^+) = -g(m_p + m_\Sigma)\left[\frac{M_{\Sigma^+p}}{(m_\Sigma - m_p)2m_p} - \frac{2fM_{\Sigma^+p}}{(m_\Sigma - m_p)2m_\Sigma}\right], \tag{49}$$

$$B(\Sigma_+^+) = \sqrt{2}\,g(m_p + m_\Sigma)\left[\frac{M_{\Sigma^+p}}{(m_\Sigma - m_p)2m_p} - \frac{fM_{\Sigma^+p}}{(m_\Sigma - m_p)2m_\Sigma} + \left(\frac{2}{3}\right)^{1/2}\frac{dM_{\Lambda n}}{(m_\Lambda - m_n)(m_\Sigma + m_\Lambda)}\right],$$

$$B(\Xi_0^0) = -g(m_\Lambda + m_\Xi)\left[\frac{(d-f)M_{\Xi^0\Lambda}}{(m_\Xi - m_\Lambda)2m_\Xi} - \left(\frac{2}{3}\right)^{1/2}\frac{dM_{\Xi^-\Sigma^-}}{(m_\Xi - m_\Sigma)(m_\Sigma + m_\Lambda)}\right],$$

where g is the πNN coupling constant ($g^2/4\pi = 14.6$), and f, d describe the SU(3) structure of the strong coupling ($f + d = 1$).[25] We will use $d/f = \frac{3}{2}$. It has been known for a long time that

P-wave amplitudes, when derived from experimental S-wave vertices using PCAC and only baryon poles, as in Eq. (49), are too small by roughly a factor of 2.[24]

In addition to baryon poles, it is easy to see that kaon poles must also contribute. Indeed, kaon poles are needed together with baryon poles in order to obtain a vanishing $\langle B'\pi|H|B\rangle$ amplitude when $H=\bar{d}s$, as required by the Coleman-Glashow theorem.[26] A typical diagram is shown in Fig. 3(c). The strength of the kaon poles is governed by the K-π transition matrix elements which are studied in Sec. V. Using the same notation as Eq. (48), their form is

$$\begin{aligned} B(\Lambda_0^0) &= -g\,\frac{(d+3f)}{\sqrt{3}}\,\frac{M_{K^0\pi^0}}{m_K{}^2-m_\pi{}^2}\,,\\ B(\Sigma_0^+) &= -\sqrt{2}\,g(d-f)\,\frac{M_{K^0\pi^0}}{m_K{}^2-m_\pi{}^2}\,,\\ B(\Sigma_+^+) &= 0\,,\\ B(\Xi_0^0) &= -2g\,\frac{(d-3f)}{\sqrt{3}}\,\frac{M_{K^0\pi^0}}{m_K{}^2-m_\pi{}^2}\,. \end{aligned} \tag{50}$$

A final possible contribution arises from the anomalous-commutator terms $\mathcal{O}_{5,6}^{(c)}$ which we identified in Eq. (18). For $\mathcal{O}_{5,6}$, the commutator of Eq. (47) is replaced by

$$\begin{aligned} \frac{-i}{F_\pi}\langle B'|[F_3^5,\mathcal{O}_{5,6}]|B\rangle &= \frac{i}{2F_\pi}\langle B'|\mathcal{O}_{5,6}^{\rm PV}|B\rangle-\frac{i}{F_\pi}\langle B'|\mathcal{O}_{5,6}^{(c)}|B\rangle\\ &= \frac{-i}{F_\pi}\langle B'|\mathcal{O}_{5,6}^{(c)}|B\rangle\,. \end{aligned} \tag{51}$$

As our discussion of these terms in the S-wave case would suggest, the result obtained in this manner corresponds to the vacuum-insertion method proposed by SVZ. The matrix elements of $\mathcal{O}_{5,6}^{(c)}$ can be readily obtained provided we know $\langle B'|\bar{d}\gamma_5 s|B\rangle$. One way to calculate this matrix element assumes dominance of the kaon pole:

$$\begin{aligned} \langle B'|\bar{d}\gamma_5 s|B\rangle &= \langle 0|\bar{d}\gamma_5 s|K\rangle\frac{1}{q^2-m_K{}^2}\langle KB'|B\rangle\\ &= \frac{F_K m_K{}^2}{m_d+m_s}\,\frac{1}{m_K{}^2}\,g_{KB'B}\bar{u}\gamma_5 u\,. \end{aligned} \tag{52}$$

Another uses the quark equation of motion:

$$\begin{aligned} \langle B'|\bar{d}\gamma_5 s|B\rangle &= \frac{-i\partial_\mu}{m_d+m_s}\langle B'|\bar{d}\gamma^\mu\gamma_5 s|B\rangle\\ &= g_A^{B'B}\left(\frac{M_B+M_{B'}}{m_d+m_s}\right)\bar{u}\gamma_5 u\,. \end{aligned} \tag{53}$$

The two evaluations are completely equivalent if one uses the generalized Goldberger-Treiman relation. (Again we note the sensitivity to the quark masses.) There is an important subtlety associated with the commutator terms. If one evaluates them using the SVZ method and employes Eq. (52) one can write the total amplitude for π^0 emission as

$$\langle B'\pi^0|\mathcal{O}_5|B\rangle = \frac{g_{K^0BB'}}{q^2-m_K{}^2}\;2t_{ij}^A t_{kl}^A\langle\pi^0|\bar{d}_i\gamma_5 d_l|0\rangle\langle 0|\bar{d}_k\gamma_5 s_j|K^0\rangle\,. \tag{54}$$

