

Tests and predictions of the modified cascade model for hadron-, photon-, and lepton-nucleus collisions

Monowar Hossain and Don M. Tow

Center for Particle Theory, Department of Physics, University of Texas, Austin, Texas 78712

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The recently proposed space-time model, the modified cascade model, for hadron-nucleus collisions is investigated further and also extended to apply to photon- and lepton-nucleus collisions. Dependences of the multiplicity on incident particle, nucleus, rapidity, and energy, as well as the A dependences of the total inelastic cross sections, are calculated with the model's only free parameter fixed by a normalization as determined in our previous paper. Our results are compared and found to be in agreement with published experimental data from p , π , K , μ , and ν nucleus collisions. Several representative predictions of the model are also presented.

I. INTRODUCTION AND SUMMARY OF MODEL

Recently Bialkowski, Chiu, and Tow (BCT) formulated a new space-time model for high-energy hadron-nucleus collisions.¹ This is basically a multiple-scattering or cascade model because the produced secondaries can interact with subsequent nucleons in the nucleus, but modified by the important concept of immaturity. The immaturity concept says that particles at the moment of production are bare or immature, i.e., they have small inelastic collision probability, and after a certain characteristic time (in their own rest frames) they become dressed or mature (spontaneous maturity), and this maturity process is enhanced in the presence of other hadronic matter (induced maturity). Physically one can picture that a particle at the moment of creation is not accompanied by a quark-antiquark sea or gluon cloud (or meson cloud in more traditional language). After a certain characteristic time, they are dressed up with this $(q\bar{q})$ sea or gluon cloud. This dressing up process is enhanced in the presence of other hadronic matter where there is an abundance of $(q\bar{q})$ sea or gluon cloud. We call this model the modified cascade model.

In the modified cascade model, the incident particle, after averaging over impact parameter, encounters an average number of $\bar{K} \approx 0.8 A^{1/3}$ nucleons in a nucleus with a large number A of nucleons. The value of \bar{K} is the same for all incident particles. However, the probability P_i that an incident particle i collides inelastically with a nucleon depends on the type of incident particle; Ref. 1 used the disk approximation to relate P_i to σ_T and σ_{el} . One then uses the hadron-nucleon multiplicity rapidity distribution as input and calculates the hadron-nucleus multiplicity rapidity distribution as output. The model has one free parameter λ , the "mean induced maturity path," which was fixed

at $\lambda = 2.4$ by normalizing the model calculation to the 200-GeV proton-nucleus data for one heavy nucleus.² The parameter-free model is then used to calculate the dependences of the multiplicity on incident particle, nucleus, rapidity, and energy, as well as the A dependences of σ_{inel}^{hA} .

In Ref. 1, BCT provided plausibility arguments for the immaturity concept, formulated the one-parameter model in a quantitative manner, calculated the results for p and π nucleus collisions, and found good agreement with experimental data.^{3,4}

Since then kaon and more p and π nucleus data,⁵ as well as ν - ($\bar{\nu}$ -) Ne data,⁶ have been published. Furthermore, because of the present availability of high-energy hadron, photon, and lepton beams, more such data should become available in the near future. In this paper we calculate several new hadron-nucleus results of the BCT model and compare with the new data of Ref. 5, as well as present several representative predictions. Furthermore, we extend the BCT model to apply to photon- and lepton-nucleus collisions, compare with the new results of Ref. 6, and present several representative predictions for γ , μ (e), and ν nucleus collisions.

The new results for hadron-nucleus collisions are presented in the next section. The extension and results for photon- and lepton-nucleus collisions are discussed in Sec. III. In Sec. IV we end with a short conclusion and mention some additional interesting applications of the modified cascade model.

