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SO(2N) grand unification in an SU(N) basis

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We discuss the algebra and the representations of SO(2N) groups used in the construction of grand unified theories in a basis in which its connections with the SU(N) grand unification is most transparent. Specializing to the case of $N=5$, we discuss the problem of fermion masses for various Higgs representations. Applying our considerations to SO(12) grand unification, we comment on the nature of weak interactions of the extra generation of fermions present in the 32-dimensional spinor representation of this group.

I. INTRODUCTION

In recent days, a great deal of attention has been focused on the construction of grand unified theories[1] (GUT) of weak, electromagnetic, and strong interactions using SU(N) (Ref. 2) as well as SO(M) (Ref. 3) groups as the local symmetry groups of the GUT Lagrangian. Prior to spontaneous breakdown, all interactions are described by one coupling constant. The local invariance is then broken down by the Higgs mechanism to the residual local invariance group $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}$ of strong and electromagnetic interactions. This is usually done in stages, each stage being characterized by a mass parameter and a residual local symmetry group. The simplest of these theories is the SU(5) model of Georgi and Glashow,[2] in which the fundamental fermions (the quarks and the leptons) of each generation (e.g., u,d,ν,e) are assigned to the $\{\bar{5}\}$- and $\{10\}$-dimensional representations. Another attractive possibility is to consider the gauge group SO(10) (Ref. 3) and assign all fermions of one generation of both chiralities to the $\{16\}$-dimensional spinor representation of this group. The SU(5) decomposition of this $\{16\}$-dimensional representation is $\{10\}+\{5^*\}+\{1\}$. Thus it unifies all fermions into a single representation, apparently a more appealing feature than that of SU(5). Furthermore, in order to accommodate more generations within a single representation, higher SU(N) as well as the corresponding SO(2N) gauge groups have been considered. In view of the intimate connection between the SU(N) and SO(2N) grand unified theories, it is worthwhile to present a discussion of SO(2N) algebra and representations in terms of an SU(N) basis. Apart from providing a very simple way to handle SO(2N), this formalism becomes most useful if the SO(2N) group is first broken down to the SU(N) group, which subsequently breaks down to the $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}} \times G_{\mathrm{fl}}$, where G_{fl} is the gauge group of flavor dynamics.

The purpose of this article is to discuss first the SO(2N) algebra and relevant Higgs representations and couplings in terms of an SU(N) basis. We then apply this to discuss the fermion masses in the context of the SO(10) gauge group, using the Higgs fields belonging to $\{10\}$-, $\{120\}$-, and $\{126\}$-dimensional representations. We also apply our method to discuss the $\{32\}$-dimensional spinor representation of SO(12). This representation contains two $\{16\}$-dimensional representations under SO(10). We show that the extra generation of fermions present have $V+A$ structure for their weak interactions.

II. SO(2N) IN AN SU(N) BASIS

Consider a set of N operators χ_i ($i=1,\ldots,N$) and their Hermitian conjugate $\chi_i^\dagger$ satisfying the following anticommutation relations:

$$\{\chi_i, \chi_j^\dagger\} = \delta_{ij}, \tag{1a}$$

$$\{\chi_i, \chi_j\} = 0. \tag{1b}$$

We use the symbol { , } to denote anticommutation and [,] to denote commutation operations. It is well known that operators T_j^i defined as

$$T_j^i = \chi_i^\dagger \chi_j \tag{2}$$

satisfy the algebra of the U(N) group, i.e.,

$$[T_j^i, T_l^k] = \delta_j^k T_l^i - \delta_l^i T_j^k. \tag{3}$$

Now let us define the $2N$ operators Γ_μ ($\mu=1,\ldots,2N$):

