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Infrared properties of quark gas

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Infrared properties of quark gas at finite density are studied using renormalization-group-improved perturbation theory. The running coupling constant shows color charge screening in the infrared region and asymptotic freedom in the ultraviolet region. Color density correlations are finite. Instanton contributions to the partition function are estimated and found to be large at low density. Possible ambiguities of the perturbation expansion in the many-body medium are discussed.

I. INTRODUCTION

Since the observation by Collins and Perry[1] that the effective coupling constant in quantum chromodynamics (QCD) is small at high density and temperature, much work has been done on the thermodynamic properties of a gas of quarks and gluons.[2] For the most part this work has concentrated on the calculation of the thermodynamic potential in perturbation theory. The accuracy of the calculations has been improved by an application of the renormalization group whereby the masses and coupling constant are effectively replaced by ones depending on the temperature and chemical potentials. Recently it has been shown[3] that the perturbative vacuum about which the above calculations were carried out is stable against fluctuations of the color magnetic field.

A lingering question remains about the true infrared finiteness of the theory. The standard procedure is to (i) calculate the thermodynamic potential with a fixed coupling constant, (ii) subtract off the infinite vacuum contribution, and (iii) replace the fixed coupling constant by the renormalization-group running coupling constant. What if the renormalization group were applied to the many-body Green's functions and then integration over momenta were carried out to obtain the thermodynamic potential? Would not the pole in the running coupling constant, $\bar{g}^2 \sim 1/\ln(-p^2/\Lambda^2)$, cause the thermodynamic potential to be ill-defined?

A second lingering question, touched on by most papers,[2] is how to relate the scale violation parameter Λ as determined by scattering experiments to the running coupling constant in the many-body problem.

A third question concerns the role of instantons. At what density, if any, do instanton contributions to the thermodynamic potential become significant?[4]

In this paper we shall attempt to answer these questions. To do so first requires the construction of the effective running coupling constant in the material medium, $\bar{g}^2 = \bar{g}^2(M, \mu)$, where M is the subtraction point and μ is the chemical potential. This computation is presented in Sec. II. The result is that $\bar{g}^2 \to 0$ in the ultraviolet and infrared regions. (Potential ambiguities in the perturbation expansion must be kept in mind.) The connection between Λ measured in a scattering experiment and the many-body medium then becomes apparent.

As an application of Sec. II we construct the renormalization-group-improved result for the color density fluctuation/correlation function in Sec. III. Finally Sec. IV contains an estimate of the instanton contribution at moderate densities.

II. COMPUTATION OF RUNNING COUPLING CONSTANT

Our computation of the running coupling constant will follow the standard procedure.[5] Consider the gauge group SU(N_c) and N_f flavors of quark, all massless for simplicity. Each flavor i will have an associated chemical potential μ_i. Temperature is taken to be zero. Nonzero temperature complicates the algebra and is not expected to introduce any different physics. We work consistently in the Landau gauge.

The calculation begins by evaluating the two- and three-point gluon functions in the many-body system. These are shown in Fig. 1. Only the diagrams with an internal quark loop differ from those in the vacuum. For the two-point function

$$\Gamma_T^{2,0} = (p^2 g^{\mu\nu} - p^\mu p^\nu)\left[1 + \frac{g^2}{96\pi^2}(13N_c - 4N_f)\ln\left(\frac{-p^2}{M^2}\right) - \frac{1}{\vec{p}^2}\Delta\pi^{00}\right] + g^{\mu i}(p_i p_j - \delta_{ij}\vec{p}^2)g^{j\nu}\frac{1}{2\vec{p}^2}\left(\Delta\pi^\mu_\mu + \frac{3p^2}{\vec{p}^2}\Delta\pi^{00}\right), \quad (1)$$

where the matter contributions are

$$\Delta\pi^{00}=\frac{g^2}{\pi^2}\sum_i\left[-\frac{\mu_i^2}{3}-\frac{p^2\sin^2\theta}{48}\ln\left(1-\frac{8\mu_i^2\cos2\theta}{p^2}+\frac{16\mu_i^4}{(p^2)^2}\right)\right.$$
$$+\frac{\mu_i}{48(-p^2)^{1/2}\sin\theta}(4\mu_i^2+3p^2)\ln\left(\frac{[\sin\theta-2\mu_i/(-p^2)^{1/2}]^2+\cos^2\theta}{[\sin\theta+2\mu_i/(-p^2)^{1/2}]^2+\cos^2\theta}\right)$$
$$\left.+\frac{\cot\theta}{4}\left(\mu_i^2+\frac{p^2}{12}(1+2\sin^2\theta)\right)\tan^{-1}\left(\frac{\sin2\theta}{\cos2\theta-p^2/4\mu_i^2}\right)\right] \tag{2}$$

