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Kaonic hydrogen atom and $\Lambda(1405)$

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A model of the $\bar{K}N$ interaction at low energies is proposed such that the recently observed, surprisingly small energy shift associated with the $2p$-$1s$ x rays from K^-p atoms can be explained. The essential difference between this and conventional models lies in the interpretation of the origin of $\Lambda(1405)$. Our model is in accord with the quark model in which $\Lambda(1405)$ is regarded as being as "elementary" as the nucleon. It predicts that the $\bar{K}N$ scattering amplitude for $I = 0$ is very small at the threshold, but increases rapidly with energy such that the $\bar{K}N$ scattering data above the threshold can be reproduced.

Very recently the x rays from K^-p atoms have been detected, and a peak consistent with an unshifted $2p$-$1s$ transition has been found.[1] This small or no energy shift would imply that the K^-p s-wave scattering amplitude at the threshold, i.e., the scattering length,[2] is very small. On the other hand, the $\bar{K}N$ scattering lengths a_0 and a_1 (for $I=0$ and 1, respectively) have been estimated from scattering data.[4] Results of such recent estimates[5,6] are given in Table I, and $a(K^-p)=(a_0+a_1)/2$ is compared with the estimate based on the K^-p atom result. The discrepancy between the K^-p-atom and scattering results is striking.[7] We recall, however, that the scattering data at very low energies, say $k_{1ab} \lesssim 100$ MeV/c, still have some uncertainty, and hence the apparent discrepancy between the K^-p-atom and scattering results need not necessarily imply a real contradiction. We assume a point of view that the K^-p scattering amplitude (actually we have its $I=0$ part in mind) depends strongly on the energy around the threshold, that is, the amplitude is very small at the threshold, increases rapidly as the energy increases, and reaches a maximum beyond which it gradually decreases.

In addition to the data at and above the threshold, we have to remember $\Lambda(1405)$ which is about 27 MeV below the K^-p threshold. Conventionally, $\Lambda(1405)$ is interpreted as a quasibound state of K^- and p due to a strong attractive interaction between them in the $I=0$ s state. However, if the K^-p interaction is that strong, how can the K^-p scattering amplitude at the threshold be so small? A successful model of the $\bar{K}N$ interaction at low energies has to incorporate the following three features: (i) the scattering data above the threshold,[8] (ii) $\Lambda(1405)$, and (iii) the very small scattering amplitude at the threshold. The purpose of this note is to propose such a model. We assume that nothing unusual happens in the $I=1$ state and consider only the $I=0$ state. We ignore the n-p and K^0-K^- mass differences.

Before introducing our model let us note that the analysis of the $\bar{K}N$ scattering data has always been done, to our knowledge, by means of the K-matrix method with the assumption that the K-matrix elements are constants (zero-range approximation) or at most slowly varying functions of the energy.[9] The few explicit models for the $\bar{K}N$ interaction that have been proposed, all of which assume some potential between $\bar{K}$ and N, also lead to such slowly varying K-matrix elements.[10-12] This approach has been quite successful in correlating features (i) and (ii),[5] but feature (iii) cannot be incorporated in it unless some singularity is assumed in the K-matrix elements. Our model leads to such a singularity which is related to the interpretation of the origin or structure of $\Lambda(1405)$.

The basic idea that underlies our model is as follows. The quark model regards $\Lambda(1405)$ as a three-quark system in a unitary-singlet configuration, as "elementary" as the nucleon or any other baryon. This is in contrast to the conventional models in which the nucleon is treated as being more elementary than $\Lambda(1405)$, the latter being regarded only as an outcome of the $\bar{K}N$ interaction—a quasibound $\bar{K}N$ state. If instead the assertion of the quark model is taken seriously, then in a model of the $\bar{K}N$ interaction there should be a "bare" Λ, which we denote by Λ_0, corresponding to $\Lambda(1405)$. Similar considerations would apply to the πN interaction and $\Delta(1236)$.