This is very similar to the K pole

$$\langle B'\pi^0|\mathcal{O}_5|B\rangle = \frac{g_{K^0BB'}}{q^2-m_K{}^2}\,{-2}t_{ij}^A t_{kl}^A\langle\pi^0|\bar{d}_i(1-\gamma_5)d_l\bar{d}_k(1+\gamma_5)s_j|K^0\rangle\,. \tag{55}$$

In fact, Eq. (54) is just one contribution to Eq. (55). Indeed, a complete set of intermediate states can be inserted into the latter between the quark densities, and Eq. (54) is obtained by retaining only the vacuum intermediate state. The bag evaluation of the K-π matrix element provides one way of summing over all intermediate states, and thereby includes Eq. (54). To include both equations would constitute double counting. We feel that the bag evaluation of the K pole is the more complete and that the factorization (or commutation) term should therefore not be included. This approach is similar to that taken by Shrock and Treiman in their discussion of $\langle K^0|H_w|\bar{K}^0\rangle$.[27] The problem of double counting will surface again in Sec. V.

In this framework, the $\Delta I=\frac{1}{2}$ rule is obtained for the P-wave amplitudes if it is obeyed for $\langle B'|\mathcal{H}_w|B\rangle$ and $\langle\pi|\mathcal{H}_w|K\rangle$. In the former it follows from the Pati-Woo theorem. In Sec. V we will argue that the $\Delta I=\frac{1}{2}$ rule is understandable in meson matrix elements if c_5 is large enough. If this is true, the $\Delta I=\frac{1}{2}$ rule holds for the parity-conserving hyperon amplitudes also. For this reason, we leave the discussion of $\Delta I=\frac{1}{2}$ dominance for the appropriate other sections, and concentrate below on the SU(3) structure.

The matrix elements which are required for our analysis are evaluated in Secs. III and V. We list the three contributions by mode in Table IV. As an indication of the relative size of its effect the commutator term has been included in the

TABLE IV. Contributions to the hyperon parity-conserving amplitudes. The units are 10^{-7} and each of the theoretical numbers should be multiplied by c_1 or c_5 before comparing with experiment. As discussed in the text, the "commutator" column and the "kaon pole" column should not be both included simultaneously as they are different evaluations of the same effect.

Mode	Baryon pole $\mathcal{O}_1$	Baryon pole $\mathcal{O}_5$	Kaon pole $\mathcal{O}_1$	Kaon pole $\mathcal{O}_5$	Commutator
$\Lambda \to n\pi^0$	2.2	8.8	−1.3	−44	−15
$\Sigma^+ \to p\pi^0$	5.0	1.1	−0.4	−12	−4.4
$\Sigma^+ \to n\pi^+$	−8.8	2.6			
$\Xi^0 \to \Lambda\pi^0$	2.3	−4.9	0.4	16	5.9

table, although we have argued above that it is contained in the kaon pole evaluation and should not be included in a final analysis. Table V shows the resulting amplitudes for $c_1 = 2.5$ and $c_5 = 0$, -0.25, and -0.50, along with the experimental data. Although individual amplitudes can be reproduced by a judicious choice of c_5, there are no reasonable values of c_1 and c_5 which bring the overall pattern into agreement with experiment. However, the effect of c_5 is on the whole beneficial.

The decay $\Sigma^+ \to n\pi^+$ is a case with reduced theoretical uncertainty, and therefore it is useful to examine its failure a little closer. The reason for our theoretical confidence in case of this mode is the lack of kaon pole or commutator terms, leaving only baryon matrix elements. One way to demonstrate the problem in $\Sigma^+ \to n\pi^+$ is to form the ratio of it to an S-wave quantity, such as $B(\Sigma_+^+)/A(\Sigma^+ p)$. If there were not the anomalous-commutator contribution to $A(\Sigma^+ p)$, this could be written in terms of experimental quantities

$$\frac{B(\Sigma_+^+)}{A(\Sigma^+ p)} = \frac{11A(\Sigma^+ p) + 9(\Lambda n)}{A(\Sigma^+ p)}, \tag{56}$$

which yields a value of 4.5 compared to the empirical value

$$B(\Sigma_+^+)/A(\Sigma^+ p) = 12.9 .$$

However, the anomalous commutator contributes

TABLE V. Predictions and experimental data for the hyperon parity-conserving decays in units of 10^{-7}. Values are given at fixed $c_1 = 2.5$ and variable c_5.

Mode	$c_5 = 0$	$c_5 = -0.25$	$c_5 = -0.5$	Experiment
$\Lambda \to n\pi^0$	2	11	20	16.1 ±1.2
$\Sigma^+ \to p\pi^0$	12	14	17	26.6 ±1.2
$\Sigma^+ \to n\pi^0$	−22	−23	−23	−42.2 ±0.3
$\Xi^0 \to \Lambda\pi^0$	7	4	1	13.1 ±2.2

to $A(\Sigma^+ p)$ while it should not be included in the baryon matrix element involved in $B(\Sigma_+^+)$. When calculated directly using the procedure of Sec. II, we find

$$\frac{B(\Sigma_+^+)}{A(\Sigma^+ p)} = 7.4\left(\frac{c_1 - 0.3c_5}{c_1 - 0.9c_5}\right) \tag{57}$$

or

$$\frac{B(\Sigma_+^+)}{A(\Sigma^+ p)} \approx 6 - 7$$

for values of c_1 and c_5 which appear reasonable in the S waves. There appears to be no reasonable way to change this within our framework.

The failure to explain the P-wave data may not be the fault of the weak Hamiltonian which we are using. Indeed, it is hard to see how the Hamiltonian could be changed to improve the agreement. Rather, it appears that the fault lies in the calculational framework. Comparing S and P waves using PCAC and the pole model does not work. Le Yaouanc *et al.*[28] have suggested that very strong PCAC breaking could help. Alternatively, the pole model may be inadequate. Perhaps as quark model techniques become better in the future, they will be able to better decide where the blame for this failure lies.