$$\Gamma_{2j-1} = -i(\chi_j - \chi_j^\dagger)$$

and

$$\Gamma_{2j} = (\chi_j + \chi_j^\dagger), \quad j=1,\ldots,N. \tag{4}$$

It is easy to verify using Eq. (1) that

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu}. \tag{5}$$

Thus, the Γ_μ's form a Clifford algebra of rank $2N$ (of course, $\Gamma_\mu = \Gamma_\mu^\dagger$). Using the Γ_μ's we can construct the generators of the SO(2N) group as follows:

$$\Sigma_{\mu\nu}=\frac{1}{2i}[\Gamma_\mu,\Gamma_\nu]. \tag{6}$$

The $\Sigma_{\mu\nu}$ can be written down in terms of χ_j and $\chi_j^\dagger$ as follows:

$$\begin{aligned}\Sigma_{2j-1,2k-1}&=\frac{1}{2i}[\chi_j,\chi_k^\dagger]-\frac{1}{2i}[\chi_k,\chi_j^\dagger]+i(\chi_j\chi_k+\chi_j^\dagger\chi_k^\dagger),\\ \Sigma_{2j,2k-1}&=\tfrac{1}{2}[\chi_j,\chi_k^\dagger]+\tfrac{1}{2}[\chi_k,\chi_j^\dagger]-(\chi_j\chi_k-\chi_j^\dagger\chi_k^\dagger),\\ \Sigma_{2j,2k}&=\frac{1}{2i}[\chi_j,\chi_k^\dagger]-\frac{1}{2i}[\chi_k,\chi_j^\dagger]-i(\chi_j\chi_k+\chi_j^\dagger\chi_k^\dagger).\end{aligned} \tag{7}$$

It is well known that the spinor representation of SO(2N) is 2^N dimensional. To write it in terms of the SU(N) basis, let us define a "vacuum" state $|0\rangle$ which is SU(N) invariant. The 2^N-dimensional spinor representation is then given in Table I.

This representation can be split into the 2^{N-1}-dimensional representations under a chiral projection operator. We now proceed to construct this operator. Define

$$\Gamma_0=i^N\Gamma_1\Gamma_2\cdots\Gamma_{2N}. \tag{8}$$

Also define a number operator $n_j\equiv\chi_j^\dagger\chi_j$. Using Eq. (4), Γ_0 can be written as follows:

$$\begin{aligned}\Gamma_0&=[\chi_1,\chi_1^\dagger][\chi_2,\chi_2^\dagger]\cdots[\chi_N,\chi_N^\dagger],\\ &=\prod_{j=1}^{N}(1-2n_j).\end{aligned} \tag{9}$$

Using the property of the number operators $n_j^2=n_j$, one can show that $1-2n_j=(-1)^{n_j}$ and so we get

$$\Gamma_0=(-1)^n,\ n=\sum_j n_j. \tag{10}$$

It is then easily checked that

$$[\Sigma_{\mu\nu},(-1)^n]=0. \tag{11}$$

The "chirality" projection operator[4] is therefore given by $\frac{1}{2}(1\pm\Gamma_0)$. Each irreducible "chiral" subspace is therefore characterized by an odd or even number of the χ particles. To make it more explicit, let us consider $N=5$ and define a column vector $|\psi\rangle$ as

$$\begin{aligned}|\psi\rangle=&|0\rangle\psi_0+\chi_j^\dagger|0\rangle\psi_j+\tfrac{1}{2}\chi_j^\dagger\chi_k^\dagger|0\rangle\psi_{jk}\\ &+\tfrac{1}{12}\epsilon^{jklmn}\chi_l^\dagger\chi_m^\dagger\chi_n^\dagger|0\rangle\overline{\psi}_{jk}\\ &+\tfrac{1}{24}\epsilon^{jklmn}\chi_k^\dagger\chi_l^\dagger\chi_m^\dagger\chi_n^\dagger|0\rangle\overline{\psi}_j+\chi_1^\dagger\chi_2^\dagger\chi_3^\dagger\chi_4^\dagger\chi_5^\dagger|0\rangle\overline{\psi}_0,\end{aligned} \tag{12}$$

where $\overline{\psi}_i$ is not the complex conjugate of ψ_i but an independent vector. We will denote the complex conjugate by an asterisk. The generalization of Eq. (12) to the case of arbitrary N is obvious if we write

$$\psi\equiv\begin{bmatrix}\psi_0\\ \psi_j\\ \psi_{jk}\\ \overline{\psi}_{jk}\\ \overline{\psi}_j\\ \overline{\psi}_0\end{bmatrix}. \tag{13}$$