Following the above arguments we assume that the free part of our model Hamiltonian H_0 contains Λ_0 in addition to $\bar{K}$ and N. For the interaction Hamiltonian H_1, we assume two terms which are depicted in Fig. 1. Term (a) is a usual separable potential for $\bar{K}N \to \bar{K}N$. Term (b) is a Yukawa interaction for $\Lambda_0 \to \bar{K}N$. Note that $\bar{K}$ and Λ_0 are both of odd parity. All interactions other than term (b), such as the ρ and ω exchanges between $\bar{K}$ and N,[12] are represented by term (a). For simplicity, we do not include the $\pi\Sigma$ channel explicitly. Instead we assume that the coupling constant for term (a) is complex. There is no reason why the coupling constant in term (a) is

TABLE I. The $\bar{K}N$ s-wave scattering lengths in fm. The subscripts refer to isospin $I=0$ or 1. Statistical errors are not shown.

Ref.	a_0	a_1	$a(K^-p)=\frac{1}{2}(a_0+a_1)$
4[a] A	$-1.70+0.71i$	$0.00+0.61i$	$-0.85+0.66i$
4[a] B	$-1.60+0.75i$	$0.08+0.69i$	$-0.76+0.72i$
5	$-1.66+0.75i$	$0.35+0.66i$	$-0.66+0.71i$
1 (K^-p atom)			$0.10+0.00i$

[a] Set B incorporates below-threshold constraints, and hence is thought to be more reliable than set A.

complex while that in term (b) is real. This choice is for simplicity. We make no further reference or appeal to the quark model, carrying from it only the idea that Λ(1405) and N are equally "elementary."

First we work in the static approximation in which the kinetic energies of N and Λ_0 are ignored, and later we will take account of the recoil corrections of the baryons. We assume the Hamiltonian $H=H_0+H_1$:

$$H_0=m_N N^\dagger N+m_0\Lambda_0^\dagger\Lambda_0+\int d\vec{k}\,\omega_k a_k^\dagger a_k\,, \tag{1}$$

$$H_1=g\left(\Lambda_0^\dagger N\int d\vec{k}\,u_k a_k+\text{ H.c.}\right) -G\int\int d\vec{k}\,d\vec{k}'\,u_k u_{k'}\,a_k^\dagger a_{k'}\,. \tag{2}$$

Here m_0 is the mass of Λ_0, $\omega_k=(\mu^2+k^2)^{1/2}$, μ is the kaon mass, and $u_k=(2\pi)^{-3/2}(2\omega_k)^{-1/2}v_k$, where v_k is the form factor of the interaction source and is normalized by $v_0=1$. The same u_k appears in the two terms of H_1; this is to simplify the solution to the Schrödinger equation.

It is straightforward to solve the Lippman-Schwinger equation[13] and find the scattering amplitude f, which is related to the phase shift δ by $f=k^{-1}e^{i\delta}\sin\delta$, to be

$$f_0=\lambda_k v_k^2/(1-\lambda_k J_k)\,, \tag{3}$$

where the subscript 0 indicates the isospin state

$$4\pi\lambda_k=G+g^2/(\Delta-\omega_k)\,, \tag{4}$$

with $\Delta=m_0-m_N$, and

$$J_k=\frac{1}{\pi}\int_\mu^\infty\frac{p v_p^2 d\omega_p}{\omega_p-\omega_k-i\epsilon}\,. \tag{5}$$

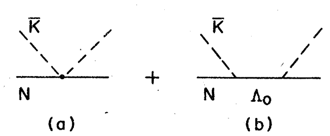

FIG. 1. Diagram of the interactions. Λ_0 is the bare Λ(1405).

We then include the baryon-recoil correction by replacing ω_k in the denominators in Eqs. (4) and (5) with $\tilde{\omega}_k=\omega_k+(k^2/2m_N)$, and Eq. (3) with

$$f_0=\frac{m_N}{m_N+\omega_k}\frac{\lambda_k v_k^2}{1-\lambda_k J_k}\,, \tag{6}$$

where λ_k and J_k are the modified ones described above. The resonance energy, which is to be identified with the mass of Λ(1405), is determined from

$$\text{Re}(1-\lambda_k J_k)=0\,. \tag{7}$$

Of course, this energy is different from m_0.

