Neutrino-induced heavy-lepton pair production at high energies and multimuon events

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Total cross sections for the process $\nu_\mu + {}^{56}\text{Fe} \to \mu^- + L^+ + L^0 + {}^{56}\text{Fe}$ have been calculated in conventional $V - A$ weak-interaction theory. For multi-GeV/c^2 mass assignments for L^+, the cross section is $\sim 10^{-6}$ of the observed rate of charged-current events production by neutrinos at Fermilab. However, for $m_{L^+} = m_{\mu^+}$, and $m_{L^0} \sim 1$ to 2 GeV/c^2, the cross section is $\sim 10^{-1}$ of the reported trimuon rates. Allowing L^0 to decay into muons + anything, we find that the leptons in the final states may be very energetic.

Since the observation of μe final states in e^+e^- annihilation,[1,2,3] evidence for the existence of heavy leptons has been considered compelling.[4] The recent observation of trimuon and four-muon final states in high-energy neutrino interactions has triggered speculation that heavy leptons are being produced there as well.[5,6] The observed trimuon rate $\sim 10^{-4}$–10^{-5} $[\sim(\alpha/\pi)^2]$ of the single-muon rate suggests that one possible source of such events might be the coherent production of heavy-lepton pairs by neutrinos in the Coulomb field of heavy nuclei via the Bethe-Heitler process, such as in

$$\nu_\mu + {}^{56}\text{Fe} \to \mu^- + L^+ + L^0 + {}^{56}\text{Fe}\ , \tag{1}$$

followed by the subsequent decays

$$L^+ \to \overline{L}^0 + X^+ \ \ (L^- \to L^0 + X^-) \tag{2a}$$

and

$$L^0 \to l^- + Y^+ \ \ (\overline{L}^0 \to l^+ + Y^-)\ , \tag{2b}$$

where $X^\pm$, $Y^\pm = \pi^\pm$, $\rho^\pm$, $K^\pm$, $(\mu^\pm, \nu_\mu)$, or $(e^\pm, \nu_e)$ pairs, and $l^\pm = \mu^\pm$ or $e^\pm$, etc., and where (L^-, L^0) and $(L^+, \overline{L}^0)$ are heavy leptons and their associated antiparticles, respectively. Following conventional assumptions, the heavy leptons may carry their own lepton numbers and obey the usual additive lepton-number conservation law. However, we still allow for the possibility that the heavy leptons may decay into μ's (or e's). This may occur either because the lepton-number conservation is only an approximate symmetry which may be violated (Sec. I), or because the heavy leptons have the same quantum numbers as the muon (or electron) as in certain renormalizable gauge theories (Sec. II).[7]

Neutrino-induced leptonic trident production via $\nu_1 + Z \to l_1 + \overline{l}_2 + \nu_2 + Z$, where $l_{1,2} = \mu^-$ or e^-, has been studied extensively by Czyz, Sheppey, and Walecka (CSW)[8] as well as by many others.[9] More recently, heavy-lepton pair production has also been considered by Ng[10] and by Barshay.[11] We extend the calculations of CSW to include the case where the neutral lepton L^0 may be massive.

In terms of conventional $V-A$ point-vertex four-fermion weak interactions, the heavy leptonic current $J_L^\lambda = \overline{\psi}_L(x)\gamma^\lambda(1-\gamma_5)\psi_{L^0}(x)$ is coupled to the muonic current $J_\mu^\lambda = \overline{\psi}_\mu(x)\gamma^\lambda(1-\gamma_5)\psi_{\nu_\mu}(x)$ through

$$\mathcal{L} = \frac{G}{\sqrt{2}}\,(J_L^\lambda J_{\mu\lambda}^+ + \text{H.c.})\ . \tag{3}$$

Here G is taken to be $G_F = 1.025\times 10^{-5}\, m_p^{-2}$, and J_μ^λ are charged currents.