Under chirality,

$$\psi=\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix},$$

where

$$\psi_\pm=\tfrac{1}{2}(1\pm\Gamma_0)\psi$$

and

$$\psi_+=\begin{bmatrix}\psi_0\\ \psi_{ij}\\ \overline{\psi}_i\end{bmatrix},\quad \psi_-=\begin{bmatrix}\overline{\psi}_0\\ \overline{\psi}_{ij}\\ \psi_i\end{bmatrix}. \tag{14}$$

For the $N=5$ case, $\overline{\psi}_i$ and ψ_{ij} represent $\{\overline{5}\}$- and $\{10\}$-dimensional representations of SU(5) and ψ_0 is the singlet. All the fermions are assigned to ψ_+. It is then easy to write down the gauge interaction of the fermions. We further note that for the case of SO(10), the formula for electric charged Q is given by

$$Q=\tfrac{1}{2}\Sigma_{78}-\tfrac{1}{6}(\Sigma_{12}+\Sigma_{34}+\Sigma_{56}). \tag{15}$$

We next tackle the problem of the spontaneous breakdown of symmetry and generating fermion masses and mixings.

TABLE I. Construction of the states belonging to the spinor representation of SO(2N) and their SU(N) dimensionality.

SO(2N) spinor state	SU(N) dimension
$\|0\rangle$	1
$\chi_j^\dagger\|0\rangle$	N
$\chi_j^\dagger\chi_k^\dagger\|0\rangle$	$\frac{N(N-1)}{2}$
$\chi_j^\dagger\chi_k^\dagger\chi_l^\dagger\|0\rangle$	$\frac{N(N-1)(N-2)}{6}$
$\cdots$	
$\chi_1^\dagger\chi_2^\dagger\cdots\chi_N^\dagger\|0\rangle$	1
	Total 2^N

III. FERMION MASSES AND THE "CHARGE-CONJUGATION" OPERATOR

As is well known in the framework of gauge theories, at present, the fermion masses arise from Yukawa couplings of fermions to Higgs bosons and subsequent breakdown of the gauge symmetry by nonzero vacuum expectation values (vev) of the Higgs mesons. In general grand unified theories, both particles and antiparticles belong to the same irreducible representation of the gauge group. So, to generate all possible mass terms, one must write down gauge-invariant Yukawa couplings of the form

$$\tilde{\psi}BC^{-1}\Gamma_\mu\psi\phi_\mu\,,\quad \tilde{\psi}BC^{-1}\Gamma_\mu\Gamma_\nu\Gamma_\lambda\psi\phi_{\mu\nu\lambda},\ldots\ ,$$

where $\tilde{\psi}$ stands for transpose of ψ, B is the equivalent of the charge-conjugation matrix for SO(10), and C is the Dirac charge-conjugation matrix. The ϕ_μ, $\phi_{\mu\nu\lambda}$, etc., are the Higgs mesons belonging to irreducible representations of appropriate dimensions of SO($2N$), i.e., ϕ_μ is $\{2N\}$ dimensional, $\phi_{\mu\nu\lambda}$ is $\{2N(2N-1)(2N-2)/6\}$ dimensional, etc. (for $N=5$, ϕ_μ, $\phi_{\mu\nu\lambda}$ are, respectively, $\{10\}$ and $\{120\}$ dimensional). To see the need for inserting B, we note that under the group transformation

$$\begin{aligned}\delta\psi&=i\epsilon_{\mu\nu}\Sigma_{\mu\nu}\psi\,,\\ \delta\psi^\dagger&=-i\epsilon_{\mu\nu}\psi^\dagger\Sigma_{\mu\nu},\\ \delta\tilde{\psi}&=i\epsilon_{\mu\nu}\tilde{\psi}\tilde{\Sigma}_{\mu\nu}\,.\end{aligned}\qquad(16)$$

