PHYSICAL REVIEW D VOLUME 21, NUMBER 7 1 APRIL 1980

Possible evidence for fluctuations in the hadronic temperature

Saul Barshay
Instituut voor Theoretische Fysica, Universiteit Leuven, Leuven, Belgium

H. Braun, J. P. Gerber, and G. Maurer
Centre de Recherches Nucléaires and Université Louis Pasteur, Strasbourg, France
(Received 31 May 1979)

We describe ways in which one can test the idea that the temperature parameter relevant for the description of the thermal-like emission spectra observed in hadronic collisions is related to the very large transverse relative acceleration that occurs between projectile and target in the course of these collisions through $kT = a/2\pi$. As a consequence, the typical temperature should increase with a decrease in the typical impact parameter, and also significant fluctuations should occur in the temperature, on an event-by-event basis. We perform the first analysis of data to look for such fluctuations, and we find an indication that they are present in antiproton annihilation reactions.

The idea has been presented[1] that the large transverse relative acceleration a between projectile and target, which occurs in the course of a high-energy hadronic collision, is connected with the thermal emission[2,3] of particles at a temperature given by $kT=a/2\pi$. A temperature $kT \sim 100$ MeV corresponds to an acceleration $a\sim 3\times 10^{29}$ km/sec^2. Such enormous accelerations occur only in the course of particle collisions because a second is an extremely long time on the characteristic time scale for strong interactions, $\sim 10^{-23}$ sec. As a direct consequence[1] of this hypothesis, an average temperature is related to an average impact parameter between projectile and target according to $\langle kT\rangle \propto 1/\langle b\rangle$. This implies that in the hypothetical collision of two particles with a much smaller size than that of the usual projectiles and targets (i.e., nucleons, pions, kaons), for example, bound states of a heavy quark and antiquark with size of order of $1/m_q$, an order-of-magnitude increase in the typical temperature would occur. This increase would be roughly on the ratio of the heavy-quark effective mass to that of the ordinary light quarks.[1] Another implication, which is more readily tested by experiment, is that significant *fluctuations* in the temperature parameter must occur, *event by event.* Until now,[4] analyses to look for such fluctuations have not been performed, although they can readily be performed with much *existing* data on multiparticle production. In this paper we discuss some possible procedures for studying the event-by-event fluctuations in the temperature parameter, and we carry out such a study for the first time by applying one procedure to an analysis of 7650 events involving antinucleon-nucleon annihilation into pions. We find an indication that the dispersion of the temperature parameter, event by event, is significantly different from zero.

It is now established that inclusive single-particle transverse-momentum distributions near center-of-mass rapidity $y^*\sim 0$, in particular for pion production in pion-proton[4,5] and proton-proton collisions,[6] are quantitatively well described by the form[7,8]

$$\frac{d^2\sigma}{d^2\vec{p}_T} \propto \exp[-\beta(p_T{}^2+m^2)^{1/2}]\,, \tag{1}$$

where m is the mass of the particle and β defines the temperature parameter in this thermal-emission form $kT=\beta^{-1}$. The most straightforward way in which to look for event-by-event fluctuations in this parameter is as follows. Take a given class of events defined[9] by having n charged[10] pions measured in the central region ($y^*\sim 0$), where n is sufficiently large so that a transverse-momentum distribution can be formed for each event. Fitting each distribution to the form in Eq. (1), one obtains a temperature parameter for *each* event. The dispersion divided by the average for this parameter is then given by

$$\left(\frac{\langle T^2\rangle}{\langle T\rangle^2}-1\right)^{1/2}=(a-1)^{1/2}\,, \tag{2}$$

where the averages are performed over all events. Of course, added to this is a statistical fluctuation, which can be isolated because it must go down as $1/\sqrt{n}$. If essentially a single temperature is relevant for all events, then $a\to 1$. If the temperature fluctuates significantly, for *example*, with impact parameter, the quantity in Eq. (2) can be as large as the similar quantity for the impact parameter if the production of a given class n receives significant contributions from many impact parameters.[11] The dispersion (divided by the average) of the impact parameter can be obtained from the standard impact-parameter resolution of the inelastic cross section[12]; for example, for pp collisions it is about 0.5, which is about the same as the dispersion divided by the average for the charged-prong multiplicity distribution.[11] This procedure for studying event-by-event fluctuations requires a large sample of very-high-multiplicity events (with say $n \gg 10$). The study of such events in this manner may become possible at the highest-energy accelerators, in conjunction with the refined detectors in operation.[13]

