Mass mixing and CP nonconservation in neutral-B-meson systems

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We calculate the complex mass and decay matrix parameters δm, $\delta\Gamma$, Γ, $\mathrm{Im}m_{12}$, and $\mathrm{Im}\Gamma_{12}$ for B^0-$\bar{B}^0$ mixing in the sequential six-quark model.

With the prospect that B mesons can soon be observed in e^+e^- experiments, the question of mass mixing and CP nonconservation in the B^0-$\bar{B}^0$ system becomes of immediate interest. From rough theoretical calculations[1,2] it has been estimated that both mixing and CP nonconservation could be large for neutral-B-meson systems. The purpose of this article is to make more precise the quantitative expectations for these phenomena.

The wave function at time t for a state which is B^0 at time $t=0$ is[3]

$$|\psi(t)\rangle = a_+(t)|B^0\rangle + (1-\rho)/(1+\rho)a_-(t)|\bar{B}^0\rangle\,, \tag{1}$$

where

$$a_\pm(t) = \tfrac{1}{2}(e^{im_St-\Gamma_St/2} \pm e^{im_Lt-\Gamma_Lt/2})\,. \tag{2}$$

In Eq. (1) ρ is the CP parameter

$$\rho = (i\,\mathrm{Im}\,m_{12} + \tfrac{1}{2}\mathrm{Im}\Gamma_{12})/(i\tfrac{1}{2}\delta\Gamma - \delta m)\,, \tag{3}$$

with $\delta\Gamma = \Gamma_S - \Gamma_L \simeq \Gamma_1 - \Gamma_2$ and $\delta m = m_S - m_L \simeq m_1 - m_2$. The time-integrated measure[1,4] of $B^0-\bar{B}^0$ mixing for small ρ is

$$\Delta \equiv \frac{\int_0^\infty dt\,|a_-(t)|^2}{\int_0^\infty dt\,|a_+(t)|^2} = \frac{(\delta m/\Gamma)^2 + (\delta\Gamma/\Gamma)^2/4}{2 + (\delta m/\Gamma)^2 - (\delta\Gamma/\Gamma)^2/4}\,, \tag{4}$$

where $\Gamma = (\Gamma_S + \Gamma_L)/2$. For $e^+e^- \to \gamma \to B^0\bar{B}^0$ production followed by semileptonic decays $B^0 \to l^-\nu X^+$, $\bar{B}^0 \to l^+\bar{\nu}X^-$ the numbers of events with lepton charges $(++)$, $(--)$, and $(+-)$ are related by

$$\begin{aligned}(N^{++} + N^{--})/N^{+-} &= 2(N^{++}N^{--})^{1/2}/N^{+-} \\ &= 2\Delta/(1+\Delta^2)\,.\end{aligned} \tag{5}$$

The mixing is maximal for $\Delta = 1$ (which is approximately the case for K^0-$\bar{K}^0$). The asymmetry due to CP violation is

$$(N^{++} - N^{--})/(N^{++} + N^{--}) = -(1-r^2)/(1+r^2)$$

or (6)

$$(N^{--}/N^{++})^{1/2} = 1/r\,,$$

where

$$r = |1-\rho|^2/|1+\rho|^2 \simeq 1 - 4\,\mathrm{Re}\rho\,. \tag{7}$$

We make calculations of all mass and decay matrix quantities needed to predict ρ from Eq. (3) and Δ from Eq. (4). The mass terms δm and $\mathrm{Im}\,m_{12}$ are evaluated from an effective four-quark Lagrangian[5] for the B^0-$\bar{B}^0$ transition. The width terms $\delta\Gamma$, Γ, and $\mathrm{Im}\Gamma_{12}$ are evaluated with the heavy-quark decay model including gluon corrections.[6,7] Our calculations are based on the standard six-quark model[8-10] with three left-handed doublets of quarks and leptons. We incorporate the constraints on the weak current mixing angles and CP parameter obtained from recent analyses of mixing and CP nonconservation in the K^0-$\bar{K}^0$ system.[11,12] Our assumption throughout is that CP violation arises solely from the phase angle in the quark mixing matrix.

The charged weak current of the sequential six-quark $SU(2)_L \times U(1)$ model[8-10]

$$J_\mu = 2(\bar{u}\ \bar{c}\ \bar{t})_L \gamma_\mu U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{8}$$

is specified by the three rotation angles θ_i and the

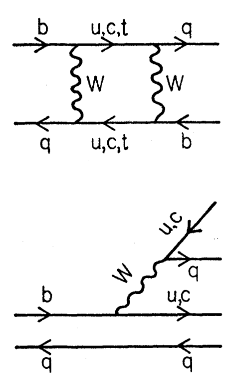

FIG. 1. Quark diagrams for mass mixing and decay mixing.