Before determining the parameters of the model to fit experimental data, let us make some general observations concerning the effect of Λ_0. If we set $g=0$, then λ_k becomes a constant and our model is reduced to a simple separable-potential model (SPM). The crucial difference between SPM (or any conventional models) and our model is that our λ_k has a pole at $\tilde{\omega}_k=\Delta$ which gives rise to a pole in the K matrix in the vicinity of $\tilde{\omega}_k=\Delta$. This can be seen as follows. Our f_0 is identical to the T matrix, to which the K matrix ($K=k^{-1}\tan\delta$) is related by $T^{-1}=K^{-1}-ik$. If we use Eq. (3) rather than Eq. (6) for simplicity, we obtain

$$K=\frac{\lambda_k v_k^2}{1-\lambda_k\text{Re}J_k}\,. \tag{8}$$

The denominator of K has a zero which is due to the rapid variation of λ_k around $\tilde{\omega}_k=\Delta$. If we choose $\Delta\approx\mu$ as we will do later, the behavior of our K near the $\bar{K}N$ threshold becomes very different from that of the K of SPM. In all the conventional models the K is found to be a slowly varying function of k.[10-12]

There is another important difference between our model and the conventional ones. Let us assume for the moment that G is also real. If G is sufficiently large, there is a bound state in SPM which can be identified with Λ(1405). According to Levinson's theorem, $\delta(\tilde{\omega}=\mu)-\delta(\infty)=\pi$; δ starts from π at the threshold and tends to zero as $\tilde{\omega}\to\infty$.[14] For a usual form factor v_k, δ monotonically decreases; hence, the scattering length is negative.[4] In our model (with $g \neq 0$) again we have a bound state, but the pole in λ_k at $\tilde{\omega}=\Delta$ modifies Levinson's theorem to $\delta(\mu) - \delta(\infty) = 0$.[15] The scattering length is expected to be positive.

The above observation is qualitatively valid even if G is complex unless ImG dominates. The phenomenological a_0 with Re$a_0 < 0$ shown in Table I is consistent with the existence of $\Lambda(1405)$ within the usual K-matrix approach. Our model, however, would not be compatible with Re$f_0 < 0$. At this point we would like to point out that the sign of Ref_0 has not been determined completely independently of fitting $\Lambda(1405)$. The sign of Ref_0 can be determined from the Coulomb interference in the differential cross section, but the data so far available do not seem to be sufficient to do so unambiguously.[16] Therefore, Ref_0 could well be positive, and we assume so in fitting the scattering data above the threshold.

Figure 2 shows our f_0 for the set of parameters (in units of μ): $g=0.465$, $G=19.06+1.42i$, $\Delta=0.989$, and $k_c=2$.[17] Here k_c is a cutoff parameter in v_k which we assumed to be $v_k=\theta(k_c - k)$. In choosing the parameters we imposed the following conditions: (a) $f_0(k=0)\approx 0$, (b) $f_0 = -(f_0$ of solution B Chao *et al.*$)^*$ at $k_{\text{c.m.}}=0.3\mu$ ($\tilde{\omega}-\mu=33$ MeV), and (c) $\text{Re}(1-\lambda_k J_k)=0$ at $m_N+\tilde{\omega}=1405$ MeV. At the K^-p threshold we obtained $f_0=0.001+0.029i$ (fm). Figure 2 also shows f_0 of solution B of Chao *et al.* Our f_0 is not very different from $-f_0^*$ of Chao *et al.* for $\tilde{\omega}-\mu \gtrsim 10$ MeV. With a slight readjustment of f_1, the K^-p scattering data above the threshold can be fitted quite well.[16] At the threshold, if we combine our f_0 with the $a_1=0.08+0.69i$ of Chao *et al.*, we obtain $a(K^-p)=0.04+0.36i$ which is consistent with the K^-p atom result. For the width of $\Lambda(1405)$ we obtain $\Gamma\approx 8$ MeV, which is much too small as compared with the experimental value of $\Gamma\approx 40$ MeV. It is possible to increase Γ by relaxing conditions (a) and (b), but we have not pursued this thoroughly. We plan to extend the model so that the $\pi\Sigma$ channel is explicitly taken into account. We expect that the fit can be significantly improved by such an extension.