Thus, $\tilde{\psi}$ does not transform like a conjugate spinor representation of SO($2N$). However, if we introduce a $2^N\times2^N$ matrix B such that[5]

$$B^{-1}\tilde{\Sigma}_{\mu\nu}B=-\Sigma_{\mu\nu}\,,\qquad(17)$$

then

$$\delta(\tilde{\psi}B)=-i\epsilon_{\mu\nu}(\tilde{\psi}B)\Sigma_{\mu\nu}\,.\qquad(18)$$

Thus, $\tilde{\psi}B$ has the correct transformation property under SO($2N$). It is easy to see that Eq. (17) requires that

$$B^{-1}\tilde{\Gamma}_\mu B=\pm\Gamma_\mu\ .\qquad(19)$$

We will choose the negative sign on the right-hand side. Since the Γ_μ's are represented by symmetric matrices for even μ in the spinor basis of Table I, one obvious representation of B in this spinor space is

$$B=\prod_{\mu=\text{odd}}\Gamma_\mu\ .\qquad(20)$$

Using Eqs. (12) and (20), we conclude that

$$B\begin{bmatrix}\psi_0\\ \psi_{ij}\\ \bar{\psi}_i\\ \psi_i\\ \bar{\psi}_{ij}\\ \bar{\psi}_0\end{bmatrix}=i\begin{bmatrix}\bar{\psi}_0\\ -\bar{\psi}_{ij}\\ \psi_i\\ -\bar{\psi}_i\\ \bar{\psi}_{ij}\\ \psi_0\end{bmatrix}\ .\qquad(21)$$

Since

$$\tilde{\psi}BC^{-1}\Gamma_\mu\psi=\langle\psi^*|BC^{-1}\Gamma_\mu|\psi\rangle\,,$$

the Yukawa coupling (for $N=5$) of fermions with Higgs mesons ϕ_μ will be written as

$$\langle\psi^*|BC^{-1}\Gamma_\mu|\psi\rangle\phi_\mu\equiv\sum_{j=+,-}\langle\psi_j^*|BC^{-1}\Gamma_\mu|\psi_j\rangle\phi_\mu\ ,\qquad(22)$$

where all quantities are listed in this and the previous sections. Note that in writing Eq. (22), we used the fact that $[\Gamma_0, B\Gamma_\mu]=0$. To get fermion masses, all we have to do is set $\langle\phi_\mu\rangle\neq0$ for appropriate μ and evaluate $\langle\psi_+^*|BC^{-1}\Gamma_\mu|\psi_+\rangle$ using the anticommutation relations of the χ_j's and the fact that $\chi_j|0\rangle=0$. In the next section, we give explicit examples for the case of the SO(10) grand unified group.

IV. FERMION MASSES IN SO(10): AN APPLICATION

As as explicit application of our techniques, we will calculate the fermion masses for SO(10) theory with ϕ Higgs mesons belonging to both $\{10\}$-dimensional (ϕ_μ) and $\{120\}$-dimensional ($\phi_{\mu\nu\lambda}$) representations.[6] Before doing that, we would like to identify the various particle states belonging to the $\{16\}$-dimensional spinor representation of SO(10). We identify

$$\psi_0=\nu_L^c,\quad\bar{\psi}_i=\begin{bmatrix}d_1^c\\ d_2^c\\ d_3^c\\ e^-\\ \nu\end{bmatrix}_L\,,$$
$$\psi_{ij}=\begin{bmatrix}0&u_3^c&-u_2^c&u_1&d_1\\ -u_3^c&0&u_1^c&u_2&d_2\\ u_2^c&-u_1^c&0&u_3&d_3\\ -u_1&-u_2&-u_3&0&e^+\\ -d_1&-d_2&-d_3&-e^+&0\end{bmatrix}_L\,.\qquad(23)$$

