Second-order mass-breaking effects on hadron masses

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SU(4) second-order mass-breaking effects on charmed hadron masses are studied in a dynamical consideration. We consider the process $S + B \to B + S$ in the s, t, and u channels, where S is the first-order symmetry-breaking spurion. Assuming that the nonexotic intermediate states dominate, several mass sum rules are obtained. Charmed isomultiplet masses are also estimated.

I. INTRODUCTION

The conventional approach to mass splitting in SU(4) is to assume that the mass-breaking Hamiltonian transforms like the $T_3^3 + yT_4^4$ component of the $\underline{15}$ representation.[1] Hadron mass relations up to the first-order perturbation have been obtained by many authors.[2] Since SU(4) symmetry is badly broken, contributions from higher orders of perturbation are expected to be significant. In the case of charmed hadrons, not much experimental information is available, but the observed charmed mass spectrum seems to require a nonzero contribution from higher orders of perturbation. Observed masses[3] of the charmed mesons do not satisfy first-order mass sum rules in quadratic form and favor linear mass formulas poorly. Even in the uncharmed sector, $\pi^+ \neq \pi^0$ and $\Sigma^+ - \Sigma^0 \neq \Sigma^0 - \Sigma^-$ indicate the presence of a higher-order symmetry-breaking interaction.[4] Second-order mass breaking has been considered earlier in the current-algebra framework.[5] Two of us (RVC and MPK)[6] have also derived higher-order mass formulas by assuming that the second-order mass-breaking Hamiltonain transforms like the $\underline{20}''$ and $\underline{84}$ representations of SU(4).

In the present work, we study the second-order mass-breaking contribution to various hadron multiplets in a dynamical consideration. We assume that the second-order mass breaking arises through the process $S + B \to B + S$, where S is the first-order mass-breaking spurion. We consider the transition in all three s, t, and u channels and express the amplitude for the process in terms of eigenamplitudes corresponding to the intermediate states present in these channels. We obtain constraints on the matrix elements by assuming that the nonexotic intermediate states contribute dominantly.

We find that the t-channel contribution obeys the mass relations of the first-order perturbation. This is an obvious consequence of the fact that only the $\underline{15}$ and singlet intermediate states are allowed to appear in the t channel. However, the s- and u-channel contributions are different from first-order mass breaking. We express the second-order mass parameters in terms of the masses of hadrons and thereby relate the discrepancies present in the first-order mass formulas. First, we neglect the electromagnetic interaction and obtain relations among various isomultiplets. Then the electromagnetic (em) mass differences among different charge states of isomultiplets are related.

II. MASS RELATIONS

The first-order mass-breaking spurion is assumed (neglecting electromagnetism) to transform like the $T_3^3 + yT_4^4$ component of the $\underline{15}$ representation.[1] The inclusion of the em mass breaking modifies the interaction to

$$S_b^a \sim aT_1^1 + bT_3^3 + cT_4^4 . \tag{1}$$

A. $J^P = \frac{1}{2}^+$ baryons

The parity-conserving mass-breaking Hamiltonian for the process

$$S + B(\tfrac{1}{2}^+) \to B(\tfrac{1}{2}^+) + S \tag{2}$$

has the form

$$\begin{aligned} & a_1(\overline{B}{}^m_{[c,d]} B_m^{[c,d]} S_a^b S_b^a) + a_2(\overline{B}{}^d_{[c,a]} B_e^{[c,b]} S_d^e S_b^a) \\ & + a_3(\overline{B}{}^b_{[c,d]} B_a^{[c,e]} S_e^d S_b^a) + a_4(\overline{B}{}^c_{[d,a]} B_c^{[e,b]} S_e^d S_b^a) \\ & + a_5[(\overline{B}{}^c_{[e,d]} B_c^{[e,b]} S_a^d S_b^a) + (\overline{B}{}^c_{[e,a]} B_c^{[e,d]} S_d^b S_b^a)] \\ & + a_6[(\overline{B}{}^c_{[e,d]} B_a^{[e,b]} S_c^d S_b^a) + (\overline{B}{}^b_{[e,a]} B_c^{[e,d]} S_d^c S_b^a)] \\ & + a_7[(\overline{B}{}^c_{[e,d]} B_a^{[e,d]} S_c^b S_b^a) + (\overline{B}{}^b_{[e,d]} B_c^{]e,d]} S_d^c S_b^a)] , \end{aligned} \tag{3}$$

