Comment on the flavor grand unification

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Georgi's theory of flavor unification in big SU(N) groups, in particular SU(11), is shown to be asymptotically strong. A possible implication of this theory in the early hot universe is the ultimate temperature of the universe, of the order of 10^{17}–10^{20} GeV.

Phenomenologically successful quantum chromodynamics, based on the gauge group SU(3), and the standard electroweak theory,[1] based on the gauge group SU(2)× U(1), were grandly unified in an SU(5) gauge theory[2] five years ago. Since then, the beauty of the standard grand unified theory[3] has been known: One generation consists of 15 chiral fields (e,ν_L,u,d), a neutrino is massless, the electroweak charged current is left-handed, the charge is quantized $Q_p=-Q_e$, the b-quark mass is roughly 3 times the τ mass, $\sin^2\theta_W\sim 0.2$, there is baryon-number violation and a plausible mechanism for baryon asymmetry of the universe. This standard theory, however, gives no explanation for the repetition of generations (e, μ, τ). To guide flavor unification in large SU(N) gauge groups which include SU(5), Georgi[4] imposed three laws of flavor unification (or generation unification) and found that SU(11) is the minimal gauge group[5] incorporating 3 generations in the scheme. If this scheme is correct, low-mass fermions are completed by the t quark and nothing happens between 1000 to 10^{14} GeV. We take Georgi's SU(11) theory as a guiding example for flavor unification in SU(N) groups and explore its qualitative consequences.

Quantum chromodynamics is asymptotically free[6] up to the energy scale so far probed. The reason is that quark flavor is ≤16 for currently available energies. The number of fermions in SU(11) theory, however, is enormous and changes the common view of decreasing coupling constants at higher mass scale. The high-energy behaviors of coupling constants are usually studied by the Callan-Symanzik β function which, in the lowest-order non-Abelian gauge theories, is[6]

$$\beta(g)=\frac{g^3}{16\pi^2}\left[-\tfrac{11}{3}C_2(G)+\tfrac{1}{3}I_F(R)+\text{scalars}\right], \qquad (1)$$

where $C_2(\mathrm{SU}(N))=N$ and the fermion contribution $I_F(R)$ is the index of the fermion representation R. The completely antisymmetric representation with m indices in SU(N) denoted as $[N,m]$ has

$$I_F([N,m])={}_{N-2}C_{m-1}\ \text{for}\ N\geq 2,\ \ m\geq 1\,. \qquad (2)$$

Usually, one does not include fermions whose threshold is higher than the energy scale under consideration. Four representations of Ref. 4, $[11,4]\oplus[11,8]\oplus[11,9]\oplus[11,10]$, give, neglecting scalar contributions,

$$\frac{g^3}{16\pi^2}\left[-\tfrac{121}{3}+\tfrac{1}{3}(84+36+9+1)\right]=+\frac{g^3}{16\pi^2}\times 3\,,$$

which allows increasing the SU(11) coupling constant above SU(11) unification. More importantly, above SU(5) unification there are twenty $\underline{5}$, twenty-three $\underline{5^*}$, sixteen $\underline{10}$, and thirteen $\underline{10^*}$ representations, and we obtain from (1) and (2), neglecting Higgs interactions,

$$\beta(g)=\frac{g^3}{16\pi^2}\left[-\tfrac{55}{3}+\tfrac{1}{3}(43+3\times 29)\right]$$
$$=\frac{g^3}{16\pi^2}\times\tfrac{75}{3} \qquad (3)$$

above SU(5) unification but below SU(11) unification. The $\underline{5}$ and $\underline{24}$ of Higgs fields would have contributed $\frac{1}{6}$ and $\frac{5}{6}$, respectively, in units of $g^3/16\pi^2$.