Following CSW, the evaluation of the cross section of reaction (1) $\sigma(1)$ was carried out analytically considering only the two lowest-order Feynman diagrams, shown in Fig. 1, and then numerically with a straightforward Monte Carlo method. The validity and accuracy of our calculations were checked out by reproducing the results of CSW for the coherent process $\nu_\mu + {}^{56}\text{Fe} \to \mu^- + \mu^+ + \nu_\mu + {}^{56}\text{Fe}$ to within 10% for neutrino energies ranging from 50 m_μ to 10 000 m_μ. We summarize below the major results of two kinds of mass assignments to the (L^+, L^0) pair of reaction (1).

I. $m_{L^+} = 1.85$ GeV/c^2, $m_{L^0} = 0.3$ GeV/c^2

Identifying $L^\pm = \tau^\pm$ of Perl *et al.*,[4] and choosing $m_{L^0} = 0.3$ GeV/c^2, which is the largest mass that does not violate the experimental observations in $e^+e^- \to \tau^+\tau^-$ (Ref. 12) and yet allows (2b) to happen, we find that $\sigma(1)$ is too small to account for any of the currently observed phenomena at Fermilab and/or at the CERN SPS. Curve a of Fig. 2 shows the energy dependence of $\sigma(1)$ for E_ν ranging from 50 to 400 GeV. The smallness of $\sigma(1)$ and the data which strongly suggest that the τ family may well possess its own quantum number which forbids (2b) make this scheme very unattractive

II. $m_{L^+} = m_{\mu^+}$, $m_{L^0} \leqslant 2.0$ GeV/c^2

Most renormalizable gauge theories contain heavy leptons.[7] In the context of these theories,

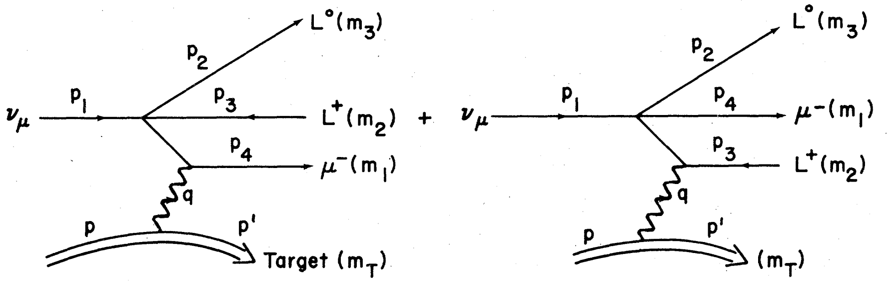

FIG. 1. Feynman diagrams for reaction (1): $\nu_\mu + {}^{56}\mathrm{Fe} \rightarrow \mu^- + L^+ + L^0 + {}^{56}\mathrm{Fe}$.

the heavy leptons are M^+ and M^0 (E^+ and E^0), $J=\frac{1}{2}$ fermions with the same lepton-number assignment as the $\mu^-(e^-)$. The identification of $L^+=\mu^+$ and $L^0=M^0$ for reaction (1) is perfectly legitimate. In other words, μ^- couples to L^0 with full strength G_F.[13] Choosing $m_{L^0}=2.0$ GeV/c^2 (Ref. 14) and 1.0 GeV/c^2, we find the energy dependence of $\sigma(1)$ shown in curves c and d, respectively, of Fig. 2. It is remarkable that $\sigma(1)$ for the latter mass assignment lies less than a factor of 2 lower than the cross section for the coherent process $\nu_\mu + {}^{56}\mathrm{Fe} \rightarrow \mu^- + \mu^+ + \nu_\mu + {}^{56}\mathrm{Fe}$ at $E_\nu > 150$ GeV.