and

$$\Delta\pi^{\mu}{}_{\mu}=\frac{g^2}{\pi^2}\sum_i\left\{-\frac{\mu_i^2}{2}-\frac{(-p^2)^{1/2}}{8\sin\theta}\left[\mu_i\ln\left(\frac{[\sin\theta-2\mu_i/(-p^2)^{1/2}]^2+\cos^2\theta}{[\sin\theta+2\mu_i/(-p^2)^{1/2}]^2+\cos^2\theta}\right)\right.\right.$$
$$-\frac{(-p^2)^{1/2}\sin\theta}{2}\ln\left(1-\frac{8\mu_i^2\cos2\theta}{p^2}+\frac{16\mu_i^4}{(p^2)^2}\right)$$
$$\left.\left.+(-p^2)^{1/2}\cos\theta\tan^{-1}\left(\frac{\sin2\theta}{\cos2\theta-p^2/4\mu_i^2}\right)\right]\right\}\ . \tag{3}$$

Here the angle θ is defined by

$$\tan\theta=i|\vec{p}|/p^0\ . \tag{4}$$

We are using the Minkowski metric in a many-body system, so p^0 is pure imaginary: $-i\infty<p^0<i\infty$. The most notable property of Eq. (1) is the appearance of a non-Lorentz-covariant tensor. This occurs because there is a preferred frame of reference, the c.m. of the medium.

The standard procedure for massless quarks in the vacuum is to take

$$\left(M\frac{\partial}{\partial M}+\beta\frac{\partial}{\partial g}+2\gamma_A\right)\Gamma_T^{2,0}=0\ , \tag{5}$$

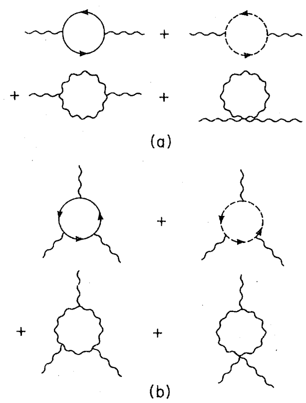

FIG. 1. One-loop contributions to the (a) gluon propagator and (b) three-gluon vertex. Only the quark loops give a contribution different from the vacuum at finite density but zero temperature.

evaluate at $p^2=-M^2$, and solve for γ_A in terms of β or vice versa. Now there is a slight complexity because an explicit scale, μ_i, enters the problem. The procedure is analogous to that required to handle massive quarks. Define an auxiliary coupling constant by

$$f_i=\mu_i/M\ . \tag{6}$$

The many-body Green's function has the functional form

$$\Gamma(p,g,p^2/M^2,\mu_i^2/p_0^2,\mu_i^2/\vec{p}^2)=\Gamma(p,g,p^2/M^2,f_i^2M^2/p_0^2,f_i^2M^2/\vec{p}^2)\ . \tag{7}$$

One can now go through the standard analysis to obtain the renormalization-group-improved Green's function. Scaling the initial momentum p by λ so that $k=\lambda p$ and $p^2=-M^2$ results in

$$\left(-\lambda\frac{\partial}{\partial\lambda}-\sum_i f_i\frac{\partial}{\partial f_i}+\beta\frac{\partial}{\partial g}+\gamma\right)\Gamma=0\ . \tag{8}$$

Then

$$\lambda\frac{\partial\bar{g}}{\partial\lambda}=\beta(\bar{g},\mu_i/M\lambda),\quad \bar{g}(\lambda=1)=g\ , \tag{9}$$

and the renormalization-group-improved Green's function is

$$\Gamma(k,\bar{g}(-k^2),\ \mu_i^2/k_0^2,\ \mu_i^2/\vec{k}^2)\times\exp\left[\int_M^{(-k^2)^{1/2}}\gamma(\bar{g}(M'^2))\frac{dM'}{M'}\right]\ . \tag{10}$$

