

Can $SU(2)_L \times U(1)$ and $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories be distinguished at high Q^2 ?

T. Rizzo and D. P. Sidhu

Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

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It is known that the low-energy weak-interaction experiments do not distinguish between the standard $SU(2)_L \times U(1)$ and $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories. In this paper, we investigate the question of distinguishability of the two classes of gauge theories in high- Q^2 weak-interaction experiments which are planned for the future.

I. INTRODUCTION

Weak-interaction experiments during the last few years have given considerable support to the unified-gauge theory view of weak and electromagnetic interactions. The first unified gauge model based on the $SU(2)_L \times U(1)$ gauge group¹ (hereafter called the standard model) has had a remarkable success so far. Its prediction of a new neutral force in eN interactions which violates parity has been confirmed in polarized-electron-deuteron scattering experiments² at SLAC. Recent model-independent determinations³ of the neutrino-quark weak-neutral-current couplings also agree with the predictions of the standard model. It has been shown⁴ recently by one of us (D.P.S.) that an appropriately constructed unified gauge model based on the $SU(2)_L \times SU(2)_R \times U(1)$ gauge group can essentially duplicate all the low-energy predictions of the standard model and that no low-energy weak-interaction experiment gives the hope of distinguishing between the two models. In this paper, we investigate whether it would be possible at all to distinguish between the two rival gauge theories in future high- Q^2 weak-interaction experiments.

In the calculation reported in this paper, we need the weak couplings of quarks and leptons and the neutral-gauge-boson masses in the two classes of models which are given below.

$SU(2)_L \times U(1)$ gauge theory

The neutral-current Lagrangian of this model is

$$\mathcal{L}_{\text{int}} = -iZ_\mu J_\mu - ieA_\mu \bar{\Psi} \gamma_\mu \Psi, \quad (1.1)$$

where

$$J_\mu = (g^2 + g'^2)^{1/2} \sum_{i=e,\nu,u,d,\dots} \bar{\Psi}_i \gamma_\mu \left[\left(\frac{1+\gamma_5}{2} \right) T_{3L}^i + \left(\frac{1-\gamma_5}{2} \right) T_{3R}^i - Q^i \sin^2 \theta_w \right] \Psi_i. \quad (1.2)$$

The electric charge e and the Weinberg angle θ_w are defined through the relations

$$e = g \sin \theta_w = g' \cos \theta_w, \quad (1.3)$$

where g (g') is the coupling constant of the $SU(2)$ [$U(1)$] subgroup. In (1.2), Q^i is the charge on the fermion Ψ_i measured in units of the electric charge e . Note that in the standard model $T_{3R}^i = 0$ for all i since all the right-handed chiral fields are assigned to the singlet representations of the group. The neutral- Z -boson mass is given by

$$M_Z = 37 \text{ GeV} / \sin \theta_w \cos \theta_w. \quad (1.4)$$

$SU(2)_L \times SU(2)_R \times U(1)$ gauge theory

For details of this model, we refer the reader to Ref. 4. The neutral-current Lagrangian of this model is

$$\begin{aligned} \mathcal{L}_{\text{int}} = & -ieA_\mu \bar{\Psi} \gamma_\mu \Psi - \frac{ie}{2 \sin \theta} (\cos \phi Z_{1\mu} - \sin \phi Z_{2\mu}) \sum_{i=e,\nu,u,d,\dots} \bar{\Psi}_i \gamma_\mu \gamma_5 (T_{3L}^i + T_{3R}^i) \Psi_i \\ & - \frac{ie}{\sin 2\theta} (\cos \phi Z_{2\mu} + \sin \phi Z_{1\mu}) \sum_{i=e,\nu,u,d,\dots} \bar{\Psi}_i \gamma_\mu [\cos^2 \theta (T_{3L}^i + T_{3R}^i) - 2 \sin^2 \theta Y^i] \Psi_i, \end{aligned} \quad (1.5)$$