The contribution of $\sigma(1)$ to multimuon final states can be expressed in terms of R, where

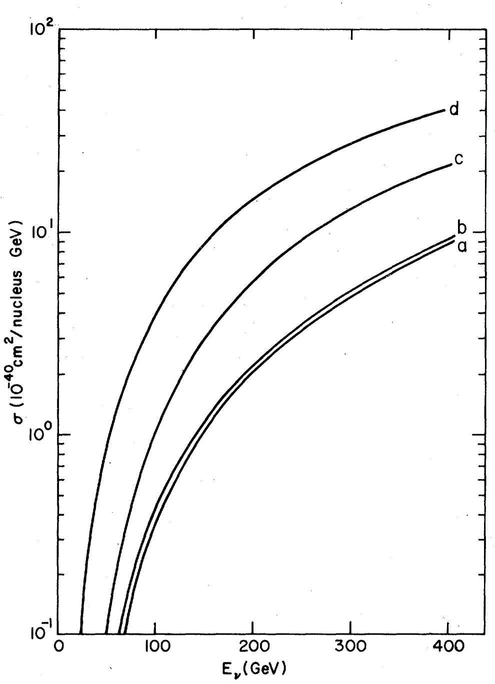

FIG. 2. Total cross sections $\sigma(1)$ for the coherent process $\nu_\mu + {}^{56}\mathrm{Fe} \rightarrow \mu^- + L^+ + L^0 + {}^{56}\mathrm{Fe}$ for the following mass assignments: Curve (a) $m_{L^+}=1.85$, $m_{L^0}=0.3$, (b) $m_{L^+}=1.85$, $m_{L^0}=0.0$, (c) $m_{L^+}=m_{\mu^+}$, $m_{L^0}=2.0$, (d) $m_{L^+}=m_{\mu^+}$, $m_{L^0}=1.0$ GeV/c^2.

$$R \equiv \frac{\text{Rate (multimuon final states)}}{\text{Rate (single-muon final states)}} = \frac{B\sigma(1)}{\sigma_{\text{tot}}\,(\text{charged current}) \times 56} \quad . \tag{4}$$

Here B is the branching ratio for L^0 to decay into states containing at least one muon and

$$\sigma_{\text{tot}}\,(\text{charged current}) = 0.8[E_\nu \text{ (in GeV)}] \times 10^{-38}\ \text{cm}^2/\text{nucleon}.$$[15]

Let us assume that L^0 is created with little polarization and that its branching ratios appropriate for $1 \leq m_{L^0} \leq 2$ GeV/c^2 are given by[7,16]

$$B = 0.6 \text{ for } L^0 \rightarrow \mu^- + \text{hadron(s)}^+ \ ,$$

$$B = 0.2 \text{ for } L^0 \rightarrow \mu^- + \nu_e + e^+ \ ,$$

and

$$B = 0.2 \text{ for } L^0 \rightarrow \mu^- + \nu_\mu + \mu^+ \ .$$

When R is weighted by the energy spectrum of the Fermilab quadrupole-triplet (QT) neutrino beam above 100 GeV (Ref. 17), then trimuon final states of the type $\mu^-\mu^-\mu^+$ will occur at a rate

$$\langle R \rangle_{\text{QT}} \sim 1 \times 10^{-5} \text{ for } E_\nu > 100 \text{ GeV} \ . \tag{5}$$

A more realistic calculation, assuming[16] $B=0.3$ for $L^0 \rightarrow \mu^- + \pi^+$ and $B=0.3$ for $L^0 \rightarrow \mu^- + \rho^+$ and including the contribution of the incoherent process involving individual protons and neutrons in ^{56}Fe as well as the Fermilab-Harvard-Pennsylvania-Rutgers-Wisconsin (FHPRW) experimental cut on acceptance at $E_\mu > 4$ GeV, shows that the latter two corrections cancel one another and yield the same value for $\langle R \rangle_{\text{QT}}$.[18] A similar computation using the spectrum of wide-band (WB) neutrinos of the SPS at CERN and including the incoherent cross section and acceptance cut $E_\mu > 4.5$ GeV (Ref. 17) gives

$$\langle R\rangle_{WB} \sim 0.7\times 10^{-5}, \quad E_\nu > 100 \text{ GeV}$$
$$\sim 0.2\times 10^{-5}, \quad E_\nu > 4.5 \text{ GeV}. \qquad (6)$$