II. NEW RESULTS FOR HADRON-NUCLEUS COLLISIONS

Here we present the results of the model for KA collisions as well as some more results for pA and πA collisions. Some of these results have data⁵ to be compared with, and some are predic-

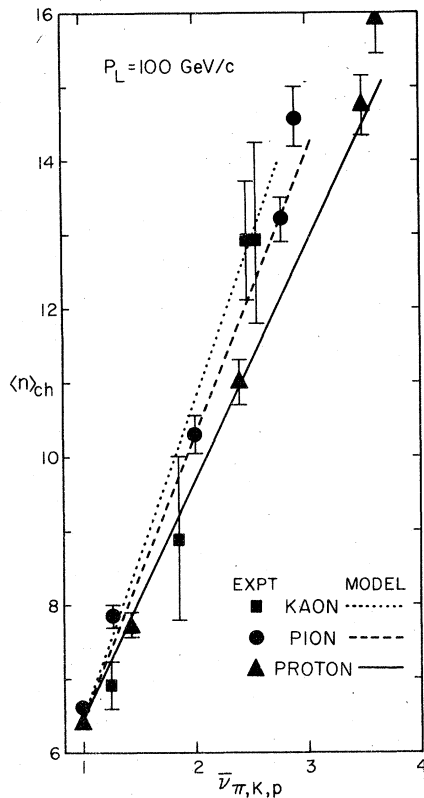


FIG. 1. $\langle n \rangle_{\text{ch}}$ vs $\bar{\nu}_i$ for K , π , and p nucleus collisions at $P_L = 100 \text{ GeV}/c$. Data points are from Ref. 5. To avoid overcrowding at $\bar{\nu} = 1$, the kaon point was not drawn; its value is very close to that of the pion, but with error bars four times larger.

tions to be checked by future data.

The input inelastic differential multiplicity distribution for KN collisions is the same as that for pN and πN collisions.¹ The inelastic collision probability P_K as determined from the $\sigma_{\text{inel}}^{Kp}$ and σ_{ol}^{Kp} data⁷ is $P_K \approx 0.44$. The same data give $P_{p(\bar{p})} \approx 0.60$ and $P_{\pi} \approx 0.49$.⁸ Therefore, for the same A and energy, the multiplicity is largest for $p(\bar{p})A$ and smallest for KA .

Experimentalists like to present their results in terms of the variable $\bar{\nu}_i$ instead of the variable A , where $\bar{\nu}_i \equiv A\sigma_{\text{inel}}^{ip}/\sigma_{\text{inel}}^{iA}$ with i being the incident particle. In Fig. 1, the average charged multiplicity $\langle n \rangle_{\text{ch}}$ as a function of $\bar{\nu}_i$ is plotted for p , π , and K induced reactions at $P_L = 100 \text{ GeV}/c$. At high energy, the model does not distinguish particle from antiparticle and gives the same results for p and \bar{p} , K^+ and K^- , π^+ , and π^- . The data points are the p , π^+ , and K^+ data of Ref. 5. In Fig. 2, the pseudorapidity multiplicity distribution ratio (normalized by the hadron-proton distribution) $r(\eta)$ is plotted for p , π^+ , and K^+ induced reactions

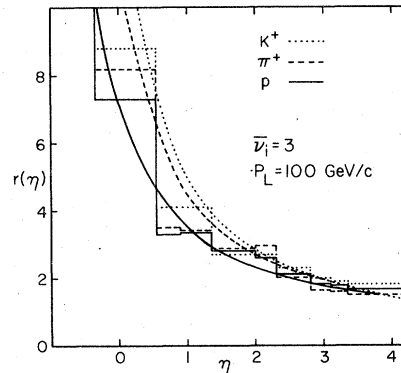


FIG. 2. Differential multiplicity ratio $r(\eta)$ vs pseudorapidity η for K , π , and p nucleus collisions with $\bar{\nu}_i = 3$ at $P_L = 100 \text{ GeV}/c$. Histograms are from Ref. 5. Predictions for π and K are the same for $3.0 < \eta < 4.3$ and all three cases coincide for $3.8 < \eta < 4.3$.

for $\bar{\nu}_i = 3$ at $P_L = 100 \text{ GeV}/c$. The histograms are the data of Ref. 5.⁹ In both cases, the results of the model are in reasonable agreement with the data, although better data are needed to reach a conclusive answer.

The A dependence of $\sigma_{\text{inel}}^{KA}$ in the modified cascade model is energy independent at high-energy and is given by¹⁰

$$\sigma_{\text{inel}}^{KA} = C_K A^{2/3} [1 - (1 - P_K)^{\bar{K}}], \quad (2.1)$$

where C_K is a constant independent of A . The result is plotted in Fig. 3 and may be parametrized as $\sigma_{\text{inel}}^{KA} \propto A^{0.76}$; the data points are for K^+A at $P_L = 100 \text{ GeV}/c$ and are inferred from the $\bar{\nu}_{K^+}$ values given in Ref. 5 and $\sigma_{\text{inel}}^{K^+p}$ of Ref. 7. The agreement is good.