where $Y^u = Y^d = \frac{1}{6}$ and $Y^e = Y^\nu = -\frac{1}{2}$. The electric charge e and the mixing angle θ are defined through the relations

$$g_L = g_R = 2e / \sin \theta, \quad g' = e / \cos \theta, \quad (1.6)$$

where g_L , g_R , and g' are the coupling constants for $SU(2)_L$, $SU(2)_R$, and $U(1)$ subgroups respectively. The masses of the two neutral bosons of the model are

$$\begin{pmatrix} M_{Z_1}^2 \\ M_{Z_2}^2 \end{pmatrix} = \frac{M_{W_L}^2}{2[1 - (1 - \epsilon)\xi]} \left\{ (1 - \epsilon)\sec^2\theta + (1 + \epsilon) \mp \left\{ [(1 + \epsilon) - (1 - \epsilon)\sec^2\theta]^2 + 4(1 - \epsilon)^2\xi^2\sec^2\theta \right\} \right\}, \quad (1.7)$$

where $0 \leq \epsilon \leq 1$ and $-1 < \xi < +1$ are the parameters of the model. Note that in the limit $\epsilon \rightarrow 0$ and $\xi \rightarrow +1$, the structure of low-energy weak-neutral currents becomes identical to that of the standard $SU(2)_L \times U(1)$ model provided $\sin^2\theta = 2 \sin^2\theta_W$. The consistency with the present experiments does not call for going to this extreme limit. Good agreement with these experiments is achieved for $\epsilon \approx 0$, $\sin^2\theta \approx 0.50$, and $\xi \geq 0.70$. In the subsequent discussion, we set $\epsilon = 0$ for simplicity.

II. ELECTRON-NUCLEON INTERACTIONS

In an effort to distinguish between the predictions of the $SU(2) \times U(1)$ Weinberg-Salam model and the $SU_L(2) \times SU_R(2) \times U(1)$ left-right-symmetric model we need to examine a Q^2 range where the effects of the gauge-boson propagator(s) will be felt. One way to do this is to examine high-energy deep-inelastic interactions such as $\nu N \rightarrow \nu X$ and $eN \rightarrow eX$; it is the second of these processes that we will examine in this section.

To contrast the two models under discussion here we must extract from the total cross section for $eN \rightarrow eX$ that part which is due to the weak interactions; at low Q^2 , this process is, of course, completely dominated by single-photon exchange. A convenient way to do this is to examine various asymmetries⁵ produced by the parity and charge conjugation violating neutral currents of both of these theories. Assuming that we can easily produce electrons and positrons of either helicity, there are four possible total cross sections which can be measured:

$$\sigma(e_{L,R}^\pm N \rightarrow e_{L,R}^\pm X). \quad (2.1)$$

To extract the weak contributions, we consider the following asymmetry parameters⁶:

$$A^\pm \equiv \frac{\sigma(e_R^\pm) - \sigma(e_L^\pm)}{\sigma(e_R^\pm) + \sigma(e_L^\pm)}, \quad (2.2)$$

$$B^\pm \equiv \frac{\sigma(e_R^\pm) - \sigma(e_L^\mp)}{\sigma(e_R^\pm) + \sigma(e_L^\mp)}, \quad (2.3)$$

$$C_{L,R} \equiv \frac{\sigma(e_{L,R}^+) - \sigma(e_{L,R}^-)}{\sigma(e_{L,R}^+) + \sigma(e_{L,R}^-)}. \quad (2.4)$$

Note that although there are six of these parameters, only four are independent since there are only four cross sections actually being measured. The asymmetry A^- is exactly that measured by the

recent SLAC-Yale experiment⁷ at low Q^2 ($\leq 2 \text{ GeV}^2/c^2$).

Given the various couplings of the gauge boson(s) to the fermions, the calculation of the cross sections (2.1) is straightforward using the quark-parton model. In the results presented here we have used the quantum-chromodynamics (QCD) corrected distribution function of Buras and Gaemers⁸ in the Q^2 region between 10 and $10^4 \text{ GeV}^2/c^2$.