We have considered a second procedure for examining the question of fluctuations, one which can readily be applied to much existing bubble-chamber data.[4,5] Consider again a given class of events defined by having the transverse momentum measured for n charged pions (with $y^* \sim 0$). Compute the variable $x = \langle p_T \rangle$ *for each event.* Plot the number of events $N(x_0)$ in some interval $(x_0 - \Delta) \leqslant x \leqslant (x_0 + \Delta)$ versus x_0. Is this a broad curve? Once again, the answer to this question may be reached by calculating the quantity $(D/\langle x \rangle)$ from averaging over all events, where D is the dispersion $D = (\langle x^2 \rangle - \langle x \rangle^2)^{1/2}$. If there is a single temperature for all events, then one expects, with neglect of all kinematic constraints on these central pions,[14] simply

$$\left(\frac{D}{\langle x \rangle}\right)_n = \frac{(c_n - 1)^{1/2}}{\sqrt{n}}\,, \tag{3}$$

where

$$\begin{aligned} c_n = c(\beta_n) &= \frac{\int d^2\vec{p}_T\, p_T^{\,2} \exp[-\beta_n(p_T^{\,2} + m_\pi^{\,2})^{1/2}] \int d^2\vec{p}_T \exp[-\beta_n(p_T^{\,2} + m_\pi^{\,2})^{1/2}]}{\{\int d^2\vec{p}_T\, p_T \exp[-\beta_n(p_T^{\,2} + m_\pi^{\,2})^{1/2}]\}^2} \\ &= \frac{4}{\pi}\, \frac{K_{5/2}(\beta_n m_\pi) K_{3/2}(\beta_n m_\pi)}{K_2^{\,2}(\beta_n m_\pi)}\,. \end{aligned}$$

We have assumed that the measured single-pion distribution within any class n will turn out to be approximately described by a fit to the thermal form

$$\frac{d^2\sigma}{d^2\vec{p}_T} \propto \exp[-\beta_n(p_T^{\,2} + m_\pi^{\,2})^{1/2}]\,, \tag{4}$$

where β_n^{-1} is the effective temperature parameter for that class.[15] On the other hand, if there are significant fluctuations in the temperature, event by event, then Eq. (3) is changed to[14]

$$\left(\frac{D}{\langle x \rangle}\right)_n \cong \left(\frac{(c_n - 1)}{n} a_n + (a_n - 1)\right)^{1/2}, \tag{5}$$

where $a_n = (\langle T^2 \rangle / \langle T \rangle^2)_n$. The quantity $(a_n - 1)^{1/2}$ can thus be extracted from the data and is a measure of the temperature fluctuations (in the class n). This may be compared with a similar quantity for the impact parameter or, more directly, with the dispersion divided by the average for the charged-prong multiplicity distribution.[11] The physical implication of Eq. (5) is clear: Events with very large n have little fluctuation in the average $\langle p_T \rangle$ *unless* the temperature fluctuates significantly event by event.

We have applied a form of the above procedure to an analysis of the temperature fluctuations in several channels of $\bar{p}n$ annihilations to pions at 5.55 GeV/c. We obtain values for $(a - 1)^{1/2}$ of 0.25–0.33. These can be compared with the dispersion divided by the average for the charged-prong multiplicity distribution for $\bar{p}p$ annihilations at 6.9 GeV, which has been estimated[16] to be 0.30 $\pm$0.05. We thus have an indication of significant event-by-event fluctuations in the temperature. In Table I, we list the reactions in which the charged-pion momenta have been measured,[17] with the number of events. In these relatively low-energy annihilation reactions, leading particle effects are small and we have therefore fit the full center-of-mass momentum distribution for a single charged pion within each reaction (class m) to the form

$$\frac{d^3\sigma_m}{d^3\vec{p}^*} \propto \exp\{-\beta_m[(p^*)^2 + m_\pi^{\,2}]^{1/2}\}\,. \tag{6}$$

The temperature parameters[15] and χ^2 are tabulated in Table I. The quality of the fits is general-

TABLE I. The reactions, number of events, effective temperatures, and χ^2 per degree of freedom for $\bar{p}n$ annihilations at 5.55 GeV/c. kT is defined by $d^3\sigma/d^3\vec{p}^* \propto \exp\{-[(p^*)^2 + m_\pi^{\,2}]^{1/2}/kT\}$.