where the tensors $B_c^{[a,b]}$ and $\overline{B}{}^a_{[b,c]}$ represent baryon ($\underline{20}'$) and antibaryon ($\underline{\overline{20}}'$) multiplets, respectively, and S_b^a is the spurion transforming like a member of the $\underline{15}$ representation.

Thus the second-order mass breaking involves seven parameters, yielding no useful relation. In order to reduce the number of parameters we assume:

(i) The nonexotic intermediate states appearing in the direct product $4\otimes4\otimes4$ contribute dominantly to the process. This leads to the conditions

$$2a_1=-a_4=-a_5=-a_6=2a_7\,, \tag{4}$$

for the s and u channels, and

$$a_2=a_3=a_4=a_6=0\,, \tag{5}$$

for the t channel.

(ii) The process is symmetric in the s and u channels. Physically speaking, we essentially assume the identity of reduced matrix elements in s and u channels.[7] This leads to

$$a_2=a_3\,. \tag{6}$$

Therefore, the s and u channels effectively involve two independent matrix elements.

1. Mass relations excluding electromagnetism

The t-channel contribution obeys the first-order mass-breaking relations.[2] This result is similar to the one in the nonleptonic decays,[8] where the t-channel contribution to the decay amplitude is negligibly small in the parity-conserving mode. Here, the process being parity conserving, there is effectively no contribution to the second-order mass breaking due to the t channel.

The s- and u-channel contributions relate the discrepancies present in the first-order mass sum rules as follows:

$$(\Xi_1-\Sigma_1)-(\Omega_1-\Xi_1)=\tfrac{1}{2}[3\Lambda+\Sigma-2(N+\Xi)]\,, \quad (0.012\text{ GeV}) \tag{7}$$

$$(\Omega_1-\Sigma_1)-(\Omega_2-\Xi_2)=(\Xi-\Sigma)\,, \quad (0.125\text{ GeV}) \tag{8}$$

$$(\Xi_1'-\Lambda_1')+2(\Omega_2-\Xi_2)-3(\Xi_1-\Sigma_1)=2(\Sigma-\Lambda)\,, \quad (0.156\text{ GeV}) \tag{9}$$

$$\left[\frac{2(\Sigma_1-\Sigma)-2\Xi_2-\Xi_1+3\Xi_1'}{2(N+\Xi_2)-3\Lambda_1'-\Sigma_1}+1\right]=\left[\frac{2(\Sigma_1-\Sigma)-2\Xi_2-\Xi_1+3\Xi_1'}{2(N+\Xi)-3\Lambda-\Sigma}+\frac{2(N+\Xi_2)-3\Lambda_1'-\Sigma_1}{2(N+\Xi)-3\Lambda-\Sigma}\right]^{-1}. \tag{10}$$

Relations (7) and (8) have been derived earlier by two of us, assuming that the second-order mass breaking lies in the $\underline{84}$ representation.[6]

At present, there is some evidence for the existence of a few charmed baryon states. Peaks observed in the BNL neutrino-proton collision[9]

$$\nu p\to\mu^-\Lambda\pi^+\pi^+\pi^+\pi^- \tag{11}$$

and Fermilab photoproduction data[10] on

$$\gamma+\text{Be}\to\overline{\Lambda}+\text{pions} \tag{12}$$

indicate the existence of the $J^P=\frac{1}{2}^+$ states, $\Lambda_1'^+$ (2.26 GeV) and Σ_1^0 (2.43 GeV). Using these mass values and those of the ordinary baryons, we estimate the masses of charmed baryons as displayed in Table I. Masses of isomultiplets involve parameters f and g giving the extent of the second-order SU(4)-breaking interaction.

TABLE I. Masses of charmed baryons $J^P=\frac{1}{2}^+$.