Our results are as follows: We first examine the Q^2 dependence of one of these asymmetries to show their rough numerical magnitude. Figure 1 shows the Q^2 dependence of A^-/Q^2 for Q^2 in the above range using a typical set of values of x , y , ξ , and $\sin^2\theta$ in the left-right-symmetric ($L \times R$) model. For definiteness, we consider only ep reactions in what follows. A similar figure can be drawn for the Weinberg-Salam (WS) model. In Fig. 2, we plot the ratios

$$R_{A^\pm} \equiv A_{L \times R}^\pm / A_{WS}^\pm \quad (2.5)$$

for two values of ξ . As can be easily seen, both R_{A^+} and R_{A^-} for $\xi = 0.7$ differ from unity by $\sim 20\%$ for $Q^2 < 10^3 \text{ GeV}^2/c^2$; for $\xi = 0.9$, the difference is only $\sim 6\%$ for the same Q^2 range. For $Q^2 > 10^3 \text{ GeV}^2/c^2$ both sets of ratios begin to deviate from their low- Q^2 value due to the interference of the two Z bosons. Since, for $\xi = 0.9$, Z_2 is very heavy, the turnover for this ξ value occurs at higher Q^2 values than when $\xi = 0.7$.

Figure 3 shows the ratios

$$R_{B^\pm} \equiv B_{L \times R}^\pm / B_{WS}^\pm \quad (2.6)$$

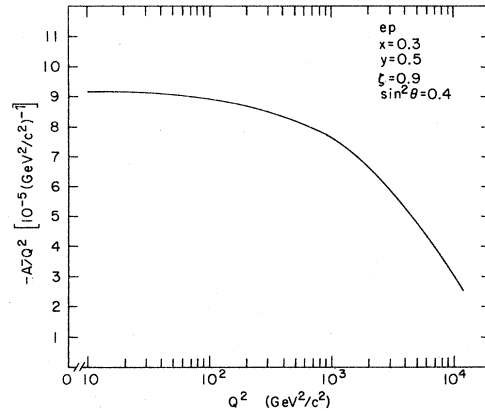


FIG. 1. $-A^-/Q^2$ as a function of Q^2 for ep deep-inelastic scattering in the left-right-symmetric model.

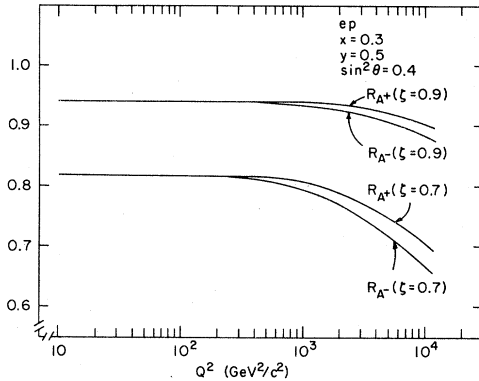


FIG. 2. The ratio R_{A^+} and R_{A^-} as functions of Q^2 ; the Weinberg-Salam calculations were performed with $x_W = 0.2$.

for the same set of parameters as used in Fig. 2. Here, for $\zeta = 0.7$, R_{B^+} and R_{B^-} differ from unity by $\sim 10\%$ and $\sim 25\%$ respectively at low Q^2 with a slight turnover beginning near 10^3 GeV^2/c^2 . For $\zeta = 0.9$, both R_{B^+} and R_{B^-} differ from unity by $\lesssim 7\%$ until the high- Q^2 range is reached.

Can any distinction be made by examining the y dependence of these ratios? To answer this question we plot the ratio C_L^{LXR}/C_L^{WS} as a function of y for two sets of Q^2 values in Fig. 4. As can be easily seen, the greatest deviation from unity occurs near $y = 0$ independent of the Q^2 value (or the value of ζ). This would suggest, for example, that knowledge of C_L in the low- y region would provide a much cleaner test than data averaged over the entire y range.