CP-violating phase δ of the Kobayashi-Maskawa mixing matrix,[8]

$$U=\begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3+s_2s_3e^{i\delta} & c_1c_2s_3-s_2c_3e^{i\delta} \\ -s_1s_2 & c_1s_2c_3-c_2s_3e^{i\delta} & c_1s_2s_3+c_2c_3e^{i\delta} \end{pmatrix}. \quad (9)$$

The notation is $c_i=\cos\theta_i$, $s_i=\sin\theta_i$, and $s_\delta=\sin\delta$ with the convention that $s_i>0$. Low-energy decay data constrain $s_1\simeq 0.23$ and $s_3<0.5$. The parameters s_2 and s_δ have been determined versus s_3 from an effective Lagrangian analysis of mass mixing and CP nonconservation in the K^0-$\bar{K}^0$ system.[11,12] Two solutions were found depending on whether $c_\delta>0$ (solution I) or $c_\delta<0$ (solution II). This ambiguity has since been resolved[13] in favor of solution I from the observed suppression[14] of $\Gamma(D^0\to\pi^-\pi^+)/\Gamma(D^0\to K^-\pi^+)$ relative to $\tan^2\theta_C=0.05$. The amount of suppression found implies the further constraint $s_3>0.35$ on solution I, but we shall present results here for smaller values of s_3 as well.

The calculation of the mass mixing in the B^0-$\bar{B}^0$ system is along the same lines as the K^0-$\bar{K}^0$ analysis.[11,12] An effective four-quark Lagrangian operator is obtained from a single-loop diagram (top graph of Fig. 1 plus its crossed graph). The $B^0(b\bar{q})\leftrightarrow\bar{B}^0(\bar{b}q)$ transition matrix element of this effective Lagrangian

$$\mathfrak{M}\equiv\langle\bar{B}^0(\bar{b}q)|-\mathcal{L}_{\rm eff}(\bar{b}q\to b\bar{q})|B^0(b\bar{q})\rangle \quad (10)$$

is given by[5]

$$\mathfrak{M}=-\sum_{i,j=u,c,t}(U_{ib}U_{iq}^*U_{jb}U_{jq}^*)A_{ij}\left(\frac{\beta f_B{}^2m_B}{3x_W}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi}\right). \quad (11)$$

Here q denotes either d or s, depending on the $B^0(b\bar{q})$ system of interest. The single-loop amplitude A_{ij} is expressed in terms of $x_i=m_i{}^2/m_W{}^2$ as[15]

$$A_{ij}=\frac{1}{(1-x_i)(1-x_j)}+\frac{1}{x_i-x_j}\left[\frac{x_i{}^2\ln x_i}{(1-x_i)^2}-\frac{x_j{}^2\ln x_j}{(1-x_j)^2}\right]. \quad (12)$$

In Eq. (11) β is a bag-model correction to a vacuum-insertion evaluation of $\mathfrak{M}$. The value $\beta=0.4$ was estimated for the K^0-$\bar{K}^0$ system.[16] In general we expect $\beta<1$. In the absence of a bag-model