V. KAON DECAY

Our intent in this section is to determine the contributions of the operators $\mathcal{O}_1, \ldots, \mathcal{O}_6$ to the $K \to \pi\pi$ decays. If the $K \to \pi\pi$ amplitudes can be successfully computed, the $K \to \pi\pi\pi$ amplitudes are well fit using current-algebra–PCAC constraints and the assumption of linear energy dependence.[29] It is useful to define $K \to \pi\pi$ isospin amplitudes f_1 and f_3 corresponding to $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ transitions, respectively,

$$\begin{aligned}
&\langle \pi^+\pi^0 | \mathcal{H}_w | K^+ \rangle = (\tfrac{3}{10})^{1/2} f_3 , \\
&\langle \pi^+\pi^- | \mathcal{H}_w | K^0 \rangle = (\tfrac{1}{3})^{1/2} f_1 + (\tfrac{1}{15})^{1/2} f_3 , \\
&\langle \pi^0\pi^0 | \mathcal{H}_w | K^0 \rangle = (\tfrac{1}{3})^{1/2} f_1 - 2(\tfrac{1}{15})^{1/2} f_3 .
\end{aligned} \tag{58}$$

From experiment[15] we find

$$f_1 = i9.46 \times 10^{-7} m_k, \quad f_3 = i6.78 \times 10^{-8} m_k , \tag{59}$$

where our overall choice of phase is made for convenience with regard to the ensuing discussion.

Calculation of S-wave hyperon decays via current algebraic techniques is fairly straightforward—continuation to the soft-pion limit is not hampered by appreciable momentum dependence. This is *not* the case for nonleptonic kaon decay, however, and our calculational confidence is thereby reduced. The sources of the momentum dependence in the kaon case are current-algebra–

PCAC constraints and the theorem of Gell-Mann and others[30] (GMPRSBB) which asserts that $K\to 2\pi$ amplitudes must vanish in the SU(3)-symmetry limit. Since, experimentally, $K\to 2\pi$ shows no sign of such suppression, it is clear that there exists a rather strong momentum dependence in the decay amplitude. This can be seen, for example, by evaluating the factorization[31] or "vacuum-intermediate-state" contribution to $K\to 2\pi$ from $\mathcal{H}_w^{(L)}$, that part of the weak Hamiltonian containing the left-left operators $\mathcal{O}_1,\ldots,\mathcal{O}_4$. We find

$$\langle\pi^+_{q_1}\pi^0_{q_2}|\mathcal{H}_w^{(L)}|K^+_k\rangle=\frac{iG_F}{2\sqrt{2}}\cos\theta_C\sin\theta_C F[\tfrac{4}{3}(3k^2+q_2^2-4q_1^2)c_4-\tfrac{2}{3}(q_1^2-q_2^2)(c_1+2c_2+2c_3)]\,, \quad (60a)$$

$$\langle\pi^+_{q_1}\pi^-_{q_2}|\mathcal{H}_w^{(L)}|K^0_k\rangle=\frac{iG_F}{2\sqrt{2}}\cos\theta_C\sin\theta_C F\left[\frac{2\sqrt{2}}{3}(k^2-q_1^2)(c_1+2c_2+2c_3)+\frac{4\sqrt{2}}{3}(k^2-3q_2^2+2q_1^2)c_4\right], \quad (60b)$$

$$\langle\pi^0_{q_1}\pi^0_{q_2}|\mathcal{H}_w^{(L)}|K^0_k\rangle=\frac{iG_F}{2\sqrt{2}}\cos\theta_C\sin\theta_C F\left[\frac{\sqrt{2}}{3}(2k^2-q_1^2-q_2^2)(c_1+2c_2+2c_3-4c_4)\right], \quad (60c)$$

where F is the meson decay constant in the SU(3) limit. In the limit $k^2=q_1^2=q_2^2$ all amplitudes are seen to vanish, as required by GMPRSBB. However, there is little evidence of this suppression in the physical limit $k^2=m_K^2$, $q_1^2=q_2^2=m_\pi^2$. Incidentally, as a check of this parametrization note that for $K^+\to\pi^+\pi^0$ in the physical limit, contributions from $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$, being $\Delta I=\frac{1}{2}$ all vanish, and only the $\Delta I=\frac{3}{2}$ contribution from $\mathcal{O}_4$ survives.

In evaluating the nonfactorization contributions which arise in the current-algebra–PCAC reduction of the $K\to\pi\pi$ amplitude, we shall eventually encounter K-to-π matrix elements of the operators $\mathcal{O}_i$. As before, we employ the MIT bag model in order to determine these matrix elements. Direct evaluation gives expressions in terms of the four-quark wave-function integrals A,B:

$$\begin{aligned}\langle\pi^+|\mathcal{O}_1|K^+\rangle&=\tfrac{1}{2}\langle\pi^+|\mathcal{O}_2|K^+\rangle=\tfrac{1}{2}\langle\pi^+|\mathcal{O}_3|K^+\rangle\\&=\tfrac{1}{2}\langle\pi^+|\mathcal{O}_4|K^+\rangle\\&=4(2m_K^2)^{1/2}\frac{R^{-3}}{4\pi}N^3N^1(A-B)\end{aligned} \quad (61a)$$

and

$$\begin{aligned}\langle\pi^+|\mathcal{O}_5|K^+\rangle&=\tfrac{16}{3}\langle\pi^+|\mathcal{O}_6|K^+\rangle\\&=-\tfrac{64}{3}(2m_K^2)^{1/2}\frac{R^{-3}}{4\pi}N^3N^1(A+B)\,.\end{aligned} \quad (61b)$$

The relative sizes of the matrix elements in (61a) and in (61b) are exact consequences of the valence-quark model. Proofs of these relations are entirely analogous to those given for the hyperon matrix elements. The corresponding $\langle\pi^0|\mathcal{O}_i|K^0\rangle$ matrix elements can be obtained via standard isospin considerations [In Eqs. (61a) and (61b) the factor $(2m_K^2)^{1/2}$ arises from the invariant normalization term $(4m_KE_\pi)^{1/2}$ which we assume continues smoothly and thus retains the value it had in the physical region $E_\pi=m_K/2$.[32]].

