PHYSICAL REVIEW D VOLUME 21, NUMBER 5 1 MARCH 1980

Dynamical effects in two-body charm decay

John F. Donoghue
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Barry R. Holstein
Physics Division, National Science Foundation, Washington, D.C. 20550
(Received 13 September 1979)

Two-body nonleptonic decays of the $D(1865)$ mesons are studied. Three mechanisms—quark-mass-dependent effects, final-state interactions, and mixing-angle effects in the six-quark model—are shown to have the capability to substantially modify naive theoretical expectations for SU(3) and quark-model relations among the various modes.

I. INTRODUCTION

The dominant decay modes of strange and charmed particles are into purely hadronic final states. The variety of available final states for charmed-meson decays make them especially interesting and challenging to study. The main purpose of this paper is to discuss possible dynamical mechanisms which may cause violations of SU(3) sum rules[1] based on the Glashow-Iliopoulos-Maiani (GIM) Hamiltonian of the Weinberg-Salam model.[2] This is of special interest in light of a recent experiment indicating possible violations of two of these sum rules.[3]

Charm decays in the four-quark GIM theory have a three-tiered hierarchy of channels. The three sets of interactions form a U-spin vector, with the "Cabibbo-favored" modes (having amplitudes proportional to $\cos^2\theta_C$) transforming as $\Delta C = \Delta S = -1$, $\Delta I = 1$, $U_z = +1$, the Cabibbo-suppressed channel (proportional to $\cos\theta_C \sin\theta_C$) obeying $\Delta C = -1$, $\Delta S = 0$, $\Delta I = \frac{1}{2}, \frac{3}{2}$, $U_z = 0$, and doubly suppressed decays (proportional to $\sin^2\theta_C$) with $\Delta C = -1$, $\Delta S = +1$, $\Delta I = 1$, $U_z = -1$. From these properties, *independent of the possible short-distance strong-interaction enhancement of quantum chromodynamics*[4] *(QCD)* [but assuming SU(3) symmetry], several sum rules can be derived relating favored and suppressed modes. Those in which we will be especially interested below are[1]

$$\Gamma(D^0 \to \pi^+ K^-) = \cot^2\theta_C \Gamma(D^0 \to K^+ K^-) \tag{1a}$$

$$= \cot^2\theta_C \Gamma(D^0 \to \pi^+ \pi^-), \tag{1b}$$

$$\Gamma(D^0 \to K^0 \bar{K}^0) = 0, \tag{1c}$$

$$\Gamma(D^+ \to \pi^+ \bar{K}^0) = 2\cot^2\theta_C \Gamma(D^+ \to \pi^+ \pi^0). \tag{1d}$$

Equations (1a) and (1b) have been tested recently and, although with large experimental errors, have been found to be violated:

$$\frac{\Gamma(D^0 \to K^+ K^-)}{\Gamma(D^0 \to \pi^+ K^-)} = (2.3 \pm 0.6)\tan^2\theta_C, \qquad \frac{\Gamma(D^0 \to \pi^+ \pi^-)}{\Gamma(D^0 \to \pi^+ K^-)} = (0.66 \pm 0.30)\tan^2\theta_C. \tag{2}$$

In the six-quark model[5] there are additional unknown mixing angles which can change these predictions. We consider the effects of extra mixing angles and of the "penguin diagram," in the context of the six-quark model, in Sec. IV. A smaller SU(3)-breaking effect is that of phase space. The predictions which follow are always for the reduced width, with corresponding phase-space factor divided out.

Not all decay rates are connected by SU(3) considerations alone. The quark model can be used to provide additional relations. For example, Cabibbo and Maiani, using the vacuum-intermediate-state method, give definite predictions for all two-body charm decay rates.[6] One such relation is

$$\frac{\Gamma(D^0 \to \pi^0 \bar{K}^0)}{\Gamma(D^0 \to \pi^+ K^-)} = 0.025$$

which will be discussed in more detail later since, in addition to discussing the violation of SU(3) sum rules, we wish also to explore the dynamics which could invalidate such quark-model predictions.

In Secs. II–IV we discuss three separate mechanisms which will affect two-body charm decay. Section II is devoted to a helicity-dependent effect in the matrix elements. Section III discusses the role of final-state interactions, and in Sec. IV we consider the effect of heavy quarks in the Cabibbo-suppressed decays. The summary, Sec. V, contains some additional brief comments and a comparison with other recent work.

II. HELICITY SUPPRESSION MECHANISM

A dynamical theory of nonleptonic processes has been long sought. Recently, techniques have been developed which allow an understanding of the $\Delta I = \frac{1}{2}$ rule in $\Delta S = 1$ nonleptonic decays.[7] For nonleptonic meson decays—$K \to 2\pi, 3\pi$—the crucial ingredients are (a) QCD radiative corrections to the weak Hamiltonian and (b) suppression of $\Delta I = \frac{3}{2}$ effects by helicity and color factors. It is naturally of interest to apply similar techniques to *charmed*-meson decays. Below we present such an evaluation and find mechanisms which break the U-spin sum rules in the experimentally found fashion. However, we first present a general discussion of the underlying physics before attempting a numerical evaluation.