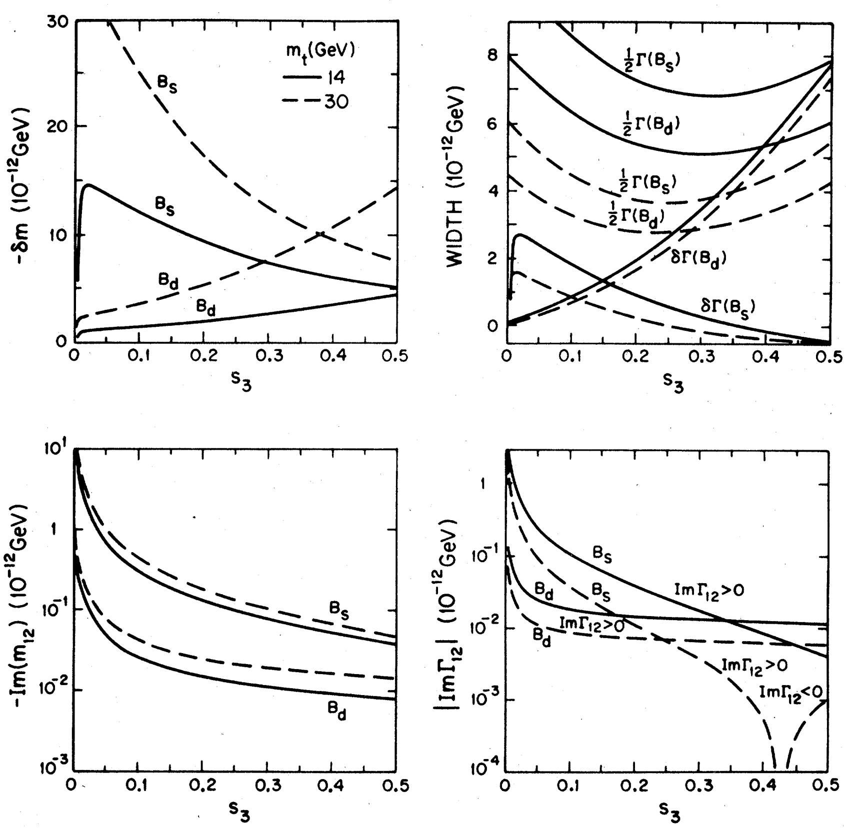

FIG. 2. Predictions versus s_3 for δm, $\delta\Gamma$, Imm_{12}, ImΓ_{12}, and $\Gamma/2$. Solid curves represent $m_t=14$ GeV and dashed curves $m_t=30$ GeV. The notation is B_d for $b\bar{d}$ and B_s for $b\bar{s}$.

calculation for the B system, we shall take $\beta = 0.4$ in this analysis. The leptonic decay constant f_B has been theoretically estimated[17] as $f_B/m_B \simeq (32\pi)^{-1/2} = 0.1$. For the Weinberg-angle factor $x_W = \sin^2\theta_W$, we take $x_W = 0.2$; then $m_W = [\pi\alpha/(\sqrt{2}\, G_F x_W)]^{1/2} = 84$ GeV. For the constituent quark masses which enter A_{ij} we take $m_u = 0.3$ GeV, $m_c = 1.5$ GeV, and consider $m_t = 14$ and 30 GeV. For the B mass in Eq. (11) we take $m_B = 5.0$ GeV for the $b\bar{d}$ state and $m_B = 5.2$ GeV for the $b\bar{s}$ state.

The mass difference δm and CP-violating mass matrix element $\mathrm{Im}\, m_{12}$ are given in terms of Eq. (11) by

$$\delta m = 2\,\mathrm{Re}\mathfrak{M}\,, \qquad \mathrm{Im}\, m_{12} = \mathrm{Im}\mathfrak{M}\,. \tag{13}$$

Results for δm and $\mathrm{Im}\, m_{12}$ are plotted versus s_3 in Fig. 2, for both $B_d \equiv b\bar{d}$ and $B_s \equiv b\bar{s}$ systems. In comparison $\delta m = -3.52 \times 10^{-15}$ GeV and $\mathrm{Im}\, m_{12}/\delta m = -3.25 \times 10^{-3}$ for the K system.

The total width for $B^0(b\bar{d})$ decay has been evaluated in Ref. 18, including gluon corrections in both hadronic and leptonic channels; we make a similar calculation of the $B^0(b\bar{s})$ total width. The lifetime difference between B_1 and B_2 mesons and the CP-violating decay matrix element $\mathrm{Im}\Gamma_{12}$ are related to the weak interaction Hamiltonian H_w by

$$\delta\Gamma = 4\pi \sum_F \rho_F \mathrm{Re}\langle \bar{B}^0 | H_w | F\rangle\langle F | H_w | B^0\rangle\,, \qquad \mathrm{Im}\Gamma_{12} = 4\pi \sum_F \rho_F \mathrm{Im}\langle \bar{B}^0 | H_w | F\rangle\langle F | H_w | B^0\rangle\,, \tag{14}$$

where the sum is over hadronic final states; ρ_F is the appropriate density of final states per unit energy. We use the heavy-quark decay model to estimate $\delta\Gamma$ and $\mathrm{Im}\Gamma_{12}$. On the right-hand side of Eq. (14), the final states reached from $b\bar{q}$ and $\bar{b}q$ must be the same. Hence the charge $-\frac{1}{3}$ quark produced in the final state must be the antiquark of the spectator, as illustrated in the lower diagram in Fig. 1. Thus the quark transitions involved in $B_{1,2}$ decays are

$$b\bar{q} \to i\bar{q}\bar{j}\bar{q}\,, \qquad \bar{b}q \to \bar{j}\bar{q}iq\,, \tag{15}$$

where i,j are charge $\frac{2}{3}$ quarks. The appropriate elements of the mixing matrix for $B_{1,2}$ decays are $(U_{ib}U^*_{jq} \pm U^*_{jb}U_{iq})/\sqrt{2}$.