We remind the reader that $\bar{\psi}_i$ and ψ_{ij} are the usual SU(5) representations of Georgi and Glashow.[2]

A. {10}-dimensional Higgs representation[5]

Since we want color symmetry unbroken, the components of ϕ_μ, which can acquire vev's, are ϕ_9 and ϕ_{10}. Let us set

$$\langle\phi_9\rangle=v_1\ ,\quad\langle\phi_{10}\rangle=v_2\,.\qquad(24)$$

We have to evaluate

$$\begin{aligned}\mathfrak{L}_{\text{mass}}=&-i\kappa v_1\langle\psi_+^*|B(\chi_5-\chi_5^\dagger)|\psi_+\rangle\\ &+\kappa v_2\langle\psi_+^*|B(\chi_5+\chi_5^\dagger)|\psi_+\rangle\,.\end{aligned}\qquad(25)$$

Using Eqs. (12) and (20) and after some algebra, we obtain

$$\mathcal{L}_{\text{mass}} = \kappa(v_2 - v_1)(\bar{d}_L d_R + \bar{e}_L e_R) + \kappa(v_2 + v_1)(\bar{u}_L u_R + \bar{\nu}_L \nu_R) + \text{H.c.} \tag{26}$$

We thus see that we get

$$m_d = m_e \text{ and } m_u = m_\nu . \tag{27}$$

It is also easily seen that if there is more than one spinor multiplet of fermions corresponding to different families of particles, the mass matrix is symmetric. We note here that the mass relations $m_d = m_e$ and $m_\nu = m_u$ follow essentially because $\langle\phi\rangle$ breaks $\text{SO}(10) \to \text{SO}(8) \equiv \text{SU}(4') \times \text{U}(1)_A$, where SU(4′) is the symmetry group involving the three colors plus the leptons, as in the Pati-Salam model.

B. {120}-dimensional case

The invariant Yukawa coupling of O(10) spinor fermions $\psi \equiv \binom{\psi_+}{\psi_-}$ to the {120}-dimensional Higgs field $\phi_{\mu\nu\lambda}$ can be written down as

$$\mathcal{L}_Y^{ab} = \kappa_{ab} \tilde{\psi}_a B C^{-1} \Gamma_\mu \Gamma_\nu \Gamma_\lambda \psi_b \phi_{\mu\nu\lambda} , \tag{28}$$

where a and b stand for the different generations of fermions. Using the fact that

$$\tilde{B} = -B \text{ and } \tilde{C} = -C ,$$

we find that

$$\mathcal{L}_Y^{ab} = -\mathcal{L}_Y^{ba} . \tag{29}$$

So, if we restrict ourselves to only one generation, it does not contribute to the fermion masses. It will, however, contribute to mixings between various generations. To analyze the kind of mixing pattern that this representation generates, we note that under SU(5), $\phi_{\mu\nu\lambda}$ breaks up as follows:

$$\{120\} = \{45\} + \{45^*\} + \{10\} + \{10^*\} + \{5\} + \{5^*\} . \tag{30}$$

Thus, one can choose either of the following pattern of vacuum expectation values:

(i) A linear combination of {45} *and* {5} *acquires* vev. This means we have

$$\langle\phi_{789}\rangle \neq 0 \text{ and } \langle\phi_{7810}\rangle \neq 0 . \tag{31}$$

Inserting this into the Yukawa couplings and proceeding with the calculation as in the case of the {10}-dimensional Higgs field, we find that the mixing between the various generations of quarks and leptons are related as follows

$$m_{d_a d_b} = 3 m_{E_a E_b} , \tag{32}$$

$$m_{u_a u_b} = 3 m_{\nu_a \nu_b} , \tag{33}$$

where $m_{\nu_a \nu_b}$ stands for mixing terms in the mass matrix between generations a and b; d_a means the $-\frac{1}{3}$-charged quark of the a generation and similarly for u, E^-, and ν.