Part of the reason for suppression of $\Delta I = \frac{3}{2}$ effects in nonleptonic kaon decays is that such transitions occur solely via an operator which is the product of two *left-handed* currents. Pseudoscalar meson matrix elements of such operators have a strong helicity suppression in that the effect of the upper components in quark wave functions are canceled by the lower components.[7] In Ref. 7, it was argued that this is a general feature of such mesonic matrix elements of $(V-A)(V-A)$ operators. A related consequence of this is that the sizes of the weak nonleptonic matrix elements are quite sensitive to the quark masses involved, since, as the quark masses decrease or increase, the helicity cancellation becomes increasingly or decreasingly complete. In charm decays, the dominant operators are again $(V-A)(V-A)$ products. However, operators which give rise to the decays $D^0 \to K^+K^-$, $D^0 \to \pi^+K^-$, and $D^0 \to \pi^+\pi^-$ involve 2, 1, and 0 strange quark fields, respectively. We shall demonstrate that increasing the average quark mass involved in the transition makes the helicity cancellation less complete, i.e., the matrix element becomes larger. This will then introduce SU(3) breaking in the form

$$\cot^2\theta_C\,\Gamma(D^0 \to K^+K^-) > \Gamma(D^0 \to \pi^+K^-) > \cot^2\theta_C\,\Gamma(D^0 \to \pi^+\pi^-)\,, \qquad (3)$$

as will be borne out by more careful analysis.

Our previous calculations of $\Delta S = 1$ kaon decay parameters utilized current-algebra and PCAC (partial conservation of axial-vector current) techniques in order to reduce $K \to 2\pi, 3\pi$ matrix elements to calculable $K \to \pi$ transitions. In the absence of reliable methods for the treatment of charm decays we shall employ similar procedures to those utilized in analysis of the kaon sector. One should be well aware of possible problems inherent in this approach—e.g., the much larger extrapolations to the soft-pion limit and previous difficulties in accommodating charm systems within the MIT bag model.[8] However, given that we are only concerned with ratios of decay rates, it is likely that many such uncertainties will tend to cancel out, such as those related to the soft-pion continuation and to normalization uncertainties within the bag model. Also, recent work by Ponce[9] has shown how charm may be accommodated within a modified bag approach. The results obtained appear reasonable and bear out our general dynamical expectations.

The existence of current-algebra–PCAC constraints[10] together with the required vanishing of the mesonic amplitudes in the SU(4)-symmetry limit[11] [the generalization of the theorem which requires $K \not\to 2\pi$ in the SU(3) limit[12]] imply a substantial momentum dependence for the decay amplitudes. If we expand to first order in momentum these constraints uniquely determine the form of the decay amplitude

$$\langle M_{q_1} M'_{q_2} | \mathcal{O} | D^0_k \rangle \propto -\frac{i}{F_{M'}}(k^2 - q_1{}^2)\langle M_{q_1} | [F^5_{M'}, \mathcal{O}] | D^0_k \rangle - \frac{i}{F_M}(k^2 - q_2{}^2)\langle M'_{q_2} | [F^5_M, \mathcal{O}] | D^0_k \rangle\,, \qquad (4)$$

where F^5_M is the axial charge which has the same SU(3) transformation properties as the meson M, of which F_M is the meson decay constant.

Via this algorithm, then, all two-body charm-decay amplitudes can be written in terms of combinations of $D \to K$ and $D \to \pi$ matrix elements, which can be calculated for a quark model in terms of standard spin and flavor counting factors and a dynamical reduced matrix element.[13] Calculation of this reduced matrix element in the MIT bag model reveals a rather small number due to cancellation between the upper and lower components of the quark wave functions—a result we term "helicity suppression."[14] A physical understanding of what is going on here can be obtained by considering $D^0 \to e\nu$. With the weak interactions constructed of a product of $V-A$ currents this amplitude is suppressed by the difficulty of forming the helicities of e and ν into a pseudoscalar state, and in fact the amplitude vanishes as $m_e \to 0$. The reduced matrix element we are dealing with is suppressed for a similar reason—except that in our case the role of $e\nu$ is played by $q\bar{q}$ quarks which form the final meson. The smaller (larger) the constituent quark masses in the $q\bar{q}$ system, the more (less) complete the helicity suppression and the smaller (larger) the value of the reduced matrix element. The $D \to K$ transition involves *two* light quarks (u,d) plus a charmed and a strange quark while the $D\to\pi$ amplitude involves a charmed quark plus *three* light quarks. Thus we expect the $D\to K$ amplitude to be larger (the helicity suppression to be less complete), and this is borne out by MIT bag model calculations, which yield

$$\eta \equiv \frac{A(D\to K)}{A(D\to \pi)} \approx 1.6\text{–}1.8 \tag{5}$$

for reasonable choices of parameters.

We can now proceed to calculate the decay amplitude. For the weak Hamiltonian we use the GIM form as modified by the strong interactions and calculated via short-distance-expansion and renormalization-group techniques[15]:

$$\begin{aligned}\mathcal{H}_w = \frac{G}{2\sqrt{2}}[&\cos^2\theta_C(c_-\mathcal{L}^{(-)}+c_+\mathcal{L}^{(+)})\\ &-\cos\theta_C\sin\theta_C(c_-\mathcal{O}^{(-)}+c_+\mathcal{O}^{(+)})\\ &+O(\sin^2\theta_C)]\,,\end{aligned} \tag{6a}$$