In order to make contact between the physical $K\to 2\pi$ amplitudes and the single-particle matrix elements, we shall *assume* that the momentum dependence of the $K\to 2\pi$ amplitude is as given by the factorization terms since, after all, these are consistent with all the current algebra and the GMPRSBB constraints. Thus, e.g., we suppose that

$$\langle\pi^+_{q_1}\pi^-_{q_2}|\mathcal{O}_1|K^0_k\rangle=\Lambda(k^2-q_1^2)\,, \quad (62)$$

where Λ is a constant to be determined. This together with continuation to the soft-pion limits

$$\langle\pi^+_{q_1}\pi^-_{q_2}|\mathcal{O}_1|K^0_k\rangle\begin{cases}\xrightarrow[q_2\to 0]{}0\\[4pt]\xrightarrow[q_1\to 0]{}\dfrac{-i}{F_\pi}\langle\pi^0_{q_2}|\mathcal{O}_1|K^0_k\rangle\end{cases} \quad (63)$$

identifies the quantity Λ as

$$\begin{aligned}\Lambda k^2&=\frac{-i}{F_\pi}\langle\pi^0_{q_2}|\mathcal{O}_1|K^0_k\rangle\\&=\frac{i2\sqrt{2}}{F_\pi}(2m_K)^{1/2}\frac{R^{-3}}{4\pi}N^3N^1(A-B)\,.\end{aligned} \quad (64)$$

There is clearly a problem in interpreting this relation—What is "k^2"? It is not clear what happens to the momenta when we take one of them to zero. If we use the value $k^2=m_K^2$; then by the same token in treating $K^+\to\pi^+\pi^0$ we should use $q_1^2=q_2^2=m_\pi^2$, which is inconsistent with the neutral kaon system. As a compromise we shall employ "k^2" $=\frac{1}{2}(m_K^2+m_\pi^2)$. This is admittedly a difficult point with which we do not currently know how to deal adequately; what is needed is a technique which allows our bag states to move.

Another complexity we must handle is the question of double counting. The point is that we wish to include both the factorization and the current-algebra amplitudes in our analysis. However, in the limit

$$\lim_{q_a \to 0} \langle \pi^a \pi^b | \mathcal{H}_w | K^n \rangle = \frac{-i}{F_\pi} \langle \pi^b | [F_a^5, \mathcal{H}_w] | K^n \rangle \qquad (65)$$

there is a contribution to the bag integral from a pion which is already included in the factorization term involving the operators $\mathcal{O}_1, \ldots, \mathcal{O}_4$, namely the contribution proportional to

$$\langle \pi^b | A^\mu | 0 \rangle \langle 0 | A_\mu | K^n \rangle .$$

This piece must be subtracted in order to avoid double counting. Adding on the factorization contribution just cancels this subtraction, so that the net result is obtained by omitting the factorization term completely. Putting all the previous considerations together, we have the following momentum-dependent amplitudes:

$$\langle \pi^+_{q_1} \pi^0_{q_2} | \mathcal{H}_w | K^+_k \rangle = \frac{8(3k^2 + q_2^2 - 4q_1^2)}{m_K^2 + m_\pi^2} c_4 T_L + \frac{2(q_1^2 - q_2^2)}{m_K^2 + m_\pi^2} [-2(c_1 + 2c_2 + 2c_3) T_L + \tfrac{32}{3} (c_5 + \tfrac{3}{16} c_6) T_R] , \qquad (66a)$$

$$\langle \pi^+_{q_1} \pi^-_{q_2} | \mathcal{H}_w | K^0_k \rangle = \frac{8\sqrt{2}(k^2 - 3q_2^2 + 2q_1^2)}{m_K^2 + m_\pi^2} c_4 T_L + \frac{2(k^2 - q_1^2)}{m_K^2 + m_\pi^2} [2\sqrt{2}(c_1 + 2c_2 + 2c_3) T_L - \tfrac{32}{3} \sqrt{2}(c_5 + \tfrac{3}{16} c_6) T_R] , \qquad (66b)$$

$$\langle \pi^0_{q_1} \pi^0_{q_2} | \mathcal{H}_w | K^0_k \rangle = \frac{2(2k^2 - q_1^2 - q_2^2)}{m_K^2 + m_\pi^2} [\sqrt{2}(c_1 + 2c_2 + 2c_3 - 4c_4) T_L - \tfrac{16}{3} \sqrt{2}(c_5 + \tfrac{3}{16} c_6) T_R] , \qquad (66c)$$

where T_L, T_R signify contributions from the left-left operators $\mathcal{O}_1, \ldots, \mathcal{O}_4$ and "left-right" operators $\mathcal{O}_5, \mathcal{O}_6$, respectively.