(ii) Only the SU(5) {45}*-dimensional Higgs field acquire* vev. This means the following fields acquire vev's:

$$\begin{aligned} \langle\phi_{789}\rangle &= -3\langle\phi_{129}\rangle = -3\langle\phi_{349}\rangle \\ &= -3\langle\phi_{569}\rangle \neq 0 , \\ \langle\phi_{78,10}\rangle &= -3\langle\phi_{12,10}\rangle = -3\langle\phi_{34,10}\rangle \\ &= -3\langle\phi_{56,10}\rangle \neq 0 . \end{aligned} \tag{34}$$

In this case, the mixing pattern is very different from case (i). We get

$$\begin{aligned} m_{E_a E_b} &= 3 m_{d_a d_b} , \\ m_{\nu_a \nu_b} &= 0 , \\ m_{u_a u_b} &\neq 0 . \end{aligned} \tag{35}$$

C. {126}-dimensional case[7]

The invariant Yukawa coupling in this case involves 5 Γ_μ matrices:

$$\mathcal{L}_Y^{ab} = \kappa_{ab} \tilde{\psi}_a B C^{-1} \Gamma_\mu \Gamma_\nu \Gamma_\lambda \Gamma_\sigma \Gamma_\alpha \psi_b \phi_{\mu\nu\lambda\sigma\alpha} . \tag{36}$$

We note first that $L_Y^{ab} = L_Y^{ba}$. Thus, this makes a symmetric contribution to various masses. We may choose the following vacuum expectation values for the Higgs field consistent with local color-SU(3) symmetry remaining exact:

$$\langle\phi_{1278\mu}\rangle = \langle\phi_{3478\mu}\rangle = \langle\phi_{5678\mu}\rangle , \tag{37}$$

where $\mu = 9$ or 10. Substituting this into the Yukawa couplings, we get for one generation $(a = b = 1)$ the following kind of mass relations:

$$m_e = 3 m_d , \quad m_u = 3 m_\nu . \tag{38}$$

V. APPLICATION TO SO(12) GRAND UNIFICATION

It has recently been suggested[8] that to incorporate more families of fermions into one representation, one may consider $\text{O}(10 + 2m)$. We analyze this proposal for $m = 1$ and discuss the nature of the weak interaction of the extra generation. As already noted by the authors of Ref. 8, we show that the second 16-plet of fermions has $V + A$ structure for weak interaction. The SO(12) case will be completely specified by

$$\chi_\alpha^\dagger , \quad \alpha = 1, \ldots , 6 .$$

As usual, we consider the algebra generated by the 12 Γ_μ's defined by

$$\begin{aligned} \Gamma_\mu &= (\chi_\alpha + \chi_\alpha^\dagger) , \quad \mu = \text{even} \\ \Gamma_\mu &= -i(\chi_\alpha - \chi_\alpha^\dagger) , \quad \mu = \text{odd} \end{aligned} \tag{39}$$

and the generators

$$\Sigma_{\mu\nu} = \frac{1}{2i}[\Gamma_\mu, \Gamma_\nu]. \tag{40}$$

We then note that as in the case of SO(10) there exists a chiral projection operator for this case, which commutes with all generators and splits the $\{64\}$-dimensional spinor representations into chiral even and odd subspaces, corresponding to even and odd numbers of "χ particles" in the state. As before, we can assign the physical particles into one of these subspaces, as before to the even subspace, which is now $\{32\}$ dimensional and is given by

$$|0\rangle, \quad \chi_\alpha^\dagger\chi_\beta^\dagger|0\rangle, \quad \chi_\alpha^\dagger\chi_\beta^\dagger\chi_\gamma^\dagger\chi_\lambda^\dagger|0\rangle, \quad \chi_1^\dagger\chi_2^\dagger\chi_3^\dagger\chi_4^\dagger\chi_5^\dagger\chi_6^\dagger|0\rangle.$$