Now an ambiguity shows up. We should insert Eq. (1) in Eq. (8), evaluate at $p^2=-M^2$ and $\lambda=1$, and obtain a relation between β and γ_A. But $\Gamma_T^{2,0}$ is not Lorentz covariant; it depends on p^0 and $|\vec{p}|$

separately, i.e., it depends on θ of Eq. (4). Thus β, and also $\bar{g}$, depend on θ. This does not seem too bad at first sight, but to determine β and γ_A we also need to evaluate a three-point function. By renormalizing the three-point function at some arbitrary momentum configuration p_1, p_2, p_3, it is not clear what angle should correspond to θ. Indeed there will be a θ_1, θ_2, θ_3 in general in addition to θ. This ambiguity is somewhat related to, but more severe than, that encountered in specifying what momentum configuration to use in the vacuum with massive quarks.[6]

We can argue that the natural way to handle this ambiguity is to subtract at $p=(iM,0,0,0)$. This is a natural choice because now p coincides with the only other (pseudo) four-vector around, $\eta=(1,0,0,0)$, which specifies the c.m. of the medium.[7] With this choice the coefficient of each non-Lorentz-covariant tensor in a Green's function should vanish. It may be verified explicitly for the $\Gamma_T^{2,0}$ of Eq. (1).

With these remarks in mind we may now find one relationship between β and γ_A using $\Gamma_T^{2,0}$:

$$\frac{g^2}{96\pi^2}(13N_c-4N_f)+\frac{g^2}{24\pi^2}(\beta/g+\gamma_A)\sum_i\left[\ln(1+4\mu_i{}^2/M^2)-4\mu_i{}^2/M^2\right]-\gamma_A=0\,. \tag{11}$$

To get a second relationship between β and γ_A we evaluate the diagrams of Fig. 1(b) with momentum p in one leg, p out a second, and 0 momentum in the third. The result is

$$\beta\left[1+\frac{g^2}{16\pi^2}\sum_i\left(\frac{4\mu_i{}^2}{4\mu_i{}^2+M^2}+\ln(1+4\mu_i{}^2/M^2)\right)\right]-\frac{g^3}{16\pi^2}(\tfrac{17}{6}N_c-\tfrac{4}{3}N_f)$$
$$+3g\gamma_A\left[1+\frac{g^2}{48\pi^2}\sum_i\left(\frac{4\mu_i{}^2}{4\mu_i{}^2+M^2}+\ln(1+4\mu_i{}^2/M^2)\right)\right]=0\,. \tag{12}$$

Solving for β perturbatively yields

$$\beta=\frac{-g^3}{16\pi^2}\left[(\tfrac{11}{3}N_c-\tfrac{2}{3}N_f)+\tfrac{2}{3}\frac{g^2}{16\pi^2}\sum_i\left[(2-\tfrac{17}{6}N_c+\tfrac{4}{3}N_f)4\mu_i{}^2/M^2+(-\tfrac{7}{2}+\tfrac{73}{12}N_c-\tfrac{7}{3}N_f)\ln(1+4\mu_i{}^2/M^2)\right.\right.$$
$$\left.\left.+(-\tfrac{3}{2}+\tfrac{13}{4}N_c-N_f)4\mu_i{}^2/(4\mu_i{}^2+M^2)\right]+\cdots\right]\,. \tag{13}$$

The effect of nonzero μ_i shows up only in order g^5 in β. Presumably a two-loop calculation would contribute vacuum terms of order g^5 and matter terms of order g^7. That this should be the case follows from the expectation that the matter contributions to Green's functions are finite and not in need of renormalization.[8] Up to order g^5 inclusive the term of order g^3 should be dominant at large M (ultraviolet), while the term of order $g^5\mu_i{}^2/M^2$ may be dominant in the infrared. Hence, to simplify our analysis, approximate β by

$$\beta\approx\frac{-g^3}{16\pi^2}\left((\tfrac{11}{3}N_c-\tfrac{2}{3}N_f)+\tfrac{8}{3}(2-\tfrac{17}{6}N_c+\tfrac{4}{3}N_f)\frac{g^2}{16\pi^2}\times\sum_i\frac{\mu_i{}^2}{M^2}\right)\,. \tag{14}$$