where

$$\begin{aligned}\mathcal{L}^{(\mp)} &= \bar{u}\gamma_\mu(1+\gamma_5)d\bar{s}\gamma^\mu(1+\gamma_5)c\\ &\mp\bar{u}\gamma_\mu(1+\gamma_5)c\bar{s}\gamma^\mu(1+\gamma_5)d\,,\\ \mathcal{O}^{(\mp)} &= \bar{u}\gamma_\mu(1+\gamma_5)d\bar{d}\gamma^\mu(1+\gamma_5)c\\ &\mp\bar{u}\gamma_\mu(1+\gamma_5)c\bar{d}\gamma^\mu(1+\gamma_5)d\\ &-\bar{u}\gamma_\mu(1+\gamma_5)s\bar{s}\gamma^\mu(1+\gamma_5)c\\ &\pm\bar{u}\gamma_\mu(1+\gamma_5)c\bar{s}\gamma^\mu(1+\gamma_5)s\,.\end{aligned} \tag{6b}$$

G ($\approx 10^{-5}m_p^{-2}$) is the conventional weak-coupling constant, θ_C ($\approx 15°$) is the Cabibbo angle, and c_-, c_+ are the usual renormalization-group enhancement factors[16]

$$c_-\approx(4)^{0.48}\,,\quad c_+\approx(4)^{-0.24}\,. \tag{7}$$

As we are only interested in ratios, we can express the two-body decay amplitudes in terms of the coefficients $c_\mp$, an overall factor A_0, and the reduced matrix element ratio η:

$$\begin{aligned}A(D^0\to\pi^+K^-) &= A_0\left[(c_--2c_+)\eta\frac{F_K}{F_\pi}+4c_+\right],\\ A(D^0\to\pi^0\bar{K}^0) &= A_0\frac{1}{\sqrt{2}}(c_--2c_+)\left(2\frac{F_K}{F_\pi}\eta-1\right),\\ A(D^+\to\bar{K}^0\pi^+) &= -A_0\left[(c_--2c_+)\eta\frac{F_K}{F_\pi}-(c_-+2c_+)\right],\\ A(D^0\to\pi^+\pi^-) &= A_0\tan\theta_C(c_-+2c_+)\frac{F_K}{F_\pi}\,,\\ A(D^0\to K^+K^-) &= A_0\tan\theta_C(c_-+2c_+)\eta\,,\\ A(D^0\to\pi^0\pi^0) &= A_0\tan\theta_C(c_--2c_+)\frac{F_K}{F_\pi}\,,\\ A(D^0\to K^0\bar{K}^0) &= 0\,,\\ A(D^+\to\pi^+\pi^0) &= A_0\tan\theta_C 2\sqrt{2}\,c_+\frac{F_K}{F_\pi}\,,\\ A(D^+\to K^+\bar{K}^0) &= A_0\tan\theta_C(c_-+2c_+)\eta\,.\end{aligned} \tag{8}$$

We note that there is a specific pattern for violation of the U-spin sum rules. For example,

$$\frac{\Gamma(D^0\to K^+K^-)}{\Gamma(D^0\to\pi^+\pi^-)}=\eta^2\frac{F_\pi^{\,2}}{F_K^{\,2}} \tag{9}$$

independent of enhancement factors. Using $F_K/F_\pi = 1.2$ (Ref. 17) and Eq. (5) this ratio is between 1.5 and 2.3. Other ratios depend on the values of c_-, c_+, and η. For example, using $c_-=2$, $c_+=1/\sqrt{2}$, and $\eta^2=3$, we find

$$\frac{\cot^2\theta_C\,\Gamma(D^0\to\pi^+\pi^-)}{\Gamma(D^0\to\pi^+K^-)}\approx 1\,. \tag{10a}$$

If we had included the extra mixing angles in the six-quark model, they would reduce this slightly:

$$\frac{2\cot^2\theta_C\,\Gamma(D^+\to\pi^+\pi^0)}{\Gamma(D^+\to\pi^+\bar{K}^0)}\approx 2.2\,. \tag{10b}$$

Both of these are rather sensitive to the interplay of η and c_+/c_-. The U-spin prediction

$$\Gamma(D^0\to K^0\bar{K}^0)=0 \tag{11}$$

remains unchanged.

This procedure also predicts relations among decays which are not related by SU(3) sum rules. For example, independent of enhancement factors or SU(3) breaking,

$$\frac{\Gamma(D^+\to K^+\bar{K}^0)}{\Gamma(D^0\to K^+K^-)}=1\,. \tag{12}$$

The Cabibbo allowed mode $D^0\to\bar{K}^0\pi^0$ has an interesting cancellation between c_+ and c_- which forces it to be reduced to the level of the Cabibbo-suppressed channels. For example,

$$\begin{aligned}\frac{\Gamma(D^0\to\bar{K}^0\pi^0)}{\Gamma(D^0\to K^+K^-)} &= \tfrac{1}{2}\cot^2\theta_C\left(\frac{c_--2c_+}{c_-+2c_+}\right)^2\left(\frac{2(F_K/F_\pi)\eta-1}{\eta}\right)^2\\ &\approx 0.05\cot^2\theta_C\\ &\approx 1\,,\end{aligned} \tag{13}$$

where the numbers correspond to $c_-=2$, $c_+=1/\sqrt{2}$, $\eta^2=3$. The main effect which leads to this strong suppression is a color-counting factor which generates the term (c_--2c_+). This is a rather general feature of quark models and has been previously noted.[6] However, in Sec. III we will show how final-state interactions can dramatically modify this result.