$$T_L = \frac{i}{2\sqrt{2}} \cos\theta_C \sin\theta_C \frac{G_F R^{-3}}{F_\pi} (2m_K^2)^{1/2} \times \frac{N^3 N^1}{4\pi} (A - B) , \qquad (67a)$$

$$T_R = \frac{i}{2\sqrt{2}} \cos\theta_C \sin\theta_C \frac{G_F R^{-3}}{F_\pi} (2m_K^2)^{1/2} \times \frac{N^3 N^1}{4\pi} (A + B) \qquad (67b)$$

and primes indicate strange-quark kinematics. Upon employing the numerical values $R = 3.26$ GeV^{-1} and $m_s R = 1.0$, we find in units of m_K

$$T_L = -i1.51 \times 10^{-8} m_K , \quad T_R = i1.02 \times 10^{-7} m_K . \qquad (68)$$

An interesting feature here, as previously noted by Donoghue and Golowich,[6] is that the matrix elements of the operators $\mathcal{O}_1, \mathcal{O}_2 \ldots \mathcal{O}_4$, associated with the dynamical integral $A - B$, are subject to large cancellations. A nice way to see this effect is to take the limit of equal quark masses. This integral, which we denote as $\overline{A} - \overline{B}$, is evaluated explicitly in the Appendix and is plotted in Fig. 2. We note that $\overline{A} - \overline{B}$ has a zero for $m \simeq 110$ MeV if $R \simeq 3.3$ GeV and never does get very large for reasonable values of quark mass. Thus, at least in this model, the matrix elements

$$\langle \pi | \mathcal{O}_i | K \rangle \quad i = 1, 2, 3, 4$$

cannot be reliably computed. However, we can infer that the matrix elements are not large, which is not an entirely negative conclusion because it means that the matrix element of the $\Delta I = \frac{3}{2}$ operator $\mathcal{O}_4$ is *suppressed*.

A heuristic demonstration of the cause of this suppression can be seen by considering $\pi \to e\nu$. When the weak interactions are due to vector and axial vector currents this process is suppressed by the difficulty of forming the helicities of the e and ν into a pseudoscalar state, with the amplitude vanishing as $m_e \to 0$. However, if the interaction is through scalar and pseudoscalar densities, no such suppression occurs. An analogous effect happens in our calculation. The left-left operators $\mathcal{O}_1 \ldots \mathcal{O}_4$ are always the product of two $V - A$ currents, and their amplitude is helicity suppressed, with the lower components in the quark wave functions canceling the upper components. However the left-right operators $\mathcal{O}_5, \mathcal{O}_6$ can be rearranged by a Fierz transformation into a product of scalar and pseudoscalar densities, whose matrix elements are *not* helicity suppressed. Thus the large cancellation which we observe should be a general feature of all models, although its precise value would change from model to model.

In view of the large cancellations in $A - B$, we shall append a factor ξ to terms containing this integral as a reminder that its precise magnitude and even its sign must be viewed as uncertain.[33] The contributions to the physical isospin amplitudes associated with the quantities T_L and T_R are thus

$$f_1 = -i1.27 \times 10^{-7} m_K (c_1 + 2c_2 + 2c_3)\xi - i4.56 \times 10^{-6} m_K (c_5 + \tfrac{3}{16} c_6) , \quad f_3 = -i5.66 \times 10^{-7} m_K c_4 \xi . \qquad (69)$$

The large contribution of $\mathcal{O}_5$ can also be partially accounted for by the large color-counting factor $\frac{64}{3}$ seen in Eq. (61b). The factor is a general feature which will be present in all quark models.

Next we consider the commutator terms $\mathcal{O}_5^{(c)}$, $\mathcal{O}_6^{(c)}$. These are equivalent to factorization processes in which the operators $\mathcal{O}_5, \mathcal{O}_6$ first undergo a Fierz transformation into an effective *SP* interaction form. The associated contributions are pure $\Delta I=\frac{1}{2}$ and thus contribute to f_1 the expression

$$iF_K \tfrac{32}{9}\sqrt{6}\,\frac{m_\pi^2}{m_u+m_d}\,\frac{m_K^2}{m_s+m_d}\times\left(\frac{1}{1-\dfrac{m_\pi^2}{m_\kappa^2}}-\frac{1}{1-\dfrac{m_K^2}{m_\sigma^2}}\right)(c_5+\tfrac{3}{16}c_6)\,, \quad (70)$$

where m_σ^2, m_κ^2 are parameters which describe the momentum dependence of the matrix elements $\langle\pi^+\pi^-|\bar{d}d|0\rangle, \langle\pi|\bar{d}s|K\rangle$. Observe that if $m_\sigma=m_\kappa$ and $m_\pi=m_K$ then $f_1=0$ as required by GMPRSBB. Numerically we have for these contributions

$$f_1=-i1.02\times10^{-6}m_K\left(\frac{m_K^2}{m_\sigma^2}-\frac{m_\pi^2}{m_\kappa^2}\right)(c_5+\tfrac{3}{16}c_6)\,,\qquad f_3=0\,. \quad (71)$$

At this point we must again take caution against double counting. We accomplish this by subtracting off from the current-algebra amplitude T_R the contribution from factorization. For example, consider the contribution to $K^0\to\pi^0\pi^0$ of the operator $\mathcal{O}_5^+$. Upon taking the soft-pion limit we obtain for the nonfactorization amplitude

$$\langle\pi_1^0\pi_2^0|\mathcal{O}_5|K_k^0\rangle_{\text{nonfact}} \xrightarrow[q_1\to0]{} \frac{-i}{F_\pi}\left[\tfrac{1}{2}\langle\pi_2^0|\mathcal{O}_5|K_{k'}^0\rangle + \tfrac{32}{3}\langle0|\bar{d}d|0\rangle\langle\pi_2^0|\bar{s}(1+\gamma_5)d|K_{k'}^0\rangle\right]. \quad (72)$$