Charmed isomultiplet		First-order breaking (GeV)	Present analysis (GeV)
$B(6)$ $C=1$	Σ_1	2.43 [a]	2.43 [a]
	Ξ_1	2.62	$2.62+f$
	Ω_1	2.81	$2.81+2f$
$B(3)$ $C=2$	Ξ_2	3.67	$3.67+g$
	Ω_2	3.92	$3.92+2f+g$
$B(3)$ $C=1$	Λ_1'	2.26 [a]	2.26 [a]
	Ξ_1'	2.49	$2.49+f/3$

[a] Inputs.

2. Mass relations including electromagnetism

Taking the electromagnetic mass breaking into consideration, we obtain the following relations.

For the t channel, the first-order mass-breaking relations are

$$(\Xi^0-\Xi^-)-(\Sigma^+-\Sigma^-)=(n-p)\,, \tag{13}$$

$$(\Xi_2^{++}-\Xi_2^+)-(\Sigma_1^{++}-\Sigma_1^0)=(n-p)\,, \tag{14}$$

$$(\Sigma_1^{++}-\Sigma_1^+)=(\Xi_1^+-\Xi_1^0)=(\Sigma^+-\Sigma^0)=(\Sigma^0-\Sigma^-), \quad (0.003\text{ GeV})\ (0.005\text{ GeV}) \tag{15}$$

$$(\Xi_2^{++}-\Xi_2^+)=(\Xi^0-\Xi^-)\,, \quad (-0.007\text{ GeV}) \tag{16}$$

$$6(\Xi_1'^0-\Xi_1'^+)=(n-p)-5(\Xi^0-\Xi^-)\,, \quad (0.036\text{ GeV}) \tag{17}$$

$$m_{\Lambda_1'^+\Sigma_1^+}=m_{\Lambda\Sigma^0}=m_{\Xi_1'^0\Xi_1^0}-m_{\Xi_1'^+\Xi_1^+}=(1/\sqrt{3})[(\Sigma^0-\Sigma^-)-(\Xi^0-\Xi^-)]\,. \quad (0.0012\text{ GeV}) \tag{18}$$

Relation (13) is the well known Coleman and Glashow relation and (14) is its charmed analog.[11]

In the presence of s- and u-channel contributions, we get the following results:

(i) Coleman and Glashow relation (13) and its charmed analog (14),

(ii) $$\Sigma_1^{++}+\Sigma_1^0-2\Sigma_1^+=\Sigma^++\Sigma^--2\Sigma^0\,, \quad (0.002\text{ GeV}) \tag{19}$$

and

(iii) $$(\Xi_2^{++}-\Xi_2^+)-2(\Sigma_1^+-\Sigma_1^0)=(\Xi^0-\Xi^-)-2(\Sigma^0-\Sigma^-)\,. \quad (0.003\text{ GeV}) \tag{20}$$

Relations (13), (14), and (19) have been obtained in the general quark model[11] considerations. In fact, it has earlier been noted by two of us that these sum rules are obeyed in the presence of second-order mass breaking belonging to the $\underline{20}''$ and $\underline{84}$ representations.[6]

B. $J^P = \frac{3}{2}^+$ baryons

Here the parity-conserving transition

$$S + D(\tfrac{3}{2}^+) \to D(\tfrac{3}{2}^+) + S \tag{21}$$

involves the following components:

$$\begin{aligned} &a_1[\bar{D}^{[mnp]} D_{[mnp]} S^a_b S^b_a] , \\ &a_2[(\bar{D}^{[mna]} D_{[mnp]} S^p_b S^b_a) + (\bar{D}^{[mnp]} D_{[mnb]} S^a_p S^b_a)] , \\ &a_3[\bar{D}^{[maq]} D_{[mbp]} S^p_q S^b_a] , \end{aligned} \tag{22}$$

where the tensors $D_{[abc]}$ and $\bar{D}^{[abc]}$ represent $J^P = \frac{3}{2}^+$ baryons and antibaryons lying in the $\underline{20}$ and $\underline{\overline{20}}$ representations, respectively.