III. FINAL-STATE INTERACTIONS

Unfortunately, although the results given in Sec. II are simple and reproduce the experimentally observed pattern for the violation of the SU(3) sum rules [Eqs. (1a) and (1b)], this picture is too naive to provide other than rough semiquantitative guidance concerning the relative rates for these decay modes because of the omission of another important dynamical mechanism of SU(3) breaking—final-state-interaction effects. In order to see how crucial such effects can be in determining the relative decay rates consider the Cabibbo-allowed modes

$$D^0 \to K^-\pi^+,\ \bar{K}^0\pi^0$$

whose ratio in our amplitude model [Eq. (8)] is given by

$$\frac{\Gamma(D^0\to\bar{K}^0\pi^0)}{\Gamma(D^0\to K^-\pi^+)}=\frac{\frac{1}{2}[2(F_\pi/F_K)\eta-1]^2(c_- - 2c_+)^2}{[4c_+ + (c_- - 2c_+)\eta F_\pi/F_K]^2} \approx 0.02\eta^2 \ll 1\,. \tag{14}$$

It is easy to imagine that final-state interactions may be quite important in these decays. The invariant mass of the $K\pi$ system is $M_D = 1.865$ GeV, which lies in the resonance region. Indeed, there is a strong $K\pi$ S-wave resonance nearby—i.e., the κ at 1.4 GeV. Final-state interactions will allow the $K^-\pi^+$ mode to convert into $K^0\pi^0$, thereby populating the latter channel, against the predictions of the naive quark model.

The calculations in Sec. II which lead to Eq. (14) hopefully hold with some degree of credibility at the soft-pion point

$$s=(p_K+p_\pi)^2 \xrightarrow[p_\pi\to 0]{} m_K^{\,2}\,, \tag{15}$$

while the physical *decay* occurs at

$$s = m_D^{\,2}\,. \tag{16}$$

In order to connect these two points believably we cannot use the simplistic approach of Sec. II but rather we need some sort of analytic continuation which takes into account the strong interactions in the final state. As a crude model, which maintains unitarity during such a continuation, consider the forms

$$\begin{aligned} A(D^0\to\bar{K}^0\pi^0) &= \sqrt{2}\,\frac{a_3}{D_3(s)}-\frac{a_1}{D_1(s)}\,,\\ A(D^0\to K^-\pi^+) &= \frac{a_3}{D_3(s)}+\sqrt{2}\,\frac{a_1}{D_1(s)}\,,\\ A(D^+\to\bar{K}^0\pi^+) &= 3\,\frac{a_3}{D_3(s)}\,, \end{aligned} \tag{17}$$

where a_3, a_1 are (real) amplitudes for decay into the $I=\frac{3}{2},\frac{1}{2}$ channels, respectively, while $D_3(s)$, $D_1(s)$ are the corresponding Omnes functions[18] normalized to unity at the soft-pion point. We require [cf. Eq. (14)] the soft-pion condition

$$\left(\frac{\sqrt{2}\,a_3/a_1-1}{a_3/a_1+\sqrt{2}}\right)^2 = 0.02\eta^2 \ll 1\,. \tag{18}$$

However, for the physical decay amplitudes in the unitarized model we have

$$R=\frac{\Gamma(D^0\to\bar{K}^0\pi^0)}{\Gamma(D^0\to K^-\pi^+)}=\left(\frac{\sqrt{2}\,\frac{a_3}{a_1}\,\frac{D_1(s)}{D_3(s)}-1}{\frac{a_3}{a_1}\,\frac{D_1(s)}{D_3(s)}+\sqrt{2}}\right)^2 . \tag{19}$$

The $I=\frac{3}{2}$ channel has no resonances from threshold to $S=M_D^{\,2}$ and so we take

$$\frac{1}{D_3(s)}\approx 1\,. \tag{20}$$

However, the $I=\frac{1}{2}$ channel possesses a resonance—the κ—at about 1.4 GeV, whereby

$$\mathrm{Re}\,\frac{1}{D_1(s)} \tag{21}$$

has a dispersive shape and changes sign between $s<m_K^{\,2}$ and $s>m_K^{\,2}$.[19] Thus the physical ratio may well have a *very different* value than that at the soft-pion point. A recent estimate by Kaptanoglu[19] gives

$$\frac{1}{D_1(s=m_D^{\,2})}\approx -2.5\,, \tag{22}$$

which predicts

$$R=0.9\,, \tag{23}$$

in very good agreement with the recently measured value at SLAC[20]

$$R=0.8\pm 0.4\,. \tag{24}$$

Thus, charge exchange originating in final-state interactions can have an appreciable effect and *must* be included for reliable predictions of charm decays.