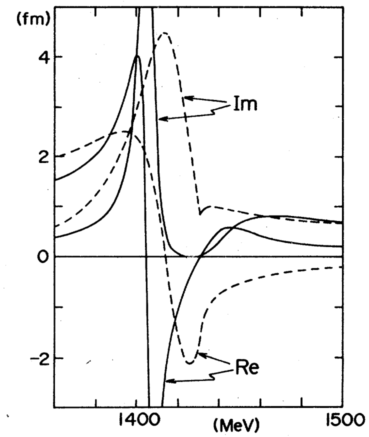

FIG. 2. The scattering amplitude f_0 in fm vs the total c.m. energy in MeV. The K^-p threshold energy is 1432 MeV. The solid lines represent our f_0. Imf_0=8.6 fm at its peak and Ref_0=−4.7 fm at its downward peak. The dashed lines are for solution B of Chao *et al.*

In summary, we have proposed a model which satisfactorily reproduces features (i), (ii), excepting the width, and (iii), which were enumerated in the beginning. Our model predicts that Re$f_0 > 0$ above the threshold. This, together with the rapid variation of f_0 just above the threshold, will be a crucial test of the model. In particular, the compatibility of this with the dispersion relation constraints discussed by Martin[6] needs to be examined. For other larger kaonic atoms, it has been known that the real part of the $\bar{K}N$ scattering length for a bound nucleon is positive, opposite in sign to that for a free nucleon.[18] This change of the sign could be achieved by a strong binding effect. However, our model may not require such a strong binding effect.

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[1]J. D. Davies, G. J. Pyle, G. T. A. Squire, D. J. Batty, S. F. Biagi, S. D. Hoath, P. Sharman, and A. S. Clough, Phys. Lett. 83B, 55 (1979).

[2]This is the Coulomb-corrected scattering length. We assume that the Coulomb correction to the $\bar{K}N$ scattering length is negligible, and we do not include the Coulomb interaction in our model calculation. For the possibility of an anomalously large Coulomb correction, see Ref. 3.

[3]A. Deloff and J. Law, Phys. Rev. C 20, 1597 (1979).

[4]We consider only s states, and use the sign convention for the scattering length: $k\cot\delta = a^{-1} + \cdots$. The effective ranges are known to be very small.

[5]Y.-A. Chao, R. W. Kraemer, D. W. Thomas, and B. R. Martin, Nucl. Phys. B56, 46 (1973).

[6]A. D. Martin, Phys. Lett. 65B, 346 (1976).

[7]Because of the np and K^0K^- mass differences the actual K^-p scattering length differs from $(a_0+a_1)/2$. But

this difference is far too small to be significant in resolving the discrepancy mentioned (Ref. 3).

[8]Here and henceforth, by "above the threshold" we mean $k_{lab} \gtrsim 100$ MeV/c.

[9]R. H. Dalitz and S. F. Tuan, Ann. Phys. (N.Y.) 3, 307 (1960).

[10]See, e.g., S. Wycech, Nucl. Phys. B28, 541 (1971) and M. Alberg, E. M. Henley, and L. Wilets, Ann. Phys. (N.Y.) 96, 43 (1976). They assumed nonlocal separable potentials for the $\bar{K}N$ interaction.

[11]A. Deloff and J. Law, Phys. Rev. C 10, 1688 (1974). Their model and that of Ref. 12 both assume local potentials.

[12]M. Atarashi, K. Hira, and H. Narumi, Prog. Theor. Phys. 60, 209 (1978).

[13]Note the similarity between our model and the Lee model: Phys. Rev. 95, 112 (1954). Alternatively one can solve the Low equation and select a solution which has one Castillejo-Dalitz-Dyson pole.

[14]At an energy which is usually very large, δ becomes $\pi/2$ and hence $K = k^{-1}\tan\delta$ blows up. This should not be confused with the singular behavior of K at low energies which was discussed in the preceding paragraph.

[15]M. Ida, Prog. Theor. Phys. 21, 625 (1959).

[16]Here we have an underlying assumption that

$$|\mathrm{Re} f_1| \ll |\mathrm{Re} f_0|\,, \text{ and } \mathrm{Re} f(K^- p\text{-}K^- p) \approx \tfrac{1}{2}\mathrm{Re} f_0.$$

[17]If we put $g = 0$, but use the same values of G, Δ, and k_c, the $\bar{K}N$ bound state disappears.

[18]R. Seki and C. E. Weigand, Annu. Rev. Nucl. Sci. 25, 241 (1975), Sec. 4.2. Their sign convention for a is opposite to ours (Ref. 4).