There are apparently three parameters expressing the second-order contribution, but effectively there is only one, i.e., a_3, since a_1 and a_2 obey the equal-spacing rule given by the first-order perturbation. The discrepancies in the equal spacing rule are related through the relations given below.

1. Among isomultiplets

We obtain the relations

$$\begin{aligned} &(\Omega - \Delta) \quad = 3(\Xi^* - \Sigma^*) , \\ &(0.443 \text{ GeV}) \quad (0.447 \text{ GeV}) \end{aligned} \tag{23}$$

$$(\Omega_3^* - \Delta) = 3(\Xi_2^* - \Sigma_1^*) , \tag{24}$$

$$(\Omega_3^* - \Omega) = 3(\Omega_2^* - \Omega_1^*) , \tag{25}$$

$$\begin{aligned} &\Omega_1^* + \Sigma_1^* - 2\Xi_1^* = \Omega + \Sigma^* - 2\Xi^* , \\ &\qquad (0.007 \text{ GeV}) \end{aligned} \tag{26}$$

$$\Omega_2^* + \Sigma^* - 2\Xi_1^* = \Xi_2^* + \Delta - 2\Sigma_1^* . \tag{27}$$

These sum rules have earlier been obtained by Hendry and Lichtenberg in the quark model.[11] Two of us have also derived these sum rules by considering the second-order effect arising from the $\underline{84}$ representation.[6]

Assuming nonexoticity of the intermediate states we get the following conditions:

$$a_1 = a_2 = 0 \quad \text{for the } s \text{ and } u \text{ channels} \tag{28}$$

and

$$a_3 = 0 \quad \text{for the } t \text{ channel.} \tag{29}$$

Here the s- and u-channel symmetry of the transition leads to no additional constraints. We notice that the t-channel contribution obeys the equal spacing rule and the s- and u-channel contributions simply lead to relations (23) to (27).

TABLE II. Masses of charmed baryons $J^P = \frac{3}{2}^+$.

Charmed isomultiplet		First-order breaking (GeV)	Present analysis (GeV)
$D(6)$ $C=1$	Σ_1^*	2.48 [a]	2.48 [a]
	Ξ_1^*	2.63	$2.63 + f'$
	Ω_1^*	2.78	$2.78 + 2f'$
$D(3)$ $C=2$	Ξ_2^*	3.73	$3.73 + g'$
	Ω_2^*	3.88	$3.88 + 2f' + g'$
$D(1)$ $C=3$	Ω_3^*	4.98	$4.98 + 3g'$

[a] Inputs.

In the Knapp photoproduction experiment,[10] there is also evidence for the $J^P = \frac{3}{2}^+$ state Σ_1^{*0} at 2.48 GeV mass value. Using this value as input we get the masses of other charmed isomultiplets as shown in Table II. Isomultiplet masses involve parameters f' and g' which give the extent of the second-order SU(4)-breaking interaction.

2. Among em mass differences

The second-order symmetry-breaking Hamiltonian (22) relates the discrepancies in the first-order em mass relations as

$$(\Delta^{++} - \Delta^-) = 3(\Delta^+ - \Delta^0) , \tag{30}$$

$$\begin{aligned} \Sigma_1^{*++} + \Sigma_1^{*0} - 2\Sigma_1^{*+} &= \Delta^+ + \Delta^- - 2\Delta^0 \\ &= \Sigma^{*+} + \Sigma^{*-} - 2\Sigma^{*0} , \\ &\quad (0.0058 \text{ GeV}) \end{aligned} \tag{31}$$

$$(\Xi^{*0} - \Xi^{*-}) + (\Delta^0 - \Delta^-) = 2(\Sigma^{*0} - \Sigma^{*-}) , \tag{32}$$

$$(\Xi_2^{*++} - \Xi_2^{*+}) + (\Delta^0 - \Delta^+) = 2(\Sigma_1^{*+} - \Sigma_1^{*0}) . \tag{33}$$

These sum rules have already been obtained by Franklin[11] and two of us[6] in quark-model and SU(4)-symmetry considerations, respectively.