In the same limit we find for the factorization amplitude

$$\langle\pi_1^0\pi_2^0|\mathcal{O}_5|K_k^0\rangle_{\text{fact}} \xrightarrow[q_1\to0]{} \frac{-i}{F_\pi}\tfrac{16}{9}\langle\pi_2^0|\bar{d}\gamma_5 d|0\rangle \times\langle0|\bar{s}\gamma_5 d|K_{k'}^0\rangle\,. \quad (73)$$

Clearly, this equals the contribution to $\langle\pi_2^0|\mathcal{O}_5|K_{k'}^0\rangle$ in Eq. (72) arising from the vacuum intermediate state. Thus we must modify the nonfactorization amplitude in the soft-pion limit to read

$$\frac{-i}{2F_\pi}\langle\pi_2^0|\mathcal{O}_5|K_{k'}^0\rangle \to \frac{-i}{2F_\pi}\left[\langle\pi_2^0|\mathcal{O}_5|K_{k'}^0\rangle - \tfrac{32}{9}\langle\pi^0|\bar{d}\gamma_5 d|0\rangle\langle0|\bar{s}\gamma_5 d|K^0\rangle\right] \quad (74)$$

or correspondingly

$$T_R\to T_R'=T_R-\frac{i}{6\sqrt{2}}G_F\cos\theta_C\sin\theta_C F_K \times\frac{m_\pi^2}{m_u+m_d}\,\frac{m_K^2}{m_s+m_u} =T_R-i3.98\times10^{-8}m_K =i6.2\times10^{-8}m_K\,. \quad (75)$$

The final result for the kaon isospin amplitudes is then

$$\frac{f_1}{m_K}=-i1.27\times10^{-7}(c_1+2c_2+2c_3)\xi -i2.77\times10^{-6}(c_5+\tfrac{3}{16}c_6) -i1.04\times10^{-6}\left(\frac{m_K^2}{m_\sigma^2}-\frac{m_\pi^2}{m_K^2}\right)(c_5+\tfrac{3}{16}c_6)\,, \quad (76)$$

$$\frac{f_3}{m_K}=-i5.66\times10^{-7}c_4\xi\,. \quad (77)$$

Here one can see that the largest contribution to the $I=\frac{1}{2}$ amplitude comes from the quark-model evaluation of the "left-right" operators $\mathcal{O}_5$, $\mathcal{O}_6$. This is in contrast with the S-wave hyperon amplitudes where the anomalous-commutator term gave the largest matrix elements. This result is gratifying in that the dependence on light-quark masses is minimized. Our absolute values for both f_1 and f_3 tend to be somewhat large. However, this is not unreasonable in view of the several assumptions which we were foced to make in order to handle the momentum dependence in $K\to2\pi$. What is important is that since f_3 receives a contribution only from $\mathcal{O}_4$, which has smaller matrix elements, we have the possibility of obtaining a $\Delta I=\frac{1}{2}$ dominance of sufficient size. For $c_4=0.4$, the ratio f_1/f_3 would agree with experiment for $c_5/\xi\approx0.9$, an entirely reasonable value.

VI. CONCLUSION

We have attempted to give as complete an evaluation of the weak nonleptonic Hamiltonian as is possible with present techniques. Current-algebra methods were used to turn two-body decay amplitudes into matrix elements between single-particle states. The latter are amenable to evaluation using quark-model techniques.

Two general types of effects were found. Given the standard current commutation relations, matrix elements of the parity-conserving Hamiltonian were calculated in the MIT bag model. In addition, for the operators $\mathcal{O}_{5,6}$ recently suggested by SVZ, we have identified an anomalous-commutator term. When evaluated using currrent algebra and the quark model, this yields the matrix elements calculated by SVZ by use of the quark equations of motion. The work of SVZ is incomplete in that it only includes this latter effect. We find that both types yield significant contributions.

In some regards our methods are crude. For example, we do not directly calculate a decay process, but use PCAC to relate it to an amplitude which we can calculate. Other mechanisms can be imagined to have an effect, such as PCAC breaking, nonperturbative phenomena,[34] or quark-sea processes.[35] In addition, there are several uncertainties inherent in our methods. However, it is hoped that our procedure would yield the dominant effects, and the reasonableness of our results suggests that they do give some insight into the nonleptonic decays.

The hyperon parity-violating amplitudes are most favorable for the use of our methods. The calculated matrix elements obey the $\Delta I = \frac{1}{2}$ rule and have an SU(3) octet property. If we only include $\mathcal{O}_1 \cdots \mathcal{O}_4$ the relative rates of the various decays are not correctly given. Including $\mathcal{O}_5$, $\mathcal{O}_6$, the anomalous commutator yields a large matrix element which tends to improve both the magnitude and the SU(3) structure of the amplitude. For $-c_5 \gtrsim \frac{1}{4}$ the improvement becomes noticeable, and a study of Table III shows that a reasonable understanding of the decays is obtained for $c_5 = -0.5 \to -1$. These values have the same sign as that suggested by SVZ, but are somewhat larger.

The parity-conserving hyperon amplitudes are the least amenable to direct calculation. We have studied them using a pole model with baryon and kaon poles. In this method, a $\Delta I = \frac{1}{2}$ rule is obtained since $\langle B' | \mathcal{H}_w^{\text{PC}} | B \rangle$ and $\langle \pi | \mathcal{H}_w^{\text{PC}} | K \rangle$ are dominated by isospin one half. The signs of the amplitudes (relative to the parity-violating ones) are correctly given. However, the SU(3) structure of the amplitudes is not accurate. We are unsure as to the source of this difficulty, but feel inclined to blame it on the pole model and on our inability to find a better calculational framework.