Under SO(10) it breaks up in the following way:

$$\begin{aligned}\psi = & |0\rangle\psi_0 + \chi_6^\dagger\chi_i^\dagger|0\rangle\psi_i' + \tfrac{1}{2}\chi_i^\dagger\chi_j^\dagger|0\rangle\psi_{ij} \\ & + \tfrac{1}{12}\epsilon^{ijklm}\chi_6^\dagger\chi_k^\dagger\chi_l^\dagger\chi_m^\dagger|0\rangle\bar{\psi}_{ij}' + \tfrac{1}{24}\epsilon^{ijklm}\chi_j^\dagger\chi_k^\dagger\chi_l^\dagger\chi_m^\dagger|0\rangle\bar{\psi}_i' \\ & + \chi_1^\dagger\chi_2^\dagger\chi_3^\dagger\chi_4^\dagger\chi_5^\dagger\chi_6^\dagger|0\rangle\bar{\psi}_0',\end{aligned} \tag{41}$$

where $i,j,k,l = 1,\ldots,5$. Here $(\psi_0, \psi_{ij}, \bar{\psi}_i)$ and $(\bar{\psi}_0', \bar{\psi}_{ij}', \psi_i')$ constitute the two $\{16\}$-dimensional spinors. Since the electric-charge formula for this case is

$$Q = \tfrac{1}{2}\Sigma_{78} - \tfrac{1}{6}(\Sigma_{12} + \Sigma_{34} + \Sigma_{56}), \tag{42}$$

we see that $\bar{\psi}_{ij}'$ and ψ_i' have opposite electric charges to ψ_{ij} and $\bar{\psi}_i$. If we want to identify the second $\{16\}$-dimensional part with the second generation (c, s, μ, ν), then it is obvious that their weak interaction will have to be right-handed, in conflict with observations. Thus, to incorporate two families, one has to go to SO(14).[8]

In conclusion, we have discussed an SU(N) basis for SO(N) grand unified gauge theories and have outlined a calculational framework which proves useful in the study of fermion masses and mixing. We have applied it to some examples in the SO(10) case. We have also discussed the nature of the weak interaction of the extra generation of fermions in the case of SO(12) grand unification. We wish to study in detail the generation puzzle in the context of SO($2N$) groups using our techniques and we also wish to study the problem of fermion masses and mixings in a future publication.

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[4]Our considerations are easily extended to the case of SO($2N+1$), where Γ_0 is treated as the ($2N+1$)th element of the algebra since $\{\Gamma_0, \Gamma_\mu\} = 0$. The spinor representation is then 2^N dimensional.

[5]The B operator and its role in construction of the fermion masses was first discussed by M. Chanowitz, J. Ellis, and M. Gaillard (Ref. 3) in a different context.

[6]The full breakdown of the symmetry down to $SU(3)_c \times U(1)_{em}$ requires additional Higgs fields belonging to $\{45\}$-dimensional representation, given by $\phi_{\mu\nu}$, with $\langle\phi_{12}\rangle = \langle\phi_{34}\rangle = \langle\phi_{56}\rangle = a$ $\langle\phi_{78}\rangle = b$ $\langle\phi_{910}\rangle \neq 0$, and others.

[7]This case has been analyzed in detail by M. Gell-Mann, P. Ramond, and R. Slansky (private communication).

[8]F. Wilczek and A. Zee, Princeton report, 1979 (unpublished).