The behaviors of coupling constants are shown in Fig. 1 for $\sin^2\theta_W=0.20$ and 0.23. We used $\alpha_{em}^{-1}=128.5$ at $\mu=M_W$ (the W-boson mass). For sim-

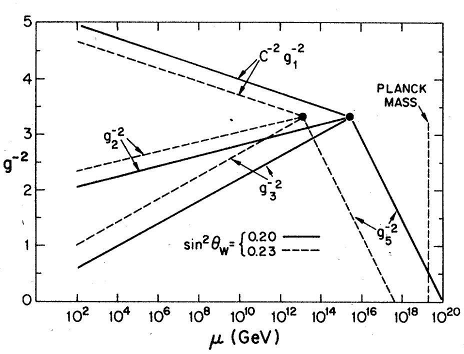

FIG. 1. The behavior of inverse coupling constants as a function of μ. A relevant coupling strength is $\alpha_i=g_i^2/4\pi$. Solid and dashed lines correspond to $\sin^2\theta_W=0.20$ and 0.23 at low energy, respectively.

TABLE I. Low-lying SU(5) singlet states. The first column shows quark and lepton contents in the SU(3) × SU(2) × U(1) convention, the second column indicates bosons (B) or fermions (F), the third column gives the SU(5)-invariant coupling from $10_L(\psi_{\alpha\beta})$ and $5_R(\psi_\alpha)$, and the fourth column indicates decay modes when SU(5) is broken. The SU(5) index is α, and the color index is $i,j,k=1,2,3$. Three ijk means cyclic combination.

Particles		Singlet combinations	Decay modes
$\nu_e\bar{\nu}_e$	B	$\psi_\alpha\bar{\psi}^\alpha$	$B\to 2Z$
$e\bar{e}, d_i\bar{d}_i$	B	$\psi_\alpha\bar{\psi}^\alpha, \psi_{\alpha\beta}\bar{\psi}^{\alpha\beta}$	$B\to 2\gamma, 2Z$
$u_i\bar{u}_i$	B	$\psi_{\alpha\beta}\bar{\psi}^{\alpha\beta}$	$B\to 2\gamma, 2Z$
$\bar{u}_i\bar{d}_j\bar{d}_k$	F	$\psi_{\alpha\beta}\bar{\psi}^\alpha\bar{\psi}^\beta$	$F\to n+(\gamma$ or $Z)$
$u_i\bar{d}_i e$	F	$\psi_{\alpha\beta}\bar{\psi}^\alpha\bar{\psi}^\beta$	$F\to W^+ + e$
$\bar{e}e\nu$	F	$\psi_{\alpha\beta}\bar{\psi}^\alpha\bar{\psi}^\beta$	$F\to W^+ + e$
$\bar{u}_i u_i\nu$	F	$\epsilon^{\alpha\beta\gamma\delta\epsilon}\psi_{\alpha\beta}\psi_{\gamma\delta}\psi_\epsilon$	$F\to(Z$ or $\gamma)+\nu$
$\bar{u}_i d_i\bar{e}$	F	$\epsilon^{\alpha\beta\gamma\delta\epsilon}\psi_{\alpha\beta}\psi_{\gamma\delta}\psi_\epsilon$	$F\to W^- + \bar{e}$
$\bar{u}_i d_i\bar{e}\bar{\nu}$	B	$\epsilon^{\alpha\beta\gamma\delta\epsilon}\psi_{\alpha\beta}\psi_\gamma\psi_\delta\psi_\epsilon$	$B\to W^- + \bar{e}+\bar{\nu}$
$u_i d_j d_k\bar{\nu}$	B	$\epsilon^{\alpha\beta\gamma\delta\epsilon}\psi_{\alpha\beta}\psi_\gamma\psi_\delta\psi_\epsilon$	$B\to n+\bar{\nu}$
$\bar{e}d_i d_j d_k$	B	$\epsilon^{\alpha\beta\gamma\delta\epsilon}\psi_{\alpha\beta}\psi_\gamma\psi_\delta\psi_\epsilon$	$B\to\Delta^- + \bar{e}$
$d_i d_j d_k\bar{e}\bar{\nu}$	F	$\epsilon^{\alpha\beta\gamma\delta\epsilon}\psi_\alpha\psi_\beta\psi_\gamma\psi_\delta\psi_\epsilon$	$F\to\Delta^- + \bar{e}+\bar{\nu}$
etc.			