To provide further tests of this model, we present in Fig. 4 the A dependences of $\langle n \rangle_{\text{ch}}$ for $p(\bar{p})$, π^{\pm} , and K^{\pm} nucleus collisions for $P_L = 50$, 400, and 1500 GeV/c .

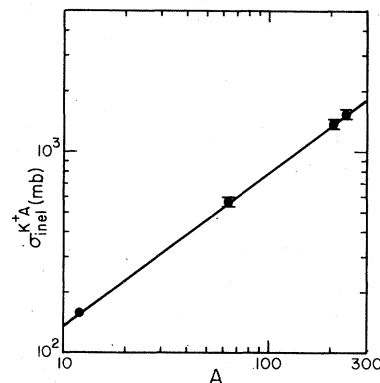


FIG. 3. A dependence of $\sigma_{\text{inel}}^{KA}$. Points are 100 GeV/c K^+A data inferred from Refs. 5 and 7.

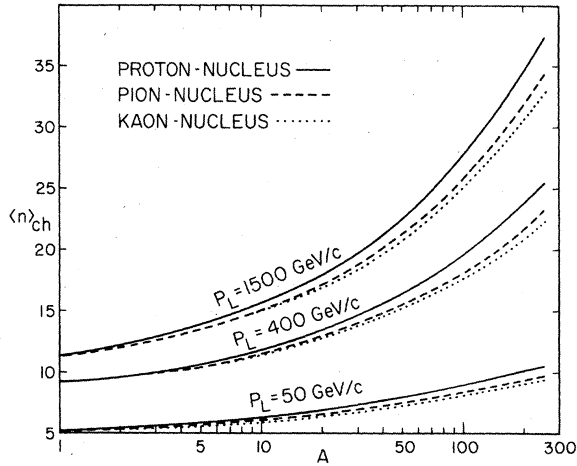


FIG. 4. Predictions for the A dependences of $\langle n \rangle_{\text{ch}}$ for p (\bar{p}), π^{\pm} , and K^{\pm} nucleus collisions at $P_L = 50, 400,$ and $1500 \text{ GeV}/c$.

III. EXTENSION TO PHOTON- AND LEPTON-NUCLEUS COLLISIONS

If we assume vector-meson dominance of the photon, then the model can easily be applied to γA collisions. The inelastic collision probability P_{γ} is determined from $\sigma_T^{\gamma p}$ (Ref. 11) and $\sigma^{\gamma p \rightarrow \rho p}$ (Ref. 12) and has the approximate value of $P_{\gamma} \approx 0.3$. The $\gamma p \rightarrow \pi^{\pm} X$ inclusive distribution normalized to σ_{inel} has approximately the same transverse momentum and rapidity distributions as those from $pp \rightarrow \pi^{\pm} X$ except that the normalization is larger by roughly a factor of 1.2.¹³ Making these two changes allows us to calculate¹⁴ $\langle n \rangle_{\text{ch}}$, dn_{ch}/dy , and $\sigma_{\text{inel}}^{\gamma A}$. Figure 5 shows the model's predictions for the A dependences of $\langle n \rangle_{\text{ch}}$ for $P_L = 10, 50,$ and $200 \text{ GeV}/c$.

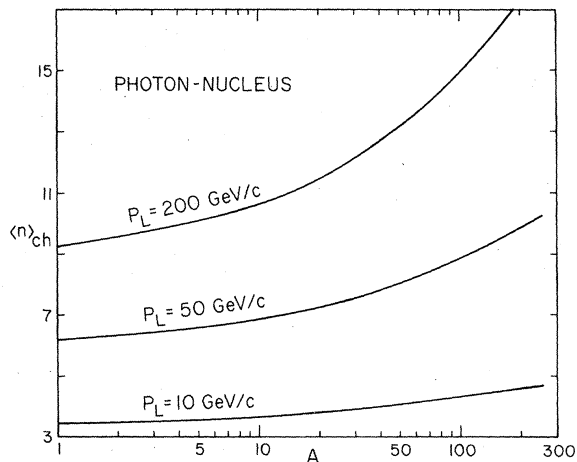


FIG. 5. Predictions for the A dependences of $\langle n \rangle_{\text{ch}}$ for γA at $P_L = 10, 50,$ and $200 \text{ GeV}/c$.