The final-state-interaction mechanism given above has an interesting consequence. Since it enhances the $I=\frac{1}{2}$ final state, it leads to a relative enhancement of D^0 modes over D^+, since D^+ final states must be pure $I=\frac{3}{2}$. Indeed, our rate prediction [corresponding to Eq. (22)] is

$$\frac{\Gamma(D^+\to\pi^+\bar{K}^0)}{\Gamma(D^0\to\pi^+K^-)}=0.11\,.$$

However, the branching ratios for the modes have been measured

$$\frac{B(D^+\to\pi^+\bar{K}^0)}{B(D^0\to\pi^+K^-)}=0.76\pm 0.17\,.$$

These are consistent only if the D^0 lifetime is shorter than that of D^+:

$$\frac{\tau_{D^0}}{\tau_{D^+}}\sim 0.14\,.$$

This lifetime difference has in fact been recently observed at SLAC[21]:

$$\frac{\tau_{D^0}}{\tau_{D^+}} < 0.17 \pm 0.05 .$$

While our calculation does not by itself explain the lifetime difference, it is consistent with it and does suggest that the explanation may simply be that $I=\frac{1}{2}$ final states are favored over $I=\frac{3}{2}$ in all channels. Rosen has detailed some of the consequences of this assumption.[22]

Of course, similar final-state interaction corrections also affect the Cabibbo-suppressed decays $D^0 \to \pi^+\pi^-, K^+K^-$. However, here the analysis is considerably more involved because of the existence of two $I=0$ 0^+ resonances coupling *both* to $\pi\pi$ and $K\bar{K}$—$S^*(980)$ and $\epsilon(1300)$. Inelasticity probably plays then a large role—$\pi\pi \to K\bar{K}$ can proceed strongly near these resonances—and a reliable analysis will require very careful and detailed study.

It is certainly possible (and likely) then that in addition to helicity suppression contributions, final-state-interaction effects also play a role in producing the observed enhancement of $D^0 \to K^+K^-$ relative to $D^0 \to \pi^+\pi^-$. Indeed, temporarily neglecting the $\epsilon(1300)$, we might speculate that since the properties of the $I=1,0$ resonances $\delta(980)$, $S^*(980)$ are quite close, the Omnes functions for $I=1$ and $I=0$ $K\bar{K}$ channels will be correspondingly similar. In this case the predicted $D^0 \to K\bar{K}$ ratio will *not* be strongly affected by final-state interactions. On the other hand, since there is no resonance in the $I=2$ $\pi\pi$ channel corresponding to the $S^*(980)$ in the $I=0$ $\pi\pi$ channel, we should find an effect similar to that seen in the Cabibbo-allowed case [Eq. (19)]—a strong mixing between the naive $D^0 \to \pi^+\pi^-$ and $D^0 \to \pi^0\pi^0$ amplitude which, since the $D^0 \to \pi^0\pi^0$ amplitude is suppressed by the strong interaction coefficient $c_- - 2c_+$, which will tend to reduce the $D^0 \to \pi^+\pi^-$ amplitude relative to $D^0 \to K^+K^-$. However, it will require a careful and detailed analysis, including both inelasticity and $\epsilon(1300)$, in order to determine the size (and direction) of this effect reliably.

In general, the effects of final-state interactions can appear similar to the other effects discussed in this paper. The only clear distinction is in the $D^0 \to K^0\bar{K}^0$ channel. In both Secs. II and IV the mechanisms considered leave intact the prediction that this branching ratio must vanish. However, SU(3) breaking and inelasticity in the final-state interactions can presumably generate this mode.

IV. THE SIX-QUARK MODEL

For $\Delta S=1$ nonleptonic decays, when W-exchange graphs are replaced by effective four-quark operators via the Wilson short-distance expansion, QCD corrections calculated via renormalization-group techniques modify the Wilson coefficients, enhancing the coefficient of the $\Delta I=\frac{1}{2}$ operator with respect to that of the operator carrying $\Delta I=\frac{3}{2}$ by the factor $c_-/c_+ \sim (4)^{0.72} \sim 3$ which is insufficient to explain the factor of 20 or so seen experimentally.[4] The effective operators generated thereby still have the canonical $(V-A)\times(V-A)$ form. However, when the charmed quark is treated as heavy and careful attention is paid to the relationship between c- and u-quark loop-diagram corrections, (see Fig. 1) new operators with the structure $(V-A)\times(V+A)$ appear, as discussed by Shifman, Vainshtein, and Zakharov.[23] These operators carry $\Delta I=\frac{1}{2}$ and although they have rather small Wilson coefficients, matrix elements of such operators between mesonic states are greatly enhanced over those of the conventional $(V-A)\times(V-A)$ variety since operators of the $(V-A)\times(V+A)$ type can be Fierz transformed into a form involving scalar and pseudoscalar densities which are not affected by the helicity cancellation mechanism described in Sec. II.[7] Estimates of this effect in the MIT bag model give

$$\frac{\langle \pi | (V-A)\times(V-A) | K\rangle}{\langle \pi | (V-A)\times(V+A) | K\rangle} \approx 0.10 \tag{25}$$

for typical bag parameters.

These $(V-A)\times(V+A)$ terms, generated by the "penguin"[24] diagram of Fig. 1, were shown in Ref. 7 to be capable of providing the extra $\Delta I=\frac{1}{2}$ enhancement needed in order to understand nonleptonic kaon decays.