Herein lies a second possible ambiguity. Our philosophy has been to compute β to some finite order in g, but at each order in g to keep all orders in μ_i/M. The consistency of this approach is not clear because terms of the form $(g^2\mu_i{}^2/M^2)^n$ may become increasingly important as $\mu_i/M\to\infty$ even though $g\to 0$. Our analysis depends on the assumption that perturbation theory is a good guide to the real physics. If that is taken from us, we are lost. At any rate we will not consider this point again in this paper. *Only if a two-loop calculation gives a qualitatively different picture should we become alarmed.*

Equation (9) is equivalent to solving

$$M\frac{\partial\bar{g}}{\partial M}=\beta\,. \tag{15}$$

Let us define an average chemical potential $\bar{\mu}$ by

$$\bar{\mu}^2=\frac{1}{\tilde{N}_f}\sum_{i=1}^{\tilde{N}_f}\mu_i{}^2\,. \tag{16}$$

The usual situation is when all nonzero μ_i are the same order of magnitude. By a redefinition of variables the equation we want to solve is

$$\frac{dy}{dx}=\frac{-y^2}{x}+\frac{y^3}{x^3}\,, \tag{17}$$

where

$$y=a\frac{\bar{g}^2}{16\pi^2}\,,\quad x=b\frac{M}{\bar{\mu}}\,,\quad a=\tfrac{2}{3}(11N_c-2N_f)\,,\quad b=a/[\tfrac{16}{3}(\tfrac{17}{6}N_c-\tfrac{4}{3}N_f-2)\tilde{N}_f]^{1/2}\,. \tag{18}$$

For SU(3) this assumes that the number of massless quarks N_f is less than five.

To our knowledge Eq. (17) cannot be solved in terms of elementary functions. Therefore, let us study various properties of the solution. As $x\to\infty$,

$$\frac{dy}{dx}\approx\frac{-y^2}{x}\ , \tag{19}$$

so

$$y\approx 1/\ln(x/x_0)\,. \tag{20}$$

As $x\to 0$,

$$\frac{dy}{dx}\approx\frac{y^3}{x^3}\ , \tag{21}$$

so

$$y\approx x\,. \tag{22}$$

Furthermore, $dy/dx=0$ at some value x_c, and the corresponding maximum value of y is $y(x_c)=x_c^2$. x_c and x_0 are related in some unknown way.

An approximate parametrization of the exact solution of Eq. (17), found to be accurate numerically to about 10%, is

$$y\approx 1/\ln\left[x/x_0+c\exp(1/x)\right]. \tag{23}$$

The parameters x_0 and c are related to x_c by

$$x_0=x_c(x_c+1)\exp(-1/x_c^2)\,,\qquad c=\frac{x_c}{x_c+1}\exp(1/x_c^2-1/x_c)\,. \tag{24}$$

This approximate solution reproduces the asymptotic forms of Eqs. (20) and (22), and also reproduces the position and value of the maximum, $y(x_c)=x_c^2$.

Transforming Eq. (23) back to the running coupling constant gives

$$\frac{\bar{g}^2}{16\pi^2}=\frac{1}{\frac{2}{3}(11N_c-2N_f)\ln\left[bM/x_0\bar{\mu}+c\exp(\bar{\mu}/bM)\right]}\ . \tag{25}$$

If we want this to agree with the vacuum result in the far ultraviolet, then we should identify

$$\frac{\Lambda}{\bar{\mu}}=\frac{x_c(x_c+1)\exp(-1/x_c^2)}{11N_c-2N_f}\times\left[2(17N_c-8N_f-12)\tilde{N}_f\right]^{1/2}. \tag{26}$$

This identification is entirely reasonable since at distances small compared to typical interparticle spacings, individual particles should know nothing about the surrounding isotropic medium. Notice that as $\bar{\mu}^2\tilde{N}_f\to\infty$, $x_c\to 0$ and as $\bar{\mu}^2\tilde{N}_f\to 0$, $x_c\to\infty$.

To get a feeling for the numbers involved, consider the case of two massless quarks, "up" and "down," which have chemical potentials equal to μ.