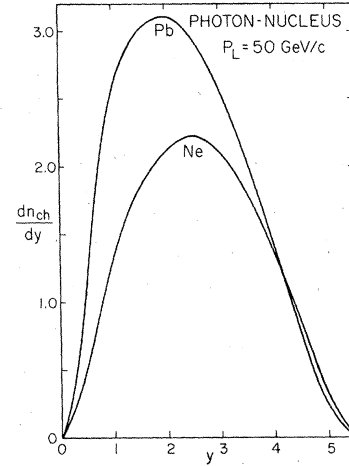


FIG. 6. Predictions for the differential multiplicity dn_{ch}/dy vs rapidity y for $\gamma\text{-Ne}$ and $\gamma\text{-Pb}$ at $P_L = 50 \text{ GeV}/c$.

200 GeV/c . Figure 6 shows the model's predictions for dn_{ch}/dy for Ne and Pb nuclei at $P_L = 50 \text{ GeV}/c$. Figure 7 shows the A dependence of $\sigma_{\text{inel}}^{\gamma A}$ at high energy and may be approximately parametrized by $\sigma_{\text{inel}}^{\gamma A} \propto A^{0.85, 15, 16}$. We caution that our input for γ -initiated reactions has larger experimental uncertainties and is based on relatively low-energy data; furthermore, the model as now formulated assumes ρ dominance of the photon. All of these will result in larger uncertainties for our γA results as compared to our hadron-nucleus results.

The extension to $\mu^{\pm} A$ (and $e^{\pm} A$) collisions is related to that of γA collisions. One can assume that once the incident μ^{\pm} (e^{\pm}) interacts with one of the nucleons in the nucleus, the outgoing μ^{\pm} (e^{\pm}) does not interact again with the subsequent nucleons, because these higher-order terms are down by factors of the fine-structure constant α . The virtual-photon-nucleon multiplicity distribu-

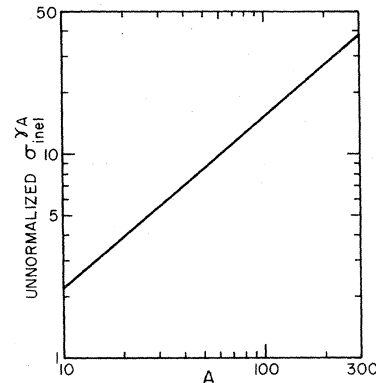


FIG. 7. Prediction for the A dependence of $\sigma_{\text{inel}}^{\gamma A}$ at high energy. Note that the vertical scale is in arbitrary units.

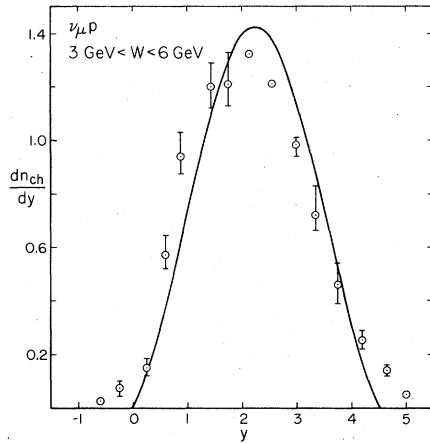


FIG. 8. Input parametrization of dn_{ch}/dy for $\nu_{\mu}p$ for $3 \text{ GeV} < W < 6 \text{ GeV}$. Data points are from Ref. 22.

tion can be approximated by the real-photon-nucleon multiplicity distribution at least for small Q^2 (mass squared of virtual photon) and most likely also for large Q^2 as present $e(\mu)N$ and $\nu(\bar{\nu})p$ multiplicity data indicate no or very little dependence on Q^2 .^{17,18} There are indications that there is less nuclear shadowing as the photon becomes more virtual.¹⁹ Within our model this would mean that P_{γ} decreases with increasing Q^2 , which is supported by the data.²⁰ However, because the quantitative behavior of this change is not clear at present and because it also seems to depend not only on Q^2 but also on W (the center-of-mass energy), we have not incorporated this change of P_{γ} with increasing Q^2 . Therefore, subject to the qualifying remark of a possible change of P_{γ} with Q^2 , $\mu^{\pm}A$ (and $e^{\pm}A$) collisions are just like A collisions with the obvious correspondence of $W_{\gamma N}$