Similar penguin operators appear for Cabibbo-suppressed charm decays (but not for the Cabibbo-allowed sector). However, the s quark cannot be treated as heavy compared to the d quark since the typical momentum flowing through the loop is of order m_c. Nevertheless, even if one could perform such a heavy-quark expansion, the Wilson coefficient would be only[25]

$$c_5' \sim \frac{g^2}{24\pi^2} \ln \frac{m_c^2 + m_s^2}{m_c^2 + m_d^2} , \tag{26}$$

to be compared to

$$c_5 \sim \frac{g^2}{24\pi^2} \ln \frac{m_c^2}{\mu^2} , \tag{27}$$

in the $\Delta S=1$ case. Thus

$$c_5' \sim 10^{-3} c_5 \tag{28}$$

and this appears much too small to have any impact for Cabibbo-suppressed charm decays.[26]

If, however, we consider a six-quark model then there appears a penquin contribution from quark loop diagrams involving the b quark. The effect of such diagrams has been considered by Abbot, Sikivie, and Wise[25] who give the Cabibbo-suppressed QCD-renormalized Hamiltonian as

$$\mathcal{H}_w = -\frac{G}{2\sqrt{2}} c_1 c_2 s_1 \left[c_- \mathcal{O}^{(-)} + c_+ \mathcal{O}^{(+)} + \left(s_3^2 + \frac{s_2 s_3 c_3}{c_1 c_2} e^{i\delta} \right) \sum_{i=1}^{6} k_i \mathcal{O}_i \right], \tag{29}$$

where c_i, s_i are the usual mixing coefficients for the six-quark model, and

$$\begin{aligned} &k_1 \cong 1.37, \quad k_2 \cong -0.42, \quad k_3 = 0.030, \\ &k_4 = 0.006, \quad k_5 = -0.037, \quad k_6 = -0.007, \end{aligned} \tag{30}$$

are Wilson coefficients for the operators

$$\begin{aligned} \mathcal{O}_1 &= \bar{u}\lambda^a \gamma_\mu (1+\gamma_5) c \bar{s} \lambda^a \gamma_\mu (1+\gamma_5) s, \\ \mathcal{O}_2 &= \bar{u}\gamma_\mu (1+\gamma_5) c \bar{s} \gamma^\mu (1+\gamma_5) s, \\ \mathcal{O}_3 &= \bar{u}\gamma_\mu (1+\gamma_5) \lambda^a c \bar{Q} \gamma^\mu (1+\gamma_5) \lambda^a Q, \\ \mathcal{O}_4 &= \bar{u}\gamma_\mu (1+\gamma_5) c \bar{Q} \gamma^\mu (1+\gamma_5) Q, \\ \mathcal{O}_5 &= \bar{u}\gamma_\mu (1+\gamma_5) \lambda^a c \bar{Q} \gamma^\mu (1-\gamma_5) \lambda^a Q, \\ \mathcal{O}_6 &= \bar{u}\gamma_\mu (1+\gamma_5) c \bar{Q} \gamma^\mu (1-\gamma_5) Q. \end{aligned} \tag{31}$$

Here the operator Q is summed over the four quark flavors u, d, s, c. We will call $\mathcal{O}_1$–$\mathcal{O}_6$ generically by the name "penquin operator," although $\mathcal{O}_1$ and $\mathcal{O}_2$ are the result of the extra mixing angles in the six-quark model and not a consequence of Fig. 1. The coefficients k_3, k_4 are small and can

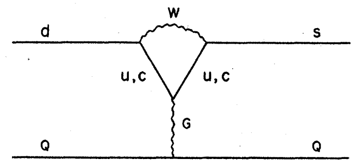

FIG. 1. The diagram which is the lowest-order contributions to the penguin terms of Sec. IV.

be neglected. However, although k_5, k_6 are also small their contribution must be retained since we find, on taking matrix elements in the MIT bag model, e.g.,

$$\frac{\langle \pi' | \mathcal{O}_5 | D^+ \rangle}{\langle \pi^+ | \mathcal{O}^{(-)} | D^+ \rangle} = -\frac{16}{3} \frac{(A+B)}{(A-B)}. \tag{32}$$

Here the factor $\frac{16}{3}$ is a color-counting coefficient and will appear in any quark-model calculation, while the terms $A-B$, $A+B$ are bag-model integrals defined in Ref. 7. For charm-decay, numerical integration gives the ratio as

$$\frac{A+B}{A-B} \approx 4, \tag{33}$$

which is sizable because $\mathcal{O}_5$ does not respect the helicity suppression, as discussed earlier. Thus, there can be a substantial contribution from the penguin operator depending on the size of the mixing angles. Evaluating the $D^0 \to K^+K^-$, $\pi^+\pi^-$ decay amplitudes as in Sec. II, we find

$$\begin{aligned} A(D^0 \to \pi^+\pi^-) &= A_0 \tan\theta_C \frac{F_K}{F_\pi} \left[(c_- + 2c_+) - \tfrac{16}{3} f (k_5 + \tfrac{3}{16} k_6) \frac{A+B}{A-B} \right], \\ A(D^0 \to K^+K^-) &= A_0 \tan\theta_C \eta \left[(c_- + 2c_+) - 8f(k_1 + \tfrac{3}{16} k_2) + \tfrac{16}{3} f (k_5 + \tfrac{3}{16} k_6) \frac{A+B}{A-B} \right], \\ A(D^0 \to K^0 \bar{K}^0) &= 0, \\ A(D^0 \to \pi^0\pi^0) &= A_0 \tan\theta_C \frac{F_K}{F_\pi} \left[(c_- - 2c_+) - \tfrac{16}{3} f (k_5 + \tfrac{3}{16} k_6) \frac{A+B}{A-B} \right], \\ A(D^+ \to \pi^+\pi^0) &= A_0 \tan\theta_C \frac{F_K}{F_\pi} 2\sqrt{2}\, c_+, \\ A(D^+ \to K^+ \bar{K}^0) &= A_0 \tan\theta_C \eta \left[(c_- + 2c_+) - 8f(k_1 + \tfrac{3}{16} k_2) + \tfrac{16}{3} f (k_5 + \tfrac{3}{16} k_6) \frac{A+B}{A-B} \right]. \end{aligned} \tag{34}$$