These predicted ratios are lower than the published experimental results: $\langle R\rangle_{CT} = (1.2 \pm 0.5) \times 10^{-4}$ for $E_\nu \geq 100$ GeV and a smaller value ($\sim 2\times 10^{-5}$) for all E_ν based on 49 trimuons [including three bare-target sign-selected (BTSS) ν events] observed by FHPRW, and $\langle R\rangle_{WB} = (3.0 \pm 0.4)\times 10^{-5}$ for $E_\nu \geq 30$ GeV from the observation of 76 $\mu^-\mu^-\mu^+$ final states in the 2.3×10^6 charged-current ν_μ interactions of the CERN-Dortmurd-Heidelberg-Saclay (CDHS) collaboration.[19] Since the model proposed here may explain as much as 10% of the observed trimuon events, it is worthwhile to examine its predictions in more detail.

This model does not require the existence of a charged heavy lepton $M^\pm$ of mass between 5 and 8 GeV/c^2 accompanied by a neutral heavy lepton M^0 of mass between 2 and 4 GeV/c^2, which a number of heavy-lepton cascade (HLC) models propose.[6] Whereas the HLC models do not predict significant production of trimuon final states until $E_\nu \geq 100$ GeV, this model allows such production by neutrinos as low as 50 GeV. For instance, for the mass assignment $m_{L^0} = 1$ GeV/c^2 and the isotropic decay mode $L^0 \to \mu^- + \pi^+$, Fig. 3 shows the products of the neutrino beam spectra of FHPRW and CDHS with the cross section $\sigma(1)$.[17] The two curves

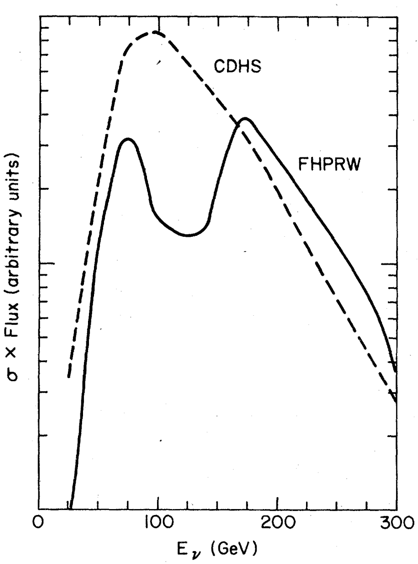

FIG. 3. Flux-times-cross-section curves for the Fermilab quadrupole-triplet neutrino spectrum (FHPRW) and the CERN SPS wide-band spectrum (CDHS) with $\sigma(1)$ for the trimuon model with $L^0 \to \mu + \pi$ and $m_{L0} = 1$ GeV/c^2. The cross section due to the incoherent process and the experimental acceptance cuts are included. The curves are not normalized with respect to each other.

shown here are not normalized with respect to each other. In this calculation and those that follow, the incoherent cross section and the experimental acceptance cuts are taken into account. Whereas the HLC models all contain neutrinos in the final states, 60% of the final states of the model presented above contain no neutrinos. Averaged over all decay modes, the visible energy in the event is ~ 90% of the initial neutrino energy. The above two predictions can be tested in narrow-band neutrino beams.

Because of the experimental acceptance cut imposed on the muon momenta, approximately 40% of the observable trimuon events due to the above process for $m_{L^0} = 1$ GeV/c^2 will arise from the $L^0 \to \mu^- + \pi^+$ decay. The remaining 60% of the events arises from the decays $L^0 \to \mu^- + \rho^+$, $\mu^- + \nu_e + e^+$, and $\mu^- + \nu_\mu + \mu^+$. We shall therefore concentrate our discussion on the $L^0 \to \mu^- + \pi^+$ decay mode. Labeling the negative muon arising at the four-fermion vertex μ_1^- and the negative muon coming from the L^0 decay μ_2^-, we find for $m_{L^0} = 1$ GeV/c^2 that the average energy $\langle E_{\mu_2^-}\rangle$ is less than or equal to $\langle E_{\mu_1^-}\rangle$ and that these average energies as well as $\langle E_{\pi^+}\rangle$ are each ~ $0.3\,E_\nu$. Table I provides more information on such average characteristics for several incident neutrino energies.