Reactions	Number of events	kT (MeV)	χ^2/DF
$\bar{p}n \to \pi^+ 2\pi^- \pi^0$	392	291 ± 22	1.47
$\bar{p}n \to \pi^+ 2\pi^- X^0$, $X^0 \geqslant 2\pi^0$	1089	156 ± 6	1.20
$\bar{p}n \to 2\pi^+ 3\pi^-$	228	235 ± 15	1.16
$\bar{p}n \to 2\pi^+ 3\pi^- \pi^0$	1348	174 ± 2	3.37
$\bar{p}n \to 2\pi^+ 3\pi^- X^0$, $X^0 \geqslant 2\pi^0$	4403	131 ± 2	1.95
$\bar{p}n \to 3\pi^+ 4\pi^-$	67	151 ± 15	1.68
$\bar{p}n \to 3\pi^+ 4\pi^- \pi^0$	173	129 ± 8	0.72
$\bar{p}n \to 3\pi^+/4\pi^- X^0$, $X^0 \geqslant 2\pi^0$	245	102 ± 2	1.66
$\bar{p}n \to \pi^\pm X$, X= pions	7645	144 ± 3	1.62

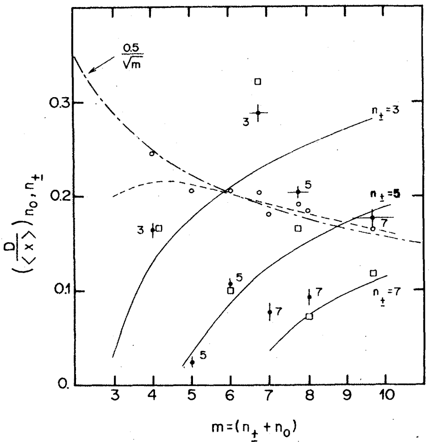

FIG. 1. The variable x is the mean magnitude of the center-of-mass momentum for the charged pions in each event and D is the dispersion of this quantity, where the averages are taken over all events in a given reaction class. The interpolated curves labeled by $n_\pm$ are the Monte Carlo results described in the text when $n_\pm$ charged pions are measured with $n_0=(m-n_\pm)$ neutrals undetected. The dashed curve is the Monte Carlo result for $(c_m-1)^{1/2}/\sqrt{m}$. The black circles are the experimental points and are labeled by $n_\pm$. This label also applies to the open circles (experimental) and open squares (described in the text) appearing at the same value of m.

ly good. With averages taken over all events within a given class m, we calculate from the data the quantity $(D/\langle x\rangle)_m$ on the left side of Eq. (5), where the variable x is now the mean magnitude of the center-of-mass momentum for the measured (i.e., charged) pions in each event. These points are shown as black circles in Fig. 1. Also shown, as open circles, is

$$\frac{\{[\langle(p^*)^2\rangle/\langle p^*\rangle^2]_m-1\}^{1/2}}{\sqrt{m}}=\frac{(c_m-1)^{1/2}}{\sqrt{m}}$$

[i.e., the quantity from Eq. (3)], as calculated from the measured overall single-charged-pion momentum distribution within each class m. Clearly, for a class in which all the pions are charged and hence measured, with neglect of the pion mass the mean momentum is simply proportional to the total energy and the dispersion $D\to 0$. For a class in which one or more neutrals are not measured, the approximate generalization of the right side of Eq. (5) which takes account of energy conservation only (and neglects the pion mass) in a class m with $n_\pm$ charged pions and $n_0=(m-n_\pm)$ neutrals is

$$\left(\frac{D}{\langle x\rangle}\right)_{n_0,n_\pm}\cong\left[\frac{n_0}{n_\pm}\,\frac{(c_m-1)}{n_\pm}a_m+\left(\frac{n_0}{n_\pm}\right)^2(a_m-1)\right]^{1/2}. \tag{7}$$