For the kaons, the standard operators $\mathcal{O}_1 \cdots \mathcal{O}_4$ involving only left-handed currents have small matrix elements. On the other hand, $\mathcal{O}_5$, $\mathcal{O}_6$, with both left- and right-handed currents, have very large amplitudes when evaluated in the quark model. This difference is due both to a large color-counting factor associated with $\mathcal{O}_{5,6}$ and also to a helicity suppression for operators with only left-handed currents. Its importance is that the $\Delta I = \frac{1}{2}$ rule becomes understandable in the kaons if c_5 is large enough. The $\Delta I = \frac{3}{2}$ effects are suppressed relative to the large purely $\Delta I = \frac{1}{2}$ contribution of $\mathcal{O}_5$. The $\Delta I = \frac{1}{2}, \frac{3}{2}$ amplitudes are reproduced in our method for $c_5 = \xi = -0.3$.

Including the new operators $\mathcal{O}_{5,6}$ with a sizable coefficient function therefore has two beneficial aspects. It improves the structure and magnitude of the hyperon S waves, and most importantly, it allows a $\Delta I = \frac{1}{2}$ rule in the kaons. This suggests that the presence of these operators should be considered seriously.

Within this framework, then, it is possible to understand the $\Delta I = \frac{1}{2}$ rule with three basic ingredients: (1) QCD radiative corrections enhance $I = \frac{1}{2}$, suppress $I = \frac{3}{2}$, and bring in the $I = \frac{1}{2}$ operator $\mathcal{O}_5$. (2) Color symmetry suppresses $I = \frac{3}{2}$ in baryon-to-baryon matrix elements (the Pati-Woo theorem). (3) $I = \frac{3}{2}$ effects are suppressed in kaons relative to $\mathcal{O}_5$ by color and helicity factors. It is interesting to note that all three factors are important and that there is no single "master-stroke" explanation of the $\Delta I = \frac{1}{2}$ rule. This suggests that in charmed-meson decay, where (3) presumably is inoperative, we would not expect the sextet rule (the analogy of the $\Delta I = \frac{1}{2}$ rule) to hold as strongly as the $\Delta I = \frac{1}{2}$ rule does for $\Delta S = 1$ decays, which seems to be consistent with experimental indications.[36]

Many past approaches to the $\Delta I = \frac{1}{2}$ rule have suggested modifying the structure of the weak-interaction theory to incorporate this unexpected selection rule. However, given the remarkable variety and consistency of the many probes of the weak current, these approaches are less plausible at present. The low-energy structure of the weak interaction is given to a high degree by the standard Weinberg-Salam theory. The problem then is to find a dynamical reason why the strong interactions should allow $\Delta I = \frac{1}{2}$ effects to be stronger than $\Delta I = \frac{3}{2}$. The combination of QCD radiative correction and quark-model matrix-element evaluations considered in this paper allow a plausible explanation of this problem.

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APPENDIX

All the four-quark integrals in our analysis can be expressed in terms of two quantities,

$$A = \int_0^1 u^2 du [\, j_0{}^2(pu) - \epsilon j_1{}^2(pu)] \times [\, j_0(pu) j_0(p'u) - (\epsilon\epsilon')^{1/2} j_1(pu) j_1(p'u)] \quad \text{(A1)}$$

and

$$B=2\epsilon^{1/2}\int_0^1 u^2du\, j_0(pu)j_1(pu)\times[\epsilon^{1/2}j_0(p'u)j_1(pu)+\epsilon'^{1/2}j_0(pu)j_1(p'u)]\,, \quad (A2)$$

where symbols accompanied by a prime pertain to kinematics of the strange quark. We also encounter the commutator terms a two-quark integral

$$C=\int_0^1 u^2du[j_0(pu)j_0(p'u)-(\epsilon\epsilon')^{1/2}j_1(pu)j_1(p'u)]\,. \quad (A3)$$

All these SU(3)-noninvariant quantities were numerically evaluated on a computer.

In the above, $\epsilon=(\omega-mR)/(\omega+mR)$ and the quark momentum p and frequency ω are related by $\omega^2=p^2+m^2R^2$. For baryons, we employ a bag radius $R=5.0$ GeV^{-1} whereas for mesons we use $R=3.3$ GeV^{-1}. The quark-field normalization factor N is given in terms of these quantities by

$$N^2=\frac{p^4}{(2\omega^2-2\omega+mR)\sin^2p}\,. \quad (A4)$$

The dynamical two-quark and four-quark integrals can be easily evaluated in the limit of SU(3) invariance. We denote the SU(3) limit by placing overbars above all the defining symbols, e.g., A,B becomes $\bar{A},\bar{B}$. The two-quark integral is easily evaluated and we find

$$\bar{C}=w[1-j_0(2p)]/[p^2(w+mR)]-j_0^2(p)/(w+mR)\,. \quad (A5)$$

The four-quark integrals can be expressed as linear combinations of the following functions:

$$I_{00}=\int_0^1 du\,u^2j_0^4(pu)=\frac{\sin^4p}{p^4}+\frac{\mathrm{Si}(2p)}{p^3}-\frac{\mathrm{Si}(4p)}{2p^3}\,, \quad (A6)$$

$$I_{01}=\int_0^1 du\,u^2j_0^2(pu)j_1^2(pu)=\frac{\sin^4p}{3p^6}+\frac{\sin^3p\cos p}{3p^5}-\frac{\sin^4p}{3p^4}+\frac{\mathrm{Si}(2p)}{3p^3}-\frac{\mathrm{Si}(4p)}{6p^3}\,, \quad (A7)$$

$$I_{11}=\int_0^1 du\,u^2j_1^4(pu)=-\frac{\sin^4p}{5p^5}+\frac{4\sin^3p\cos p}{5p^4}+\frac{14\sin^4p}{15p^3}-\frac{6\sin^2p}{5p^3}-\frac{2\sin^3p\cos p}{15p^2}+\frac{4\sin p\cos p}{5p^2}-\frac{1}{5p}+\frac{7I_{00}}{15p^3}\,. \quad (A8)$$