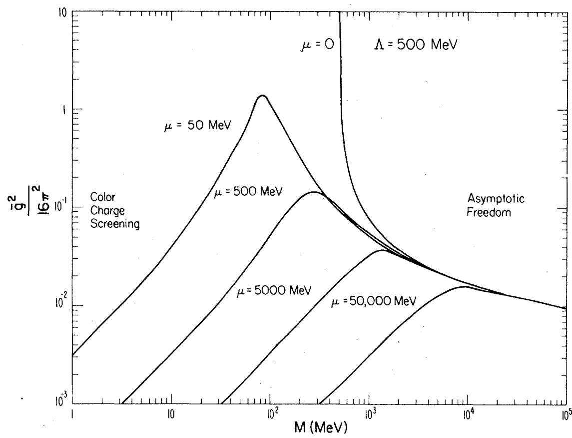

FIG. 2. Plot of the renormalization-group running coupling constant for the special case of two massless quarks with equal chemical potential μ. For $\mu\lesssim 1$ GeV the system will not be in the quark phase.

The system is analogous to nuclear matter at high density. In Fig. 2 we plot $\bar{g}^2/16\pi^2$ vs M with Λ setting the scale at 500 MeV. For $\mu=0$ the coupling has a pole at $M=500$. This is the standard vacuum result. For $\mu\neq 0$, $\bar{g}^2/16\pi^2$ is finite for all M. When $M\gg\mu$ we approach the asymptotic-freedom result. When $M\ll\mu$, $\bar{g}^2/16\pi^2$ goes to zero as M. The peak value decreases in magnitude and shifts further to the right as $\mu/\Lambda\to\infty$. This behavior is entirely reasonable. At very short distances the presence of the material medium is irrelevant. At very large distances there is sufficient matter in between so that the medium can be polarized and screen the colored charges. This is the predicted behavior of lattice gauge models at high temperature but zero quantum number densities.[9] It should be kept in mind though that if μ is too small, $\mu<$ several GeV say, then the quark gas phase will not be stable against collapse into a hadronic phase and our results will be invalidated. At least that is the hope for QCD.

An interesting side remark is in order. If we naively take the $\mu\to 0$ limit of our interpolating solution, we find

$$\frac{\bar{g}^2}{16\pi^2}\xrightarrow[\mu\to 0]{}\frac{1}{\frac{2}{3}(11N_c-2N_f)\ln(M/\Lambda+1)}\ , \tag{27}$$

which has a pole at $M=0$ in contrast to the explicit vacuum result

$$\frac{\bar{g}^2}{16\pi^2}\underset{\mu=0}{=}\frac{1}{\frac{2}{3}(11N_c-2N_f)\ln(M/\Lambda)} \tag{28}$$

which has a pole at $M=\Lambda$. This type of behavior in differential equations is well known, but its physical interpretation in this case is not understood.

III. COLOR DENSITY CORRELATIONS

As an application of the ideas presented so far let us consider color density correlations in momentum space in the one-loop approximation:

$$\pi_{ab}^{00}(p)=\int d^4x\, \exp(ip\cdot x)\langle T[J_a^0(x)J_b^0(0)]\rangle\,. \quad (29)$$

The angular brackets denote the ensemble average. J_a^μ is the color charge current.

From Eqs. (11) and (12) we find

$$\gamma_A=\frac{g^2}{16\pi^2}\left[(\tfrac{13}{6}N_c-\tfrac{2}{3}N_f)+\frac{g^2}{16\pi^2}\sum_i\left(-\tfrac{8}{3}(\tfrac{17}{6}N_c-\tfrac{4}{3}N_f-2)\frac{\mu_i^2}{M^2}+(\tfrac{73}{18}N_c-\tfrac{14}{9}N_f-\tfrac{7}{3})\ln(1+4\mu_i^2/M^2)+(\tfrac{13}{6}N_c-\tfrac{2}{3}N_f-1)\frac{4\mu_i^2}{4\mu_i^2+M^2}\right)+\cdots\right]. \quad (30)$$

Following the remarks leading to Eq. (14) we truncate γ_A to

$$\gamma_A\approx\frac{g^2}{16\pi^2}\left((\tfrac{13}{6}N_c-\tfrac{2}{3}N_f)-\tfrac{4}{9}(17N_c-8N_c-12)\tilde{N}_f\times\frac{\overline{\mu}^2}{M^2}\right). \quad (31)$$