Here c_m is determined by the fit β_m in Eq. (6), similarly to Eq. (3),

$$c_m=\frac{3\pi}{8}\,\frac{K_2(\beta_m m_\pi)K_3(\beta_m m_\pi)}{K_{5/2}{}^2(\beta_m m_\pi)} \tag{8}$$

and a_m is the fluctuation parameter to be extracted from the experimental data. The generalization in Eq. (7) is best for large m and n_0. For most of the range of temperature parameters tabulated in Table I, $(c_m-1)^{1/2}\sim 0.5$ (Fig. 1); so for $a_m\to 1$,

$$(D/\langle x\rangle)_{n_0,n_\pm}\to(n_0/n_\pm)^{1/2}(0.5/\sqrt{n_\pm})$$

[instead of Eq. (3)], with only the constraint of approximate energy conservation. These points are shown as open squares in Fig. 1. To estimate the effect of the constraints of exact energy and momentum conservation we have calculated interpolated curves for the quantity on the left side of Eq. (7) from Monte Carlo events generated according to Lorentz-invariant phase space. These curves are shown in Fig. 1 labeled by the number of measured charged pions. When this number is the total number of pions in the reaction the dispersion approaches zero as expected.[18] Also, as expected by the additional constraint of momentum conservation, the Monte Carlo curves lie below the open squares. From the Monte Carlo events we have also calculated the quantity corresponding to $(c_m-1)^{1/2}/\sqrt{m}\sim 0.5/\sqrt{m}$, when $n_\pm$ are measured and $n_0=(m-n_\pm)$ are undetected, and the values are given by the interpolated dashed curve in Fig. 1. From comparison with the open circles we see that for this quantity, Lorentz-invariant phase space fairly closely parallels the distributions in Eq. (6). The effective value of n_0 has been estimated[19] for the reactions with $n_0\geq 2$. For the six reactions with $n_0\geq 1$, the experimental points lie above the Monte Carlo curves. We are thus led to calculate a value for the fluctuation parameter a_m for each reaction using the formula

$$(D/\langle x\rangle)^{\exp}_{n_0,n_\pm}=[A_m{}^2a_m+(n_0/n_\pm)^2(a_m-1)]^{1/2}, \tag{9}$$

where A_m denotes the expected value when $a_m\to 1$ from the relevant Monte Carlo curve with exact energy-momentum conservation. The calculated values of a_m are given in Table II for each reaction. But for the class with low charged multiplicity ($\pi^+2\pi^-X^0$), where there has been the greatest experimental difficulty[20] in separating the annihilation events, a significant event-by-event fluctuation in the temperature is consistent with the data. The quantity $(a_m-1)^{1/2}\sim 0.25$–0.33 is comparable to the dispersion divided by the average for the charged-prong multiplicity distribution from annihilation events at a similar energy[16] 0.3 ± 0.05, and this latter quantity gives an estimate of the dispersion divided by the average of the impact parameter.[11,12] We consider this only an initial indication that event-by-event fluctu-

TABLE II. The fluctuation parameter $(a-1)^{1/2}$, where $a=(\langle T\rangle^2/\langle T\rangle^2)$, for each reaction.

Reactions	$(a_m-1)^{1/2}$
$\bar{p}n \to \pi^+2\pi^-\pi^0$	0.27 ± 0.04
$\bar{p}n \to \pi^+2\pi^-X^0$, $X^0 \geq 2\pi^0$	0.14 ± 0.06
$\bar{p}n \to 2\pi^+3\pi^-\pi^0$	0.28 ± 0.02
$\bar{p}n \to 2\pi^+3\pi^-X^0$, $X^0 \geq 2\pi^0$	0.25 ± 0.03
$\bar{p}n \to 3\pi^+4\pi^-\pi^0$	0.33 ± 0.06
$\bar{p}n \to 3\pi^+4\pi^-X^0$, $X^0 \geq 2\pi^0$	0.33 ± 0.07

ations in the temperature parameter occur, because of the limited charged and neutral multiplicities, and the restrictive constraint of energy conservation, in the annihilation reactions that we have analyzed. In particular, the data points for[11] $n_0=1$ are largely consistent with Eq. (7) with $a_m=1$ (Fig. 1). We have taken all of the measured charged pions and have not cut on the data; part of the fluctuation effect could be due to slight leading particle effects which are not of thermal origin. Finally, there is also the fact that annihilation is probably a relatively small impact-parameter process with relatively little fluctuation in impact parameter, as reflected in the small dispersion divided by average for the charged-prong multiplicity distribution, as compared to that for pp collisions, for example.