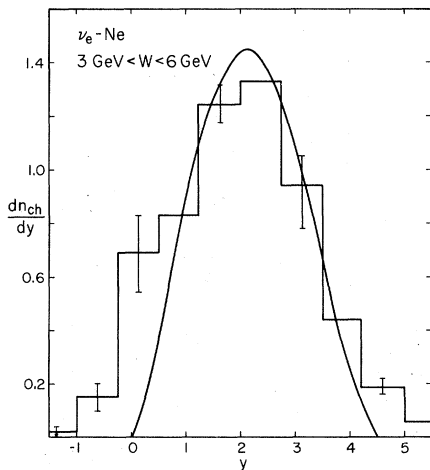


FIG. 9. dn_{ch}/dy for ν_e -Ne for $3 < W < 6 \text{ GeV}$. Histograms are from Ref. 6.

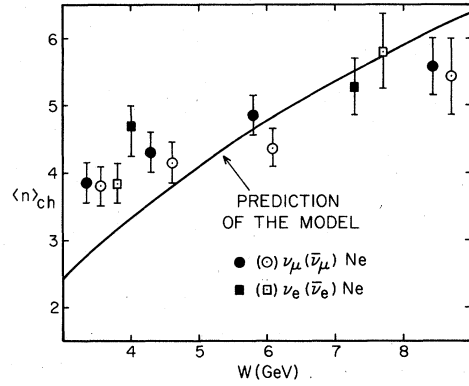


FIG. 10. $\langle n \rangle_{ch}$ vs W for neutrino-neon collisions. Data points are from Ref. 6.

$-W_{\gamma N}$. Figures 5–7 are then also applicable for $\mu^{\pm}A$ (and $e^{\pm}A$) collisions.

As an illustration of the μA results, consider A to be emulsion and $W = 10 \text{ GeV}$ (corresponding to $E_L \approx 55 \text{ GeV}/c$ for the virtual photon), where W is the c.m. energy in the virtual photon-nucleon system. Figure 5 gives for the multiplicity ratio R_A (normalized to μp data) a value of $R_A \approx 1.35$.²¹

For $\nu(\bar{\nu})A$ collisions, one does not have vector-meson dominance and therefore one does not know how to calculate the inelastic collision probability P_{ν} . However, because the neutrino (or, equivalently, the intermediate vector bosons) does not have any hadronic structure and interacts weakly, it is reasonable to assume that $P_{\nu} \ll 1$. Also, analogous to $\mu^{\pm}A$ collisions, once the incident ν interacts with one of the nucleons in the nucleus, the outgoing μ^- or ν (corresponding to charged or neutral current) does not interact again with the subsequent nucleons, because these higher-order terms are down by factors of α or G_w , the weak coupling.

Once one assumes that $P_{\nu} \ll 1$, then one can

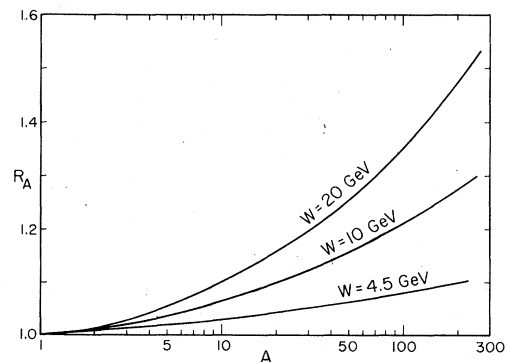


FIG. 11. Predictions for the A dependences of the integrated multiplicity ratio R_A for νA at $W = 4.5, 10,$ and 20 GeV .

easily show that the multiplicity is independent of P_ν . This is because the neutrino interacts inelastically only once, so one factor of P_ν appears in the numerator, but the inelastic probability factor which appears in the denominator as a normalization is proportional to P_ν when $P_\nu \ll 1$. We have numerically confirmed this independence when $P_\nu \lesssim 0.1$.