Inserting Eq. (25) into Eq. (31) we find that we cannot evaluate the integral in Eq. (10) in terms of elementary functions. However, we can find a (nonunique) interpolating formula which has the correct asymptotic properties:

$$\exp\left(\int_M^{(-k^2)^{1/2}}2\gamma_A(\overline{g})\frac{dM'}{M'}\right)\approx F_0\left[\frac{\Lambda}{(-k^2)^{1/2}}+\left|\ln\left(\frac{(-k^2)^{1/2}}{\Lambda}\right)\right|^K\right]. \quad (32)$$

Here F_0 is a constant, Λ and a are as before, and

$$K=(13N_c-4N_f)/3a\,. \quad (33)$$

The approximate color density correlation function is obtained by taking $\Gamma_T^{2,0}$ from Eq. (1), replacing g with $\overline{g}$, multiplying by the factor in Eq. (32), and subtracting off the free-field inverse propagator. The resulting expression is tedious to write down and not very interesting in its entirety. The interesting aspects are (i) it is finite for all $-k^2\geq 0$, (ii) its ultraviolet limit is $-k^2|\ln(-k^2/\Lambda^2)|^K$, and (iii) its infrared limit is a constant. Unfortunately its Fourier transform to position space is not well-defined because of its ultraviolet behavior. Presumably this arises because a high-frequency probe will make quark-antiquark and gluon pairs, which gives rise to strong correlations in the ultraviolet. A naive dimensional argument would say that a function whose infrared limit is a constant would behave as $1/r^4$ in position space, but such arguments are unreliable.

IV. INSTANTONS

For the most part calculations on the thermodynamics of quark gas have relied totally on perturbation theory. The exception is the series of papers by Harrington and Shepard.[4] They show that instantons have a direct physical significance in the many-body problem (which is after all a problem in Euclidean space) as pseudoparticle excitations of the medium. If these nonlinear excitations are not included in a calculation, then some essential physics may be missing. They argue on physical grounds that the integral over instanton size ρ should be cut off at $\rho_c\approx 1/T$, the inverse temperature. For the ground state of a quark gas it would be $\rho_c\approx 1/\mu$. In the vacuum the integral over ρ diverges, but with the above cutoff it converges. Hence, instanton effects are naively expected to be significant only at low energy density.

The difficulty with doing an unambiguous calculation in the manner of 't Hooft[10] is easily seen. The instanton solution in the Euclidean vacuum has an O(4) symmetry. To calculate the quantum corrections about that solution involves finding the eigenvalues of some operator, which reduces to an ordinary differential equation because of O(4) symmetry. At finite temperature we lose O(4) symmetry because the energy component of the momentum four vector is discrete. At finite chemical potential we lose O(4) symmetry because of the additional term in the Lagrangian $\mu\overline{\psi}\gamma^0\psi$. This term cannot be treated as a perturbation like a mass term can because it is precisely this term which is expected to cut off the instanton size integral. In either case we lose O(4) symmetry with the result that we must solve a complicated partial differential equation in two variables which does not factorize.

Let us recall the dilute-instanton-gas contribution in SU(3) to the generating functional in the vacuum[10,11]:

$$\ln Z^{\text{inst}} = 0.003 L^4 \int_0^\infty \frac{d\rho}{\rho^5}\left(\frac{8\pi^2}{\bar{g}^2(1/\rho)}\right)^6 \times \exp[-8\pi^2/\bar{g}^2(1/\rho)]\,. \quad (34)$$

The generating functional goes over to the partition function in the many-body problem. To get an estimate of the dominant instanton effects at moderate density let us make the following ansatz: Replace the running coupling constant in the vacuum by the running coupling constant in the quark medium. Apart from the fact that it is the most obvious modification to make, the motivation is provided by color charge screening. At high density the scale size of instantons should be naturally limited, and color charge screening limits any possible long-range correlations. The instanton contribution to the pressure is then

$$P^{\text{inst}} = 0.003 \int_0^\infty \frac{d\rho}{\rho^5}\left(\frac{8\pi^2}{\bar{g}^2}\right)\exp(-8\pi^2/\bar{g}^2)\,, \quad (35)$$