The explicit expressions for $\delta\Gamma$ and $\mathrm{Im}\Gamma_{12}$ are

$$\delta\Gamma = 2 \sum_{i,j=u,c} \mathrm{Re}(U_{ib}U^*_{jq}U_{jb}U^*_{iq})D_{ij}\,, \qquad \mathrm{Im}\Gamma_{12} = 2 \sum_{i,j=u,c} \mathrm{Im}(U_{ib}U^*_{jq}U_{jb}U^*_{iq})D_{ij}\,, \tag{16}$$

where

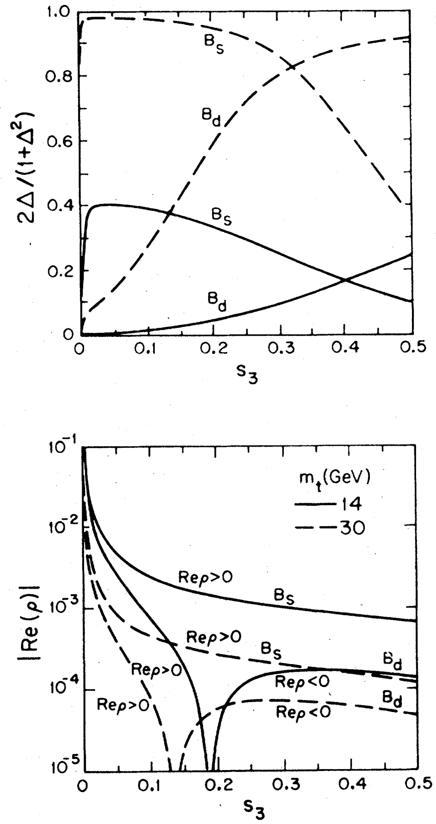

FIG. 3. Predictions for the mixing strength $2\Delta/(1+\Delta^2)$ and the CP parameter Reρ for B_d and B_s systems.

$$D_{ij} = (c_-^{\,2} + 2c_+^{\,2})\,(G_F^{\,2} m_b^{\,5}/192\pi^3) I\left(\frac{m_q}{m_b}, \frac{m_i}{m_b}, \frac{m_j}{m_b}\right). \tag{17}$$

The value of the gluon renormalization factor at the b-quark mass is estimated[18] to be $c_-^{\,2} + 2c_+^{\,2} = 3.6$. The phase-space integral I of the matrix element squared, which is normalized to $I(0,0,0) = 1$ in Eq. (17), is evaluated numerically for each channel. To ensure the appropriate kinematic boundaries of the phase space in the width calculations, we use physical meson mass assignments for the quarks: $m_b = 5.0$ for $b\bar{d}$ meson, $m_b = 5.2$ for $b\bar{s}$ meson, $m_c = 1.867$, $m_s = 0.5$, $m_u = m_d = 0.14$ GeV. In inclusive width calculations all appropriate kinematically accessible final states involving the six quarks and six leptons are summed over.

Figure 2 shows our results for $\delta\Gamma$ and $\mathrm{Im}\Gamma_{12}$, along with values of $\Gamma/2$ from Ref. 18. Predictions for the mixing strength $2\Delta/(1+\Delta^2)$ of Eqs. (4) and (5) and the CP parameter Reρ of Eqs. (3) and (7) are presented in Fig. 3.

For the favored range[13] $s_3 > 0.35$, B^0-$\bar{B}^0$ mixing can be large, with Δ reaching 0.4 for both B_d and B_s systems. The value of Reρ depends on the difference of mass-matrix and decay-matrix contributions

$$\mathrm{Re}\rho = (\mathrm{Im}\, m_{12}/\delta m - \mathrm{Im}\Gamma_{12}/\delta\Gamma)(\delta\Gamma/2\delta m + 2\delta m/\delta\Gamma)^{-1}. \tag{18}$$

In the B_d system, $\mathrm{Im}\, m_{12}/\delta m$ and $\mathrm{Im}\Gamma_{12}/\delta\Gamma$ always have the same sign so their contributions to $\mathrm{Re}\rho$ tend to cancel.[19] However, in the B_s system the contributions are of opposite sign for some regions of s_3.

In similar calculations for the D^0-$\bar{D}^0$ system, we find that Δ is very small ($< 10^{-4}$) making $\mathrm{Re}\rho$ difficult to measure. These results, as well as those for T^0-$\bar{T}^0$ systems and the details of calculations, will be presented elsewhere.

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