The model makes no distinction between ν_e and ν_μ . Only very-low-energy data are currently available. Figure 8 shows our input parametrization²² for dn_{ch}/dy for $\nu_\mu p$ for $3 < W < 6$ GeV, where W is the hadronic invariant mass. Figure 9 shows dn_{ch}/dy for ν_e -Ne for $3 < W < 6$ GeV. Figure 10 shows $\langle n \rangle_{\text{ch}}$ versus W for neutrino-neon collisions. For the last two figures the data are from Ref. 6. Except for the very-low-energy points in Fig. 10, the model is consistent with the data.²³ Figure 11 shows the prediction of the model for the A dependences of the integrated multiplicity ratio R_A (normalized to νp data) for νA collisions for $W=4.5, 10, 20$ GeV. Figure 12 shows the prediction for dn_{ch}/dy for ν -Ne collisions for $W=10$ GeV. Note that as remarked above the multiplicity is independent of P_ν , so a possible change of P_ν with Q^2 has no effect.

The A dependence (but not the normalization) of $\sigma_{\text{inel}}^{\nu A}$ is also independent of P_ν , because Eq. (2.1) gives

$$\sigma_{\text{inel}}^{\nu A} \propto P_\nu \bar{K} A^{2/3} \propto A, \quad (3.1)$$

which is just the no-shadowing result. The only data which can check Eq. (3.1) that we came across are some old low-energy ($E_\nu < 12$ GeV) data on propane (C_3H_8) and freon (CF_3Br) which showed that (3.1) is satisfied to about 35% accuracy.²⁴

Because of the no-shadowing result of Eq. (3.1),

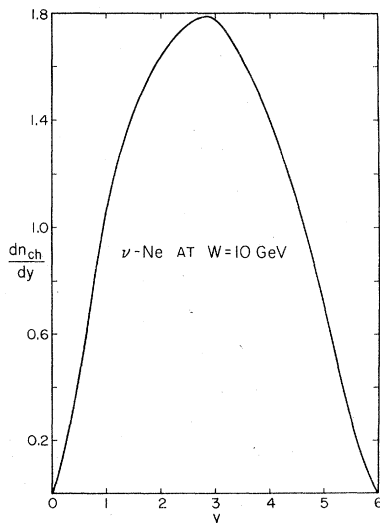


FIG. 12. Prediction for dn_{ch}/dy for ν -Ne at $W=10$ GeV.

$\bar{\nu}_\nu \equiv A\sigma_{\text{inel}}^{\nu p}/\sigma_{\text{inel}}^{\nu A}$ is expected to be unity.²⁵ This, however, does not mean that $R_A=1$, as can be seen from Fig. 11. This is contrary to the claim²⁶ that any multiple-scattering model must have $R_A=1$ for νA scattering. The reason this claim is false is that even though the ν scatters inelastically only with one nucleon of the nucleus, if it scatters with any nucleon other than the last nucleon, the produced secondaries can cascade and so results in $R_A > 1$.

IV. CONCLUSION

This paper, together with Ref. 1, shows that the modified cascade model, though simple and crude, can provide a unified description of hadron-, photon-, and lepton-nucleus collisions. The model contains only one free parameter which is fixed by normalization. The model gives then a consistent description of a surprisingly large amount of experimental results.

A crucial ingredient in the model is the immaturity concept. Many years ago, Feinberg,²⁷ basing his work on some earlier ideas of Landau and Pomeranchuk,²⁸ showed that in quantum electrodynamics if an electron underwent an interaction shortly after it underwent an inelastic interaction, then it is less likely to emit bremsstrahlung photons than if it had not undergone the first interaction. Recently, Valanju, Sudarshan, and Chiu have provided some generalization of Feinberg's argument within a general field-theoretical framework and thus provided a possible theoretical argument for the immaturity concept.²⁹ Developments along this line will shed light on the theoretical basis of the modified cascade model.

A natural question that arises is whether the model can describe the interesting large- P_T behavior found in hadron-nucleus collisions.³⁰ This problem is currently under investigation.