In Fig. 5 we examine the quantity C_R/Q^2 ; we have not plotted the ratio in this case since both C_R^{WS} and C_R^{LXR} go through a zero near $y = 0.4$. Again we see that the values of this quantity in the Weinberg-Salam and left-right-symmetric model

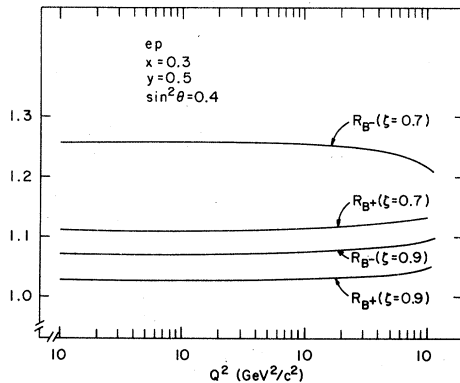


FIG. 3. The ratios R_{B^+} and R_{B^-} as functions of Q^2 ; the Weinberg-Salam calculations were performed with $x_W = 0.2$.

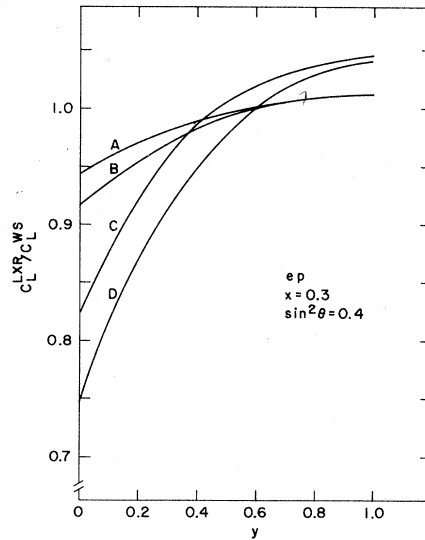


FIG. 4. The ratio C_L^{LXR}/C_L^{WS} as a function of y . A: $\zeta = 0.9$, $Q^2 = 10$ GeV^2/c^2 ; B: $\zeta = 0.9$, $Q^2 = 5000$ GeV^2/c^2 ; C: $\zeta = 0.7$, $Q^2 = 10$ GeV^2/c^2 ; D: $\zeta = 0.7$, $Q^2 = 5000$ GeV^2/c^2 . The Weinberg-Salam calculations assumed $x_W = 0.2$.

differ most for the two extreme values of y ($y = 0$ and 1).

Unfortunately, the measurements we propose here are, at present, impossible because of the large values of Q^2 which are needed and are probably unobtainable using a fixed target machine. (One could, possibly, use cosmic-ray muons.) For an electron-proton collider, $Q^2 = 4 E_e E_p xy$ such that we need something like $E_e \sim 30$ GeV/c and $E_p \sim 400$ GeV/c to analyze the Q^2 range near 10^4 GeV^2/c^2 . Such machines are possible and

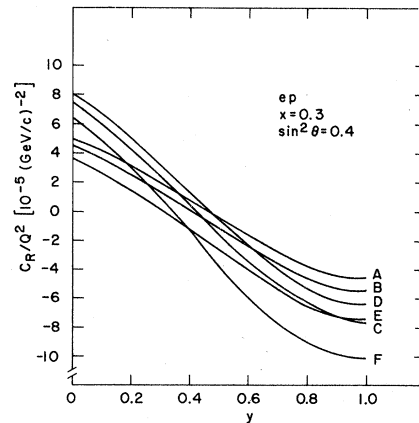


FIG. 5. C_R/Q^2 as a function of y . A: WS with $Q^2 = 5000$ GeV^2/c^2 ; B: $L \times R$ with $\zeta = 0.9$ and $Q^2 = 5000$ GeV^2/c^2 ; C: $L \times R$ with $\zeta = 0.7$ and $Q^2 = 5000$ GeV^2/c^2 ; D: WS with $Q^2 = 10$ GeV^2/c^2 ; E: $L \times R$ with $\zeta = 0.9$ and $Q^2 = 10$ GeV^2/c^2 ; F: $L \times R$ with $\zeta = 0.7$ and $Q^2 = 10$ GeV^2/c^2 . The Weinberg-Salam calculations assumed $x_W = 0.2$.

proposals already exist at CERN (CHEEP)⁹ and at ISABELLE¹⁰ for installing an electron ring for purposes such as these. These machines, however, are not expected to be running before the mid-1980's.