where $\bar{g}$ is taken from Eq. (25) and evaluated at $M = 1/\rho$. The integrand decreases as $\rho \to 0$ as

$$\rho^{6-(2/3)N_f}\left(\ln\frac{1}{\rho}\right)^6,$$

just as in the vacuum. The integrand is cut off exponentially as $\rho \to \infty$,

$$\rho \exp\left(-\frac{a}{2b}\,\bar{\mu}\rho\right).$$

This exponential falloff is exactly analogous to what happens in a weak-interaction theory.[10] There the Higgs field provides the cutoff. This mean Higgs field is really just a boson condensate,[12] i.e., a many-body system. The Higgs boson condensate and the ground-state quark gas both provide a natural cutoff to the instanton scale size.

Unfortunately, the integral in Eq. (35) cannot be done analytically. To get a feeling for the numbers consider again the example of two massless quarks with equal chemical potentials μ. The ratio of the pressure due to interactions to the pressure of a noninteracting gas is plotted in Fig. 3. The instanton contribution is positive. Also plotted are the order $\bar{g}^2$, and the order $\bar{g}^2$, $\bar{g}^4 \ln \bar{g}^2$, and $\bar{g}^4$ inclusive contributions in perturbation theory.[13] At high density clearly the instanton contribution is totally negligible. At lower density the interpretation is not so clear. The perturbative contributions are both negative. The second-order result would indicate that the total pressure is zero at $\mu \approx 0.8$ GeV, possibly indicating that the quarks will condense into hadrons. The fourth-order result would also indicate that this occurs at $\mu \approx 0.8$ GeV. Of course there is an uncertainty in the instanton estimate because of the identification $M = 1/\rho$. The safest statement to make is that when the perturbative corrections become important, i.e., of order unity, then so do the nonperturbative corrections. Depending on one's faith in the first few terms of perturbation theory (and nonperturbative perturbation theory), one might claim that instanton effects tend to stabilize the gas and so lower the density at which a phase transition to hadrons occurs. This is because the instanton corrections to the pressure are positive, while the perturbative corrections seem to be negative.

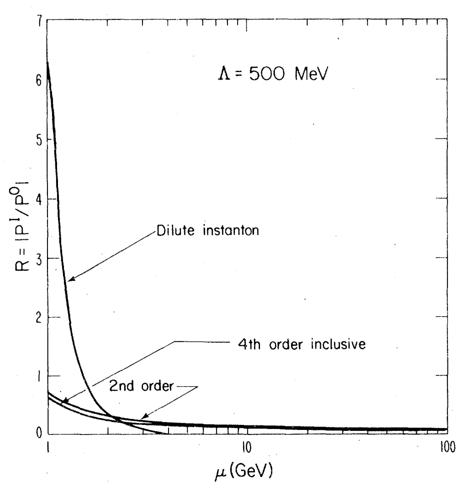

FIG. 3. Plot of the ratio of the pressure due to interactions to the pressure of an ideal gas, for the special case of two massless quarks with equal chemical potential μ. The instanton contribution is positive, and the perturbative contributions are negative.

V. CONCLUSION

In this paper we have investigated some of the infrared properties of quark gas. The two- and three-point gluon functions were evaluated in the one-loop approximation to obtain the renormalization-group β function. The running coupling constant exhibits asymptotic freedom as in the vacuum and infrared color charge screening, which is unique to quark gas. There is mild ambiguity in the choice of the subtraction point. There could be a more severe ambiguity in the definition of the perturbation expansion in g due to the appearance of the dimensionless quantity μ/M, which ranges from zero to infinity. These ambiguities are probably not related to the problem of confinement: QED should also have them. Clearly, more work can be done on this problem, especially a two-loop calculation and consideration of nonzero masses.

Many-body Green's functions are finite even in the infrared. In particular, color density correlations approach a constant in the infrared, again indicative of color charge screening.

A dense quark gas is expected to provide a natural cutoff on the instanton scale size. Unfortunately, the problem is difficult to tackle because of the reduction in symmetry from O(4) to O(3). To make a semiquantitative estimate of their importance it was conjectured that the running coupling constant in the vacuum be replaced by the running coupling constant in the quark gas. Loosely speaking, instanton corrections to ideal-gas behavior become of order unity only at low density where the perturbative corrections are of comparable size.

ACKNOWLEDGMENT

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