In our present analysis we also obtain the following relations:

$$\left(\frac{(\Xi_1^* - \Sigma_1^*) - (\Omega - \Xi^*)}{(\Xi^* - \Sigma^*) - (\Sigma^* - \Delta)} + 2\right) = \left[\frac{(\Xi_1^* - \Sigma_1^*) - (\Omega - \Xi^*)}{(\Xi_2^* - \Sigma_1^*) - (\Sigma_1^* - \Delta)} + 2\left(\frac{(\Xi^* - \Sigma^*) - (\Sigma^* - \Delta)}{(\Xi_2^* - \Sigma_1^*) - (\Sigma_1^* - \Delta)}\right)\right]^{-1} , \tag{34}$$

$$\frac{(\Sigma^{*+} - \Sigma^{*0}) - (\Sigma^{*0} - \Sigma^{*-})}{(\Xi^{*0} - \Xi^{*-}) - (\Sigma^{*0} - \Sigma^{*-})} = \frac{(\Xi^{*0} - \Xi^{*-}) - (\Sigma^{*0} - \Sigma^{*-})}{(\Xi^* - \Sigma^*) - (\Sigma^* - \Delta)} , \tag{35}$$

$$\frac{(\Xi_2^{*++}-\Xi_2^{*\,+})-(\Sigma_1^{*+}-\Sigma_1^{*0})}{(\Xi^{*0}-\Xi^{*-})-(\Sigma^{*0}-\Sigma^{*-})}=\left[\frac{(\Xi_2^{*}-\Sigma_1^{*})-(\Sigma_1^{*}-\Delta)}{(\Xi^{*}-\Sigma^{*})-(\Sigma^{*}-\Delta)}\right]^{1/2}. \tag{36}$$

Notice that the mass relations (35) and (36) relate the isomultiplet mass differences and the em mass differences. Since the masses of $J^P=\frac{3}{2}^+$ baryons are not known accurately, these relations are untestable. However, the present mass values of the left-hand side (LHS) and the right-hand side (RHS) give

LHS = 5.8 ± 6.0 MeV and RHS = 0.69 ± 4.4 MeV

for relation (35).

C. Psuedoscalar and vector mesons

We consider the mass breaking arising through the process

$$S+P\to P+S \tag{37}$$

in different channels. Including only the nonexotic intermediate states belonging to the 15-plet and singlet, the following contractions appear:

$$a_1^t(P_b^aP_a^bS_c^dS_d^c)+(a_{\underline{15}_1}^t+a_{\underline{15}_2}^t)[(P_b^mP_m^aS_n^bS_a^n)+(P_m^aP_b^mS_a^nS_n^b)] \tag{38}$$

in the t channel and

$$a_1^{s/u}(P_b^aP_c^dS_a^bS_d^c)+a_{\underline{15}_1}^{s/u}[(P_b^mP_n^bS_m^aS_a^n)+(P_m^aP_a^nS_b^mS_n^b)]$$
$$+a_{\underline{15}_2}^{s/u}[(P_b^mP_a^nS_m^aS_n^b)+(P_m^aP_n^bS_b^mS_a^n)] \tag{39}$$

in the s and u channels.

Here also the t channel obeys the first-order mass relations. At present, the masses of all the charmed mesons[3] are known. However, these mass values do not seem to satisfy the first-order mass relations in the linear form as well as in the quadratic form, e.g.,

$(F-D)$	=	$(K-\pi)$		
(0.1745 GeV)		(0.357 GeV)	linear formula	(40)
(0.6776 GeV^2)		(0.2266 GeV^2)	quadratic formula,	

indicating a large second-order mass contribution, which may appear through the s and u channels. Even in the uncharmed sector, $\pi^+\neq\pi^0$ has been indicating the presence of nonzero second-order interactions. Since s and u channels involve four parameters, no useful relation could be obtained. However, discrepancies in the sum rules are found to be

	Linear formula	Quadratic formula	
$(F-D)-(K-\pi)$	= (− 0.183 GeV)	(0.4554 GeV^2)	(41)
$3(\eta-\pi)-4(K-\pi)$	= (− 0.195 GeV)	(− 0.0568 GeV^2)	(42)
$2(\pi^0-\pi^+)$	= (− 0.010 GeV)	(− 0.0028 GeV^2)	(43)
$(K^+-K^0)+(D^+-D^0)$	= (+ 0.001 GeV)	(0.0236 GeV^2) .	(44)

Similarly for vector mesons one can see that the first-order mass relation is not satisfied, e.g.,

(F^*-D)	=	$(K^*-\rho)$		
(0.133 GeV)		(0.122 GeV)	linear formula	(45)
(0.557 GeV^2)		(0.1937 GeV^2)	quadratic formula ,	

which again shows the large second-order mass contribution, especially in the quadratic form.