Here

$$f = \left(s_3^2 + \frac{s_2 s_3 c_3}{c_1 c_2} e^{i\delta} \right), \tag{35}$$

and the relative minus sign between the $\mathcal{O}_5, \mathcal{O}_6$ and $\mathcal{O}^{(-)}, \mathcal{O}^{(+)}$ contributions to the $\pi^+\pi^-$ and K^+K^- modes arises from the fact that $\mathcal{O}_5, \mathcal{O}_6$ transform as U-spin singlets, whereas $\mathcal{O}^{(\pm)}$ are components of a U-spin vector. However, the "left-right" operators $\mathcal{O}_5, \mathcal{O}_6$ are not the most important effect of the penguin diagram. Because of its color structure $\mathcal{O}_1$ has a large color counting factor, which, combined with the sizable coefficient k_1, makes it the dominant penguin operator. The fact that the $\mathcal{O}_1$ couples only to kaons leads to a breaking of the SU(3) relations of the four-quark theory. Thus, using, for exaple, the numercial estimates of the Wilson coefficients and helicity suppression factors given earlier, we find

$$A(D^0 \to \pi^+\pi^-) = A_0 \tan\theta_C \frac{F_K}{F_\pi}(c_- + 2c_+)(1 + 0.24f),$$
$$A(D^0 \to K^+K^-) = A_0 \tan\theta_C \eta (c_- + 2c_+)(1 - 2.6f). \quad (36)$$

Thus, for values

$$|f| \gtrsim 0.1, \quad (37)$$

which are certainly not out of the question.[23] These heavy-quark QCD corrections can also play a major role in shaping the pattern of Cabibbo-suppressed charm decay. Note, however, that f needs to be negative if it is to enhance K^+K^- mode. It is difficult to generate a negative value of f of sufficient size to account, by itself, for the central values of the observed branching ratios.

V. SUMMARY AND COMPARISONS

We conclude then that any one or some combinations of the three aforementioned effects—helicity suppression, final-state interactions, and penguin diagrams—can have an appreciable effect on two-body charm decays. This makes a phenomenological study of the decays difficult. There are uncertainties inherent in the evaluation of each of the mechanisms discussed and present techniques are unable to perform a reliable calculation which includes all three contributions systematically and correctly. However, for future work to be credible, we believe it will have to carefully consider all three points.

While we have not been able to give a definite prediction for the combined result of all effects, it is important to note that the violation of the sum rules arises not from an exotic new process, but rather from conventional sources. Therefore, the experimental indications of sum-rule violation are not surprising and, unfortunately, need not signal any new physics. Likewise, the violation of the quark-model prediction for $\Gamma(D^0 \to \bar{K}^0\pi^0)/\Gamma(D^0 \to K^-\pi^+)$ need not imply anthing more dramatic than the presence of final-state interactions.

While charm decays suffer from the calculational difficulties enumerated above, several factors make it likely that the decays of t and b quarks will be considerably simpler. Final-state interactions are not very important in decays of kaons, but become stronger when the center-of-mass energy is in the resonance region, as in D decay. However, at much higher energies the final-state interactions become unimportant again, and they should be negligible in the two-body decays of t- and b-flavored mesons. In addition, the large mass scale of these heavy particles may make calculations simpler. The mass scale relevant for many of the contributions to a given decay, such as those calculated in the bag model, is that of typical hadronic size. However, any contribution that scales up with the particle mass will soon dominate over these. The factorized or vacuum intermediate state method of evaluating matrix elements[6] does scale with the mass. In charm decays it appears that the two types of contributions are still comparable,[13] but for higher-mass states it is possible that the very simple factorization method will provide a reliable calculation of two-body decays. Another favorable aspect of t and b decays is that the QCD short-distance enhancements become more reliably computed and also less important at higher energies.

Finally, we note that other recent work has also focused on these decay modes. Barger and Pakvasa[27] have calculated the two-body charm-decay rates by inserting the vacuum intermediate state between the weak currents. Whether such vacuum-state saturation is reliable is, as always, problematic. Nevertheless, results are obtained which are, in general, similar to ours as calculated in Sec. II. This is apparently because their procedure respects current-algebra–PCAC constraints and takes account of helicity suppression in a subtle fashion—the ratio F_K/F_π is sensitive to this effect.[28] In our approach the mechanism is somewhat more transparent.

Abbott, Sikivie, and Wise[25] have calculated the penguin Hamiltonian relevant for charm decay. They estimate matrix elements by inserting the vacuum state, and find that the penguin piece is too small to lead to appreciable effects in the matrix elements. The main focus of their estimate is on the $(V-A)\times(V+A)$ operators, and we also find that the effect of this piece is small. However, they did not consider the color-counting factors which, in our calculation, can lead to a sizable contribution if the mixing angles are large enough.

Work by Sanda, Hagiwara, and Fukujita[26] has also looked at the penguin mechanism. However, they do so in the context of a four-quark model which we have argued (see also Ref. 23) is unlikely to make a sizable enough contribution to have any impact on the $D^0 \to \pi^+\pi^-/D^0 \to K^+K^-$ ratio.