An extremely interesting application of the model is whether there are any astrophysical implications involving matter with nuclear density, e.g., neutron stars, black holes, the early universe. Perhaps one can test there the effects of the immaturity concept.³¹

ACKNOWLEDGMENTS

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- ¹G. Bialkowski, C. B. Chiu, and D. M. Tow, Phys. Lett. 68B, 451 (1977); Phys. Rev. D 17, 862 (1978).
- ²The formula of $\bar{K} \approx 0.8A^{1/3}$ was obtained in Ref. 1 assuming a constant nuclear density. Using a more realistic Woods-Saxon or Gaussian density increases slightly the value of \bar{K} . Replacing the disk approximation with a distribution in impact parameter may change slightly P_i . Both changes have the approximate net effect of changing slightly the value of the free parameter λ and thus result in essentially no change in the multiplicity distribution. As for the A dependence of σ_{inel}^{iA} , we have numerically checked that it is not very sensitive to small changes in the values of P_i and \bar{K} .
- ³W. Busza *et al.*, in *Proceedings of the XVIII International Conference on High Energy Physics, Tbilisi, 1976*, edited by N. N. Bogolubov *et al.* (JINR, Dubna, U. S. S. R., 1977). An updated version of these data can be found in C. Halliwell *et al.*, Phys. Rev. Lett. 39, 1499 (1977).
- ⁴J. C. Allaby *et al.*, Yad. Fiz. 12, 538 (1970) [Sov. J. Nucl. Phys. 13, 295 (1971)]; W. Busza *et al.*, Phys. Rev. Lett. 34, 836 (1975); S. P. Denisov *et al.*, Nucl. Phys. B61, 62 (1973).
- ⁵J. E. Elias *et al.*, Phys. Rev. Lett. 41, 285 (1978).
- ⁶T. H. Burnett *et al.*, Phys. Lett. 77B, 443 (1978); and *Neutrino-77*, in proceedings of the International Conference on Neutrino Physics and Astrophysics, Baksan Valley, edited by M. A. Markov *et al.* (Nauka, Moscow, 1978), Vol. II.
- ⁷D. S. Ayres *et al.*, Phys. Rev. D 15, 3105 (1977). These data also show that at high energy, there is little difference between particle and antiparticle. Thus, at high energy our model gives the same results for particle and antiparticle. Of course, at lower energies the inelastic collision probabilities that one inputs for particle and antiparticle may not be the same, and the model will give different results for particle and antiparticle.
- ⁸These values differ slightly from those used in Ref. 1 which were based on slightly older data. The calculations in the present paper used the newer data of Ref. 7.
- ⁹One should note that the "experimental histograms" for π^+ and especially for K^+ involve extrapolations of the actual data to $\bar{v}_i = 3$, as the largest \bar{v}_π and \bar{v}_K are, respectively, 2.87 and 2.55.
- ¹⁰For calculating σ_{inel}^{KA} , we have removed the approximation made in Ref. 1 that a hadron once inside the nucleus is always immature even though it may not have interacted inelastically. This is the reason why Eq. (2.1) does not have the $(1 - Q_i)$ factor which appears in Eq. (17) of Ref. 1. We have checked that removing this factor does not change appreciably σ_{inel}^{PA} and σ_{inel}^{TA} and so does not alter the good agreements with data found in Ref. 1. We are currently investigating the effect of removing this simplifying assumption altogether for the calculation of the multiplicity.
- ¹¹A. S. Belousov *et al.*, Yad. Fiz. 21, 556 (1975) [Sov. J. Nucl. Phys. 21, 289 (1975)]; G. Alexander *et al.*, Nucl. Phys. B68, 1 (1974); D. O. Caldwell *et al.*, Phys. Rev. D 7, 1362 (1973). In the energy region of overlap, these three experiments are consistent with each other. Because the highest-energy data for $\sigma^{\gamma p - \rho p}$ that we came across is 9.3 GeV, we used the 9.3-GeV data from Caldwell *et al.*, which gives $\sigma^{\gamma p} = (120 \pm 7) \mu\text{b}$.
- ¹²J. Ballam *et al.*, Phys. Rev. D 7, 3150 (1973); G. E. Gladding and J. J. Russel, *ibid.* 8, 3721 (1973). There may be some discrepancy between these experiments, although the incident energies do not exactly overlap. Furthermore, $\sigma^{\gamma p - \rho p}$ has to be extracted from the data in a model-dependent way. The value of $\sigma^{\gamma p - \rho p} = (11.8 \pm 0.5) \mu\text{b}$ that we used is the result of the Ballam *et al.* parametrization II at 9.3 GeV.
- ¹³W. Kaune *et al.*, Phys. Rev. D 11, 478 (1975). As for the leading-particle effect, real-photon-induced reactions are similar to those of hadron-induced reactions. See, e.g., J. Gandsman *et al.*, Nucl. Phys. B61, 32 (1973). The magnitude of this leading-particle effect decreases as the photon becomes more and more virtual.
- ¹⁴For the photon and lepton calculations when we re-normalize the output spectra by dividing by the inelastic probability factor and when we calculate $\sigma_{inel}^{\gamma A}$, we have also removed the simplifying assumption mentioned in Ref. 10.
- ¹⁵The value of 0.85 is fairly close to the value of 0.9 from relatively low-energy data (Caldwell *et al.*, Ref. 11).
- ¹⁶A recent higher-energy experiment [D. O. Caldwell *et al.*, Phys. Rev. Lett. 42, 553 (1979)] indicates that at least for copper target there is an increase of shadowing with increasing energy. However, more and better data for other nuclear targets are needed before one can conclude that the exponent in the A dependence does decrease with increasing energy. As to the explanation within our model of the increase of shadowing with energy, one possibility is that higher-energy photons can more easily excite higher mass vector mesons.
- ¹⁷For eN : P. H. Garbincius *et al.*, Phys. Rev. Lett. 32, 328, 955 (1974); for μp : C. del Papa *et al.*, Phys. Rev. D 13, 2934 (1976); for νp : J. W. Chapman *et al.*, Phys. Rev. Lett. 36, 124 (1976); for $\bar{\nu} p$: M. Derrick *et al.*, Phys. Rev. D 17, 1 (1978).
- ¹⁸One neglects the small changes resulting from the decrease of the leading-particle effect as the photon becomes more virtual (see Ref. 13).
- ¹⁹M. May *et al.*, Phys. Rev. Lett. 35, 407 (1975); W. R. Ditzler *et al.*, Phys. Lett. 57B, 201 (1975); E. Gabathuler, in *Proceedings of the 6th International Symposium on Electron and Photon Interactions at High Energies, Bonn, Germany, 1973*, edited by H. Rollnick and W. Pfeil (North-Holland, Amsterdam, 1974); J. E. Eickmeyer *et al.*, Phys. Rev. Lett. 36, 289 (1976).
- ²⁰K. C. Moffeit, in *Proceedings of the 6th International Symposium on Electron and Photon Interactions at High Energies, Bonn, Germany, 1973*, edited by H. Rollnick and W. Pfeil (North-Holland, Amsterdam, 1974).
- ²¹A recent Cornell-Fermilab-Krakow-MSU-Seattle μ^+ -emulsion experiment [L. Hand *et al.*, Acta Phys. Pol. (to be published)] with $\langle W \rangle = 11.1$ GeV and an experimental cut on the number of heavy tracks of $N_h \geq 3$ gives a value of $R_A = 1.47 \pm 0.14$. One expects that removing the above experimental cut ($\langle N_h \rangle = 10.2 \pm 0.7$ in this experiment) will result in an experimental value somewhat smaller than the value given above. We thank Dr. R. J. Wilkes for communicating this result to us and for an informative discussion.