plicity all heavy fermion masses are assumed to be equal to the SU(5) grand unification mass. [This may be so in SU(5) theory due to gauge-invariant terms $\bar{\psi}_{5^*}\Phi\psi_{5^*}$, $m_{5^*}\bar{\psi}_{5^*}\psi_{5^*}$, $\bar{\psi}_{10}\Phi\psi_{10}$, or $m_{10}\bar{\psi}_{10}\psi_{10}$, where Φ is the $\underline{24}$ of Higgs scalars giving X and Y bosons a mass. On dimensional grounds, these fermions' masses are of the order of the grand unification mass in the Higgs scheme but not much more than that for perturbative calculations. The remaining three generations do not acquire masses by this Φ.] The SU(3), SU(2), and U(1) coupling constants at μ below SU(5) unification mass $\tilde{M}_5$ are given[7] in terms of the SU(5) coupling constant g_5 at $\tilde{M}_5$,

$$\frac{1}{g_3^{\,2}}=\frac{1}{g_5^{\,2}}+\frac{1}{8\pi^2}(-11+\tfrac{4}{3}f)\ln\frac{\tilde{M}_5}{\mu}, \tag{4a}$$

$$\frac{1}{g_2^{\,2}}=\frac{1}{g_5^{\,2}}+\frac{1}{8\pi^2}(-\tfrac{22}{3}+\tfrac{4}{3}f+\tfrac{1}{6}N_H)\ln\frac{\tilde{M}_5}{\mu} \tag{4b}$$

$$\frac{1}{g_1^{\,2}}=\frac{C^2}{g_5^{\,2}}+\frac{C^2}{8\pi^2}(\tfrac{4}{3}f+\tfrac{1}{20}N_H)\ln\frac{\tilde{M}_5}{\mu},\quad C^2=\tfrac{5}{3}, \tag{4c}$$

with f generations and N_H Higgs doublets. We have assumed that the t-quark mass is less than M_W and $N_H=1$ and $f=3$. Above SU(5) unification, at $\mu>\tilde{M}_5$, g_5 behaves as

$$\frac{1}{g_5^{\,2}(\mu)}=\frac{1}{g_5^{\,2}(\tilde{M}_5)}-\frac{1}{8\pi^2}\times\tfrac{75}{3}\times\ln\frac{\mu}{\tilde{M}_5}. \tag{5}$$

From Fig. 1, we note that g_5^{-2} will become 0 for $\mu\approx 10^4-10^5\,\tilde{M}_5$, which means that the SU(5) coupling constant $\alpha_5=g_5^{\,2}/4\pi$ becomes infinite. A large-coupling-constant region in quantum chromodynamics is interpreted as a strong-interaction region leading to bound states of baryons and mesons. By analogy, similar bound states which are shown in Table I, presumably SU(5) singlets, will occur at 10^{17}–10^{20} GeV which depends on $\sin^2\theta_W$, but is below the Planck mass for $\sin^2\theta_W>0.21$. The following discussion will be relevant for $\sin^2\theta_W>0.21$. Gauge symmetry can be restored at high temperature, but confinement cannot be deduced from an analogy to low-energy phenomena, since it is asymptotically strong. At SU(5) strong-interaction stages, these low-lying particles can be stable. At broken SU(5) stages, these particles will decay to SU(3) singlet states which are shown in the last column of Table I. W, Z, and γ in this column are interpreted as appropriate SU(3) singlet states with these quantum numbers except the spin: W^+ means W^+, π^+, ρ^+, etc. We also note that SU(5) singlet states in Table I are neutral, a proof of which is as follows. In SU(5) theory Tr$Q=0$, for a given representation. The charge operator belongs to the group, i.e., $Q|\text{state}\rangle=q|\text{state}\rangle$ with eigenvalue q of the state in an SU(5) representation. Further, it annihilates the singlet states of SU(5), $Q|\text{singlet}\rangle=0$, because Q does not contain an SU(5) singlet piece. Therefore, all the SU(5) confined states are neutral.