In the plane perpendicular to the incident neutrino beam, the negative muon μ_1^- arising at the four-fermion vertex and the negative muon μ_2^- coming from the decay $L^0 \to \mu_2^- + \pi^+$ tend to go off in opposite directions. In order to compare the predicted results with the experimental observations, we now redefine μ_1 as the faster of μ_1^- and μ_2^-, i.e., $\mu_1 = \text{fast}\,(\mu_1^-, \mu_2^-)$. Similarly, $\mu_2 = \text{slow}\,(\mu_1^-, \mu_2^-)$ and $\mu_3 = \mu^+$. The azimuthal correlations $\Delta\phi_{i-j} = \phi_{\mu_i} - \phi_{\mu_j}$ and $\Delta\phi_{1-(2+3)} = \phi_{\mu_1} - \phi_{\mu_2+\mu_3}$ are shown in Fig. 4, where $i,j = 1, 2, 3$ for $i \neq j$, and

TABLE I. Average characteristics of the final states for the process $\nu_\mu + {}^{56}\text{Fe} \to \mu_1^- + L^+ + L^0 + {}^{56}\text{Fe}$ and $L^0 \to \mu_2^- + \pi^+$ where $L^+ = \mu^+$ and L^0 has a mass of 1.0 GeV/c^2 and decays isotropically.

E_ν (GeV)	100	200	300
$\langle E_{\mu_1^-}\rangle$ (GeV)	29	63	96
$\langle E_{\mu^+}\rangle$ (GeV)	11	22	32
$\langle E_{\mu_2^-}\rangle$ (GeV)	30	57	83
$\langle E_{\pi^+}\rangle$ (GeV)	30	58	89
$\langle\theta_{\mu_1^-\mu^+}\rangle$ (mrad)	140	110	90
$\langle\theta_{\mu_2^-\mu^+}\rangle$ (mrad)	120	90	70
$\langle\theta_{\mu_1^-\mu_2^+}\rangle$ (mrad)	80	60	50
$\langle M_{\mu_1^-\mu^+}\rangle$ (GeV/c^2)	1.2	1.5	1.8
$\langle M_{\mu_2^-\mu^+}\rangle$ (GeV/c^2)	1.1	1.4	1.6
$\langle M_{\mu_1^-\mu_2^-}\rangle$ (GeV/c^2)	1.5	1.9	2.2
$\langle M_{\mu_1^-\mu^+\mu_2^-}\rangle$ (GeV/c^2)	2.3	3.1	3.5

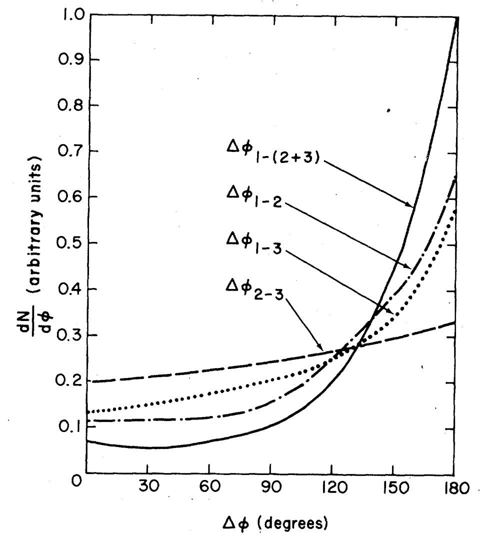

FIG. 4. The azimuthal angle between the muon systems in the plane perpendicular to the neutrino beam for the trimuon model with $L^0 \to \mu^- + \pi^+$, $m_{L^0} = 1$ GeV/c^2 and $E_\nu = 150$ GeV. We define μ_1=fast (μ_1^-, μ_2^-), μ_2=slow (μ_1^-, μ_2^-), and $\mu_3 = \mu^+$. ϕ_{2+3} is the azimuthal angle of the ($\vec{P}_{\mu_2^-} + \vec{P}_{\mu^+}$) vector.