- ²²See the second paper of Ref. 6. Except for the absence of the leading-particle effect, this parametrization is very similar to that used for hadron-initiated reactions in Ref. 1. A nucleus of course contains both protons and neutrons; in this paper we have not included the complication that the νn process may differ from the νp process; therefore, within this approximation, νA is the same as $\bar{\nu}A$.
- ²³As discussed in Ref. 1, one should not put emphasis on the first couple of bins in Fig. 9. In addition to the reasons given there, $\langle n \rangle_{\text{ch}}$ in Ref. 6 includes protons with momentum $\approx 1.0 \text{ GeV}/c$, whereas the theoretical result excludes protons.
- ²⁴I. Budagov *et al.*, Phys. Lett. 30B, 364 (1969).
- ²⁵One should remember that the coefficient in the formula $\bar{K} \approx 0.8A^{1/3}$ is applicable only when A is large.
- ²⁶Y. Afek, G. Berlad, A. Dar, and G. Eilam, Technion Report No. PH-77-83 (unpublished).
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- ²⁹P. M. Valanju, E. C. G. Sudarshan, and C. B. Chiu, Phys. Rev. D 21, 1304 (1980).
- ³⁰J. W. Cronin *et al.*, Phys. Rev. D 11, 3105 (1975).
- ³¹We thank Professor E. C. G. Sudarshan for first mentioning this possibility to us.