We emphasize that, quite apart from the hypothetical connection with impact parameter, possible nonstatistical fluctuations in the temperature parameter on an event-by-event basis are worth searching for. Event-by-event analyses of the above type looking for fluctuations in the temperature parameter can be carried out on much of the existing moderate-energy bubble-chamber data.[4,5] With current experiments at higher energies, samples of events[13] with very high multiplicities will occur and the direct analysis of individual events discussed before Eq. (2) may be possible. Clearly, it would be valuable to perform such analyses on reactions which are less phase spacelike than antinucleon-nucleon annihilation into pions and to select event samples of centrally produced pions in these reactions. Such studies could give new insight into the origin and nature of thermal-emission spectra in hadronic collisions, in particular, concerning the possible applicability to particle collisions of the general connection[2,3] between temperature and thermal-like emission and acceleration.[21] The possibility of attaining much higher temperatures in certain types of collisions[1] then follows.

One of us (S.B.) wishes to thank W. Troost for helpful discussions and the experimentalists in the hadron-physics bubble-chamber groups in Aachen, Amsterdam, and Brussels for their interest and comments on this type of analysis.

[1]S. Barshay and W. Troost, Phys. Lett. 73B, 437 (1978).
[2]W. G. Unruh, Phys. Rev. D 14, 870 (1976).
[3]W. Troost and H. Van Dam, Phys. Lett. 71B, 149 (1977).
[4]M. Deutschmann *et al.*, Nucl. Phys. B70, 189 (1974).
[5]J. Bartke *et al.*, Nucl. Phys. B120, 14 (1977).
[6]K. Guettler *et al.*, Phys. Lett. 64B, 119 (1976).
[7]S. Barshay and Y. A. Chao, Phys. Lett. 38B, 225 (1972); 38B, 229 (1972).
[8]We use this simple form although the Bose-Einstein form also yields good fits. See Ref. 4.
[9]We thank Dr. P. Sixel, in Aachen, for this suggestion.
[10]In practice, the neutrals are usually not well measured, but if they are the generalization of the suggested analyses is obvious.
[11]The broad multiplicity distribution observed in multipion production at high energies may be due to the compounding of many relatively narrow distributions, one from each contributing impact parameter. Thus, production of a given total number of pions (charged plus neutral) in the central region may occur from a limited range of impact parameters with a correspondingly small dispersion in the temperature parameter. However, within a class of events defined by having n charged pions in the central region, different numbers of neutrals occur and hence a wider range of impact parameters contribute.
[12]S. Barshay, Phys. Lett. 42B, 457 (1972).
[13]Such experiments are underway according to P. Sixel in Aachen.
[14]In this formula we have not considered the effect of energy-momentum conservation on these pions. Monte Carlo calculations similar to those described in the analysis performed below can be carried out to estimate the effect of these constraints.
[15]What is important here is the quality of the overall fit (all events together) to the thermal form, not the specific values of β_n^{-1}. These values, which are given in Table I, are essentially related through $\beta_n^{-1} \sim (\langle n\rangle/n)$ (150 MeV) where the first factor, which dominates the variation, is kinematic, with n the total number of produced pions and $\langle n\rangle$ the average (~ 7.2).
[16]S. Barshay, A. Fridman, and P. Juillot, Phys. Rev. D 15, 2702 (1977).
[17]Details of the measurements and further results are in H. Braun *et al.*, Phys. Rev. D 20, 587 (1979).
[18]We have included in the Monte Carlo calculations the spread in the energy arising from the neutron Fermi motion in the deuteron target.
[19]The mean number of neutral pions n_0 in no-fit events ($X^0 \geq 2$) has been estimated assuming the total pion

multiplicity given by $\sqrt{s}/(\langle p_\pm^*\rangle^2+m_\pi^{\,2})^{1/2}$, as it is observed in four- and one-constraint annihilation channels with $n_0=0$ and 1, respectively.

[20]The ambiguities in particle identification (π^- and $\bar{p}$) increase at low multiplicities where the competition with nonannihilation channels is high.

[21]A recent result from multipion production in K^+p interactions at 32 GeV/c gives a possible indication of the increase in the hadronic temperature with decreasing impact parameter. A good fit of the inclusive pion transverse-momentum distribution in the central region to the form in Eq. (1) yields a temperature of 157 ± 10 MeV, which is significantly larger than that for π^+p interactions at 16 GeV/c, 111 MeV (see Table II in Ref. 4). This is consistent with the empirical fact that the K^+p diffraction slope [~ 6 (GeV/c)$^{-2}$] is significantly less than that in π^+p scattering and hence a smaller average impact parameter is involved in K^+p inelastic scattering. We thank Dr. E. de Wolf and Dr. F. Verbeure in Brussels for informing us of this result prior to publication.