$\phi_{\mu_2+\mu_3}$ is the azimuthal angle of the $(\vec{P}_{\mu_2} + \vec{P}_{\mu_3})$ vector. For $m_{L^0} \sim 1$ GeV/c^2, μ_2^- originates primarily in the L^0 decay and tends to go out in the same direction as L^0 in the laboratory reference frame. Hence, the balance of momenta transverse to the incident neutrino beam requires the μ_1 and the $(\mu_2 + \mu_3)$ systems to go off back-to-back, in good agreement with the experimental observation. Although one may attribute such events to contributions made by the hadronic vertex, e.g., via muon pair production by W-nucleon interactions, etc.,[19,20] one cannot rule out the possibility that some fraction of trimuons arises from the production mechanism described above.

It has been pointed out that trimuons can also be generated from the radiative processes of the μ^- creating a muon Dalitz pair via $\nu_\mu + N \to \mu^- + $"$\gamma$"$ + X$, where "$\gamma$" $\to \mu^+ + \mu^-$ is a virtual photon.[20] Such an electromagnetic process has a comparable rate and is similar to that of reaction (1). However, it yields a $\Delta\phi_{1-(2+3)}$ distribution peaking at 0°, in sharp contrast to our model which predicts a peaking at 180°, hence providing a way to distinguish between the two processes.

Because the target nucleus (nucleons) receives in the interaction a four-momentum-transfer-squared which is less than (1 GeV/c^2)^2 on the average, the leptons are necessarily energetic and the opening angles between pairs of muons in the final state are small. In other words, the process we have described here concentrates the initial neutrino energy into the muons and not into the hadrons. This mechanism may therefore be responsible for the super trimuon events reported by the FHPRW collaboration. Since the events regarded to be "super" can be distinguished from the remaining trimuon events when one plots $E_2 + E_3$ versus $E_1 - E_{\rm had}$,[21] where once again E_1 (E_2) is the energy of the faster (slower) μ^-, we present in Fig. 5 the prediction of this production mechanism for $E_\nu = 250$ GeV. It is interesting to note that the FHPRW superevent 119 ($E_{1,2,3} = 157$, 32, and 47 GeV, $E_{\rm had} = 13$ GeV, hence $E_\nu - E_{\rm visible} \simeq 250$ GeV) lies in the most populous region of the scattered plot. However, event 281 ($E_{1,2,3} = 96$, 73, and 83 GeV, $E_{\rm had} < 30$ GeV, hence, $E_\nu - E_{\rm visible} \simeq 280$ GeV) lies in a region of much less likelihood. Although, as shown in Fig. 3, for $E_\nu \geq 250$ GeV the $\sigma\times$ flux does not exhibit the most favorable condition, its contribution to reaction (1) is not entirely insignificant.

Another consequence of small momentum and energy transfers involved in reaction (1) is the prediction of small invariant masses for the muons. Figure 6 shows the invariant-mass distributions of the muons for $E_\nu = 150$ GeV when the $L^0 \to \mu^- + \pi^+$ decay mode is considered alone. The neutrino energy is so chosen because it is near the average found when E_ν is weighted with either the FHPRW or the CDHS flux-times-cross-section