In particular, the dynamical integrals from Secs. II and III are combinations of

$$\bar{A}=I_{00}-2\epsilon I_{01}+\epsilon^2I_{11} \quad (A9)$$

and

$$\bar{B}=4\epsilon I_{01}\,. \quad (A10)$$

For example, the meson integral from Sec. III is

$$\bar{A}-\bar{B}=I_{00}-6\epsilon I_{01}+\epsilon^2I_{11}\,. \quad (A11)$$

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[8] For example, see M. K. Gaillard, in *Weak Interactions—Present and Future*, proceedings of the Sixth SLAC Summer Institute, 1978, edited by Martha C. Zipf (SLAC, Stanford, 1978), p. 397.

[9] Our definition of the operator $\mathcal{O}_1$ in Eq. (2) differs from that of Ref. 7 by a minus sign. In addition, we have removed the left and right projection operators, which makes our $\mathcal{O}_i$ four times larger than those of Ref. 7. The overall coefficient in Eq. (1) compensates for this, leaving the coefficients c_i unchanged.

[10] E. Witten, Nucl. Phys. B104, 445 (1976); B120, 189 (1977); B122, 109 (1977).

[11] We make this limiting assumption both to describe the model employed in Ref. 7 and also to highlight the effect of quark masses upon the value of b. However, other effects can be expected as well.

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[14]B. W. Lee and A. R. Swift, Phys. Rev. 136, B228 (1964).

[15]Particle Data Group, Phys. Lett. 75B, 1 (1978). Our hyperon amplitudes are defined by $\langle B\pi | H_W | B\rangle = i\bar{u}(p')(A+B\gamma_5)u(p)$.

[16]The SU(3) 27-plet reduced matrix element is quite small, and we neglect it throughout.

[17]There are several potential sources for the small $\Delta I = \frac{3}{2}$ component, and we do not consider them here.

[18]This is true when this operator is evaluated in the bag model. Use of the quark equations of motion would yield $d = -\frac{3}{4}(M_\Sigma - M_\Lambda)/\frac{1}{4}(3M_\Lambda + M_\Sigma) - M_n f = -0.3f$.

[19]For example, see S. Weinberg, in *I. I. Rabi Festschrift* (Academy of Sciences, New York, 1978); P. Langacker and H. Pagels, Phys. Rev. D 19, 2070 (1979).

[20]S. Adler, E. Colglazer, J. Healy, I. Karlinger, J. Lieberman, Y. J. Ng, and H. S. Tsao, Phys. Rev. D 11, 3522 (1975); J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Rev. D 12, 2875 (1975).

[21]We use "factorization" to denote the evaluation of a product of currents by using single current or quark density matrix elements. For example, $\langle B'\pi | V_\mu A^\mu | B\rangle \to \langle \pi | A^\mu | 0\rangle \langle B' | V_\mu | B\rangle$ or $\langle B'\pi | SP | B\rangle \to \langle \pi | P | 0\rangle \langle B' | S | B\rangle$. When dealing with operators such as $\mathcal{O}_5$ and $\mathcal{O}_6$ which have factorizations of both forms given above, we neglect that with vector and axial-vector currents in favor of the larger term with scalar and pseudoscalar densities.

[22]For example, A. Zepeda, Phys. Rev. Lett. 41, 140 (1978); C. A. Dominguez, *ibid.* 41, 605 (1978); N. G. Deshpande and D. E. Soper, *ibid.* 41, 735 (1978).

[23]In giving these rough estimates we neglect c_6.

[24]R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969).

[25]We utilize pseudovector coupling and the Goldberger Treiman relation to put Eq. (49) into the given form.

[26]S. Coleman and S. Glashow, Phys. Rev. 134, B670 (1964); J. Bajaj, M. P. Khanna, and V. Kaushal, Phys. Rev. D 18, 2526 (1978); see also G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Lett. 3, 527 (1959); J. F. Donoghue, Phys. Rev. D 19, 2772 (1979).

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[31]Here we are using factorization with vector and axial-vector currents (see footnote 21) of the form $\langle \pi\pi | V_\mu A^\mu | k\rangle = \langle \pi | A_\mu | 0\rangle \langle \pi | V^\mu | k\rangle$ in all possible ways.

[32]We thank S. Treiman for suggesting this dependence.

[33]One can trace the path of the vanishing of $A-B$. It occurs for (masses in GeV) $m_u = 0.1$ and $m_s = 0.42$, $m_u = 0.15$ and $m_s = 0.25$, etc. A very recent improvement in the treatment of mesons within the bag model [J. F. Donoghue and K. Johnson, MIT Report No. CTP 302 (unpublished)] yields quark masses in mesons $m_u = m_d = 0.033$ GeV, $m_s = 0.34$ GeV, and a radius $R = 3.4$ GeV. This reduces A-B by a factor $\xi = 0.6$.

[34]For example, see J. E. Kim, University of Pennsylvania report, 1979 (unpublished).

[35]J. F. Donoghue and E. Golowich, Phys. Lett. 69B, 437 (1977). The "totaly sea" effects described in this paper give an alternative evaluation of the diagram which yields $\mathcal{O}_5$ and $\mathcal{O}_6$ [Fig. 1(c)].

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