Certainly, this SU(11) theory is very peculiar: strong interaction at 1 GeV due to quantum chromodynamics and another strong interaction at heavy mass scale $\mu\gtrsim 10^{17}$ GeV due to SU(5) and the perturbatively calculable region in between. In this scenario we go one step further: What are the possible implications of this strong SU(5) theory?

A naive picture of the universe composed of asymptotically free quarks and leptons would not be applicable in the early state of the universe. Presumably, a better picture may be a high-energy bootstrap model due to the, so far, unexpected possible strong interactions at $>10^{17}$ GeV. In this case, one might assume the number of species of particles between m and $m+dm$ grows as fast as possible following Hagedorn, Huang, and Weinberg[8]:

$$N(m)dm\sim Am^{-B}e^{m/T_0}dm\,. \tag{6}$$

The values B and T_0 have to be determined by experiment. In Hagedorn's model, the values are $B=\tfrac{5}{2}$ and $T_0\approx 160$ MeV. In the absence of any knowledge of the values B and T_0 in the $10^{17}-10^{20}$ GeV region, we assume that

$$T_0\approx\mu_m\ \text{ with}\ \frac{g_5^{\,2}(\mu_m)}{4\pi}\approx 1 \tag{7}$$

and B is a free parameter. Weighting the distribution function with (6), the density ρ at T behaves as

$$\rho=\frac{A}{8\pi^2}\int dm\, m^{-B} e^{m/T_0}\int \frac{E d^3p}{e^{E/T}-1} \tag{8}$$

for bosons and similarly for fermions. The energy is $E=(m^2+p^2)^{1/2}$. It has been shown, for zero net baryon number by Huang and Weinberg, that ρ is

$$\rho\propto\begin{cases}(T_0-T)^{B-7/2} & \text{for } B<\frac{7}{2}\,,\\ |\ln(T_0-T)| & \text{for } B=\frac{7}{2}\,,\\ \text{constant} & \text{for } B>\frac{7}{2}\end{cases} \tag{9}$$

as T approaches T_0. Since the universe might have been symmetric (i.e., equal numbers of baryons and antibaryons and equal numbers of neutrinos and antineutrinos, etc.) at $T\sim$ grand unification mass scale, we will study the universe based on (9).

For $B\leq\frac{7}{2}$, pressure behaves in the same form as in (9) with the replacement $B\to B+1$, and hence $p\ll\rho$ as $T\to T_0$. In an adiabatic expansion, entropy per unit comoving volume is conserved,[9]

$$ds=d\left[\frac{(\rho+p)R^3}{T}\right]=0\,, \tag{10}$$

where R is the cosmic scale factor in the Robertson-Walker universe. From (9) and (10), we obtain[8]

$$T_0-T\propto\begin{cases}R^{6/(7-2B)} & \text{for } B<\frac{7}{2}\,,\\ \exp(-\text{const}/R^3) & \text{for } B=\frac{7}{2}\,.\end{cases} \tag{11}$$

Therefore, as $R\to 0$, T approaches T_0 which is less than the Planck temperature for $\sin^2\theta_W>0.21$. Then one might have an ultimate upper bound of the temperature of the universe for $B\leq\frac{7}{2}$.

On the other hand, if $B>\frac{7}{2}$, ρ and p do not diverge as T approaches T_0, but diverge as $T\to\infty$ after crossing T_0. Therefore, there is no upper bound of the temperature of the universe.

The behavior of the universe considered in this paper can happen at a much lower temperature if Georgi's fermions acquire masses below grand unification mass scale. For $\sin^2\theta_W<0.21$, the possible effect considered in this paper might happen when the gravitational interaction dominates the known elementary particle ones and hence is not predictable at present.

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