Suzuki,[29] Wang and Wilczek,[30] and Quigg[31] have studied the effects of having more than four quarks. The additional mixing angles allow extra SU(3) structures, and can accommodate the violation of the four-quark selection rules. We have given a calculation of these effects in Sec. IV. However, our work, which indicates the possibility of substantial SU(3) breaking, suggests that detailed SU(3) parametrizations of the decays may not be reliable enough to allow extraction from the data of information on the couplings of heavier quarks.

Deshpande, Gronau, and Sutherland[32] have shown how gluon exchange may modify the simple color-counting predictions given in Ref. 6.

A more exotic mechanism, involving both charged Higgs mesons and SU(3) breaking in matrix elements, has been proposed by Kane.[33] While this is possible, it is not yet required by the data.

ACKNOWLEDGMENTS

We would like to thank R. Cahn, N. Deshpande, E. Golowich, and M. Wise for discussions. J.F.D. acknowledges the hospitality of the SLAC theory group where part of this work was performed. This work was supported in part through funds provided by the U. S. Department of Energy under Contract No. EC-76-C-02-3069.

---

[1] J. F. Donoghue and L. Wolfenstein, Phys. Rev. D 15 3341 (1977); J. F. Donoghue and B. R. Holstein, *ibid.* 12, 1454 (1975).

[2] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (*Nobel Symposium No. 8*), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968). p. 367.

[3] G. S. Abrams *et al.*, Phys. Rev. Lett. 43, 481 (1979).

[4] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974).

[5] M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[6] N. Cabibbo and L. Maiani, Phys. Lett. 73B, 418 (1978).

[7] J. F. Donoghue, E. Golowich, W. Ponce, and B. R. Holstein, Phys. Rev. D 21, 186 (1980); J. F. Donoghue, E. Golowich, W. Ponce, and B. R. Holstein, MIT Report No. MIT-CTP-799, 1979 (unpublished).

[8] T. DeGrand, R. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975); J. F. Donoghue and E. Golowich, *ibid.* 14, 1386 (1976).

[9] W. Ponce, Phys. Rev. D 19, 2197 (1979); also K. Johnson (private communication).

[10] See, e.g., M. Suzuki, Phys. Rev. 144, 1154 (1966).

[11] J. F. Donoghue and B. R. Holstein, Ref. 1.

[12] M. Gell-Mann, Phys. Rev. Lett. 12, 155 (1965); S. P. Rosen, S. Pakvasa, and E. C. G. Sudarshan, Phys. Rev. 146, 1118 (1966); D. G. Boulware and L. S. Brown, Phys. Rev. Lett. 17, 772 (1966).

[13] J. F. Donoghue and E. Golowich, Phys. Rev. D 14, 1326 (1976); also Ref. 6.

[14] The helicity suppression is also present if one calculates by inserting only the vacuum state.

[15] J. F. Donoghue and B. R. Holstein, Ref. 1; N. Cabibbo, G. Altarelli, and L. Maiani, Nucl. Phys. B88, 285 (1975).

[16] Various authors have estimated for the renormalization-group factor $4 \leq 1+(g^2/16\pi^2)6\ln(M_W{}^2/\mu^2) \leq 10$. We choose the conservative lower value.

[17] Particle Data Group, Rev. Mod. Phys. 48, S1(1976).

[18] R. Omnes, Nuovo Cimento 8, 316 (1958); N. Muskhilishvili, *Singular Integral Function* (Noordhoff, Groningen, 1953); V. Alessandrini and R. Omnes, Phys. Rev. 139B, 167 (1965).

[19] S. Kaptanoglu, Phys. Rev. D 18, 1554 (1978).

[20] B. Richter, lecture at SLAC Summer School, 1979 (unpublished).

[21] V. Luth, lecture at 1979 Lepton-Photon Conference, Fermilab (unpublished); J. Kirkby, lecture at 1979 Lepton-Photon Conference, Fermilab (unpublished).

[22] S. P. Rosen, Los Alamos Reports Nos. LA-UR-79-2619 and LA-UR-79-2702 (unpublished).

[23] M. A. Shifman, A. I. Vainshtein, and V. J. Zakharov, Nucl. Phys. B120, 315 (1977); Zh. Eksp. Teor. Fiz. Pis'ma Red 22, 123 (1975) [JETP Lett. 22, 55 (1975)].

[24] M. K. Gaillard, in *Weak Interactions—Present and Future*, proceedings of the Sixth SLAC Summer Institute, 1978, edited by M. C. Zipf, (SLAC, Stanford, 1978), p. 397.

[25] L. F. Abbott, P. Sikivie, and M. B. Wise, Phys. Rev. D 21, 768 (1980).

[26] For an alternate point of view see the discussion in M. Fukugita, T. Hagiwara, and A. I. Sanda, Rutherford Laboratory report, 1979 (unpublished).

[27] V. Barger and S. Pakwasa, Phys. Rev. Lett. 43, 812 (1979).

[28] J. F. Donoghue and K. Johnson, Phys. Rev. D (to be published).

[29] M. Suzuki, Phys. Lett. 85B, 91 (1979).

[30] L. L. Wang and F. Wilczek, Phys. Rev. Lett. 43, 816 (1979).

[31] C. Quigg, Fermilab Report No. Fermilab-Pub-79/62-Thy, 1979 (unpublished).

[32] N. Deshpande, M. Gronau, and D. Sutherland, Fermilab report (unpublished).

[33] G. Kane, SLAC report, 1979 (unpublished).