III. CONCLUSION

In the present paper, we have studied the second-order mass-breaking contributions to the baryons and mesons in a dynamical model. We have assumed that the second-order mass breaking arises through the process $S+B\to B+S$ in s, t, and u channels and obtains dominant contribution from the nonexotic intermediate states. In the case of the charmed baryons, not much experimental data are available to check the presence of higher-order breaking. But in the case of the charmed mesons, $(F-D)\neq(K-\pi)$ does indicate a substantial contribution from the second-order breaking. Even the inequalities of the em mass difference among uncharmed isomultiplets, namely $\pi^+\neq\pi^0$ and $\Sigma^+-\Sigma^0\neq\Sigma^0-\Sigma^-$, have already demanded additional symmetry-breaking interaction. We notice that the t-channel contribution obeys the mass relations obtained in the first order of perturbation, as a consequence of the presence of the singlet and 15 as the intermediate states. Thus the second-order mass-breaking contribution appears effectively from the s and u channels. For the mesons, the s- and u-channel contributions involve many parameters making the estimation of mass difficult. For the $J^P=\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons, we have estimated the masses of charmed isomultiplets in Table I and II, using $\Lambda_1'^+$(2.26 GeV),

Σ_1^0(2.43 GeV), and Σ_1^{*0}(2.48 GeV) as inputs which have been observed in BNL neutrino-proton collision and the Fermilab photoproduction experiments. Among the em mass differences, we notice that the second-order perturbation preserves the Coleman and Glashow relation and its charmed analog for the $\frac{1}{2}^+$ baryons.

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[1]S. Okubo *et al.*, Phys. Rev. Lett. 34, 38 (1975); 34, 236 (1975); D. H. Boal *et al.*, *ibid.* 34, 541 (1975); J. W. Moffat, Phys. Rev. D 12, 286 (1975).

[2]D. B. Lichtenberg, Phys. Rev. D 12. 3760 (1975); 14, 1412 (1976); M. Inose and Reiji Sugano, Prog. Theor. Phys. 59, 2171 (1978); T. K. Zadoo and M. Ahmad, Can. J. Phys. 55, 783 (1977); C. A. Nelson, Phys. Rev. D 13, 2139 (1976).

[3]Particle Data Group, Phys. Lett. 75B, 1 (1978).

[4]S. Eliezer and P. Singer, J. Phys. A 6, 661 (1973); Phys. Rev. D 8, 2235 (1973).

[5]R. Simard and M. Suzuki, Phys. Rev. D 12, 2002 (1975).

[6]R. C. Verma and M. P. Khanna, Phys. Rev., D 18, 828 (1978).

[7]The identity of the *s*- and *u*-channel reduced matrix elements has led to the experimentally well satisfied Lee-Sugawara sum rule in the case of parity-conserving weak nonleptonic decays. Kohra and Nishijima (Ref. 12) were the first to arrive at this symmetry for the parity-conserving Hamiltonian.

[8]R. C. Verma and M. P. Khanna, Prog. Theor. Phys. 57, 539 (1977).

[9]E. G. Cazzoli *et al.*, Phys. Rev. Lett. 34, 1125 (1975).

[10]B. Knapp *et al.*, Phys. Rev. Lett. 37, 882 (1976).

[11]J. Franklin, Phys. Rev. D 12, 2077 (1975); A. W. Hendry and D. B. Lichtenberg, *ibid.* 12, 2756 (1975).

[12]Y. Kohra and K. Nishijima, Prog. Theor. Phys. 46, 348 (1971); 47, 648 (1972).