III. NEUTRINO INTERACTIONS

Another deep-inelastic process we can study is $(\nu)N \rightarrow (\nu)X$ which is, of course, a purely weak interaction. Hence, we do not need to examine asymmetries here and shall only examine the total cross sections. We will assume that the hadronic target N is just a proton since, for ν energies ≥ 1 TeV, the only ν source is cosmic rays and we are in the region to be studied by the deep underwater muon and neutrino detector (DUMAND).¹¹ The "target" for DUMAND is 1 km³ of water.)

As has already been shown,⁴ the low-energy effective ν coupling to quarks (and leptons) is independent of the value of ζ . Even at high energies this is a very good approximation; in fact, we find that the ν (or $\bar{\nu}$) cross section for the two values of ζ (0.7 and 0.9) considered above differ only at the 0.1% level even for $E^\nu \approx 10$ TeV.

We have calculated the total cross sections

$$\sigma(\nu) p \rightarrow (\nu) X \quad (3.1)$$

for the Weinberg-Salam model as well as for the left-right-symmetric model (with both values of ζ); we then constructed the ratios

$$R^{\nu, \bar{\nu}} \equiv \sigma_{L^*R}^{\nu, \bar{\nu}} / \sigma_{WS}^{\nu, \bar{\nu}}. \quad (3.2)$$

Our results can be found in Fig. 6; as can easily be seen, the two models differ in their predictions only at the 1-2% level for the entire energy range 10-10⁴ GeV/c. (We have included QCD corrections in our calculations as described in the previous section.) We can thus conclude that ν interactions do not provide a good testing ground to distinguish between these two models.

IV. CONCLUSION

In this paper we have investigated the possibility that high-energy deep-inelastic experiments can

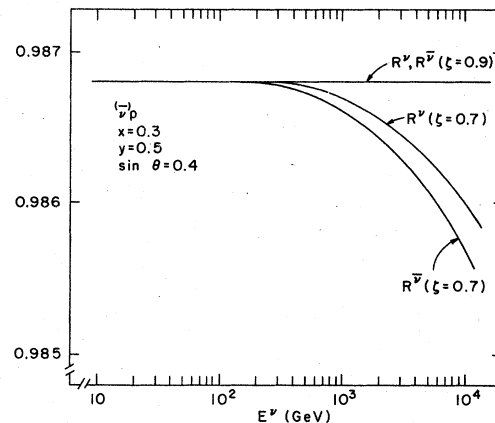


FIG. 6. The ratios $R^{\nu, \bar{\nu}}$ as functions of E^ν ; the WS calculations assumed $x_W = 0.2$.

distinguish between the standard model based on $SU_L(2) \times SU_R(2) \times U(1)$. We have found that high-energy ($E^\nu \sim 10$ TeV) neutrino experiments, such as those to be performed at DUMAND, are incapable of distinguishing between the two models for the range of model parameters studied here.

One possible means to differentiate the two models is to examine various asymmetries in deep-inelastic eN interactions at very high Q^2 values ($Q^2 > 10^3$ GeV²/c²). These asymmetries are much more sensitive to the various weak neutral currents than are the total cross sections such that even small variations are noticeable. We find that for $Q^2 > 10^3$ GeV²/c² we can reasonably expect the predictions for the various asymmetries to differ between the two models by $\geq 10\%$; this can be increased somewhat by looking in the extreme regions of y (near 0 or 1). To produce the needed Q^2 , machines such as CHEEP or ISABELLE are needed.

We conclude that high- Q^2 measurements of weak-interaction asymmetries may distinguish between these models.

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