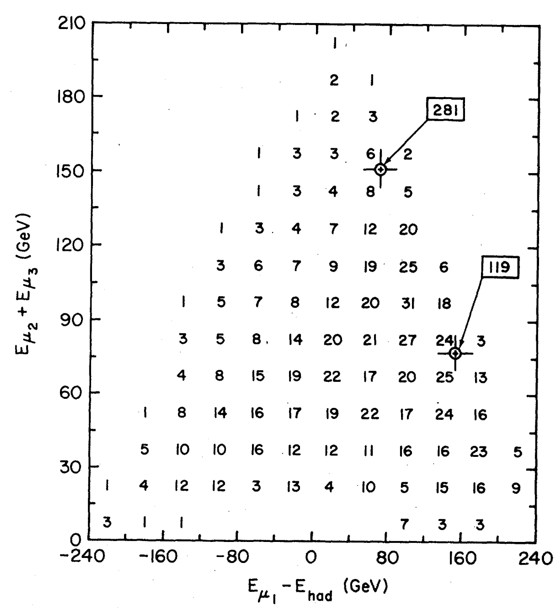

FIG. 5. Scatter plot of $E_{\mu_1} - E_{\rm had}$ versus $E_{\mu_2} + E_{\mu_3}$ for trimuon model with $L^0 \to \mu + \pi$ and $m_{L^0} = 1$ GeV/c. The crosses represent the superevents ET 119 and ET 281 of the FHPRW group. The neutrino beam energy is chosen to be 250 GeV, and the scatter plot has been normalized to 1000 events.

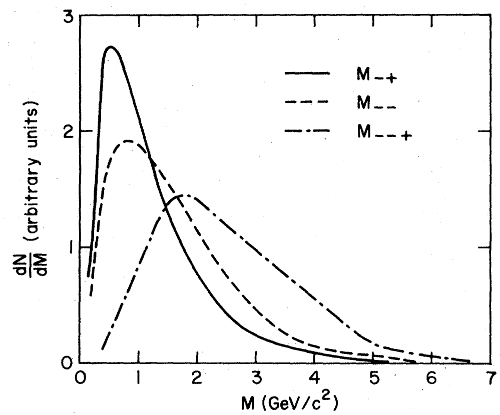

FIG. 6. Invariant-mass distributions for the trimuon model with $L^0 \rightarrow \mu + \tau$, m_{L^0}=1 GeV/c^2, and E_ν =150 GeV.

curves of Fig. 3. Folding in the neutrino energy spectrum will not change the differential distribution of Fig. 6 in any significant way. Thus, the present production mechanism predicts a peaking of the invariant-mass distributions at the lower mass ends of the spectra, which is also consistent with the experimental observations.

To conclude, assuming the existence of a massive L^0 with a moderate mass assignment, reaction (1) may contribute as much as 10% of the observed trimuon final states in high-energy neutrino reactions. Trimuon events of this kind can distinguish themselves from the electromagnetic backgrounds and from those originating from the HLC and the hadronic production mechanisms by the following features: They are characterized by (i) low E_ν threshold in production, (ii) little missing energy, (iii) small opening angle and small invariant masses for the muons, (iv) $\Delta\phi_{1-(2+3)}$ peaking at 180°, and (v) concentration of the energy among the leptons.

Finally, we wish to point out that if $L^0 \rightarrow \mu^- + \mu^+ + \nu_\mu$ occurs with $B = 0.2$ (Refs. 7 and 16) the above process will contribute a four-muon event for every four trimuon events, not including the acceptance cuts. If the proposed pair production model contributes 10% of the trimuon events, the predicted rate for four-muon events is consistent with the experimental data: one four-muon event in 76 trimuons (CDHS) and two in 49 (FHPRW).[5]

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[17]For the quadrupole-triplet neutrino beam spectrum used by FHPRW at Fermilab, see the third paper in Ref. 6. For the wide-band neutrino beam spectrum used by CDHS at CERN, see the fifth paper in Ref. 5.

[18]At $E_\nu = 50$ GeV, the cross section of the incoherent process involving protons and neutrons in ^{56}Fe is 50% that of the coherent, but is only 20% at $E_\nu = 200$ GeV. At $E_\nu = 50$ GeV, 66% of the trimuon events are not seen due to the experimental acceptance cut, but only 20% are unobserved at $E_\nu = 200$ GeV.

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