Charged- and neutral-current interference in ν_e-e scattering

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In the scattering of electronic neutrinos ν_e by electrons, charged- and neutral-current weak interactions can interfere with each other. According to all the popular gauge models that are consistent with neutral-current data, including the measurements of polarized-electron–deuteron scattering, this interference should be negative. The conditions under which a beam-dump experiment can be used to study charged- and neutral-current interference are studied in detail. It appears that if the interference is assumed to be present in the first place, then a 25% measurement of the integrated cross section will allow one to determine its sign. Greater accuracy is required to show that the interference actually does occur in ν_e-e scattering. Tests for the presence of the helicity-flipping covariants S,P,T are also discussed.

I. INTRODUCTION: THE ROLE OF ELECTRONIC NEUTRINOS

Unlike other neutral-current processes, the scattering of *electronic* neutrinos or antineutrinos from electrons should involve both charged- and neutral-current contributions, and, presumably, an interference between them. For this interference, all popular, still viable models of the weak interaction predict the same sign: *negative*. Measurement of this sign, while not distinguishing between the models, will test whether any of them is right.

One expects the cross section $\sigma(\nu_e e)$ to result from the diagrams of Fig. 1. Given μ-e universality, the charged-current (CC) diagram will contribute only to the amplitude for left-handed incoming and outgoing electrons (when the electrons are relativistic), and its size follows from the muon lifetime. Furthermore, the unseen outgoing neutrino produced by this diagram will be a ν_e, and not some new particle. Consequently, an experimental demonstration that the neutral-current (NC) and CC contributions do indeed interfere will prove that:

(a) at least some of the time the ν_{out} produced by the NC interaction is also a ν_e, and not some new particle,[1]

(b) the NC interaction couples to left-handed electrons, and

(c) at least some of the time the NC interaction preserves electron helicity. That is, at least a part of this interaction has V,A rather than S,P,T structure, since the latter flips helicity.[2]

If μ-e universality holds, the NC diagram will make identical contributions to $\nu_\mu - e$ and $\nu_e - e$ scattering.[3] From what we already know about $\sigma(\nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e)$,[4] the dominant term in $\sigma(\nu_e e)$ will then be the CC contribution, followed by the expected NC-CC interference term, a ~50% correction, followed in turn by the NC term, a ~5% correction. If one assumes the CC contribution to be known from universality, a $\sigma(\nu_e e)$ measurement of ~25% accuracy would determine the sign of the interference term, if it is *assumed* to be present. Greater precision would be necessary to prove that the interference term is there in the first place.

There is already evidence from a completed reactor study of $\sigma(\bar{\nu}_e e)$[5] that the NC-CC interference is destructive, as theory predicts, if one assumes it to be present. However, this evidence is not highly conclusive.[5] We find that the same data hint, but quite inconclusively, that the interference is indeed nonvanishing to begin with. Further ν_e-e or $\bar{\nu}_e$-e experiments would be desirable. There are plans for a ν_e-e measurement using neutrinos from the beam dump at LAMPF,[6] and we shall consider such beam-dump experiments in detail. Before doing so, we discuss the present theoretical situation.

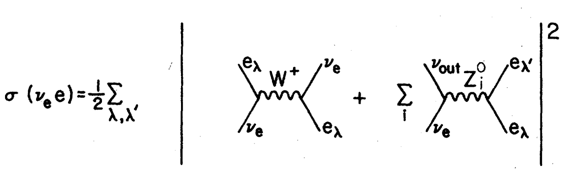

FIG. 1. Diagrams contributing to $\sigma(\nu_e e)$. Here λ, λ' are electron helicities. We allow for the possibility that there are several neutral weak bosons Z_i^0.

II. SURVIVING GAUGE MODELS AND THEIR PREDICTIONS

The number of viable models of the weak interactions has been sharply reduced by a striking increase in our empirical knowledge about neutral currents during the past year. In brief, the new information is as follows:

(1) The coupling constants for the neutral weak interactions between neutrinos and up and down quarks have been determined in a way which is independent of weak-interaction models.[7-9] This solution agrees impressively with the predictions of the Weinberg-Salam model, but the analysis which produced it, particularly the initial step in which the squares of the coupling constants were determined,[8] is not foolproof, and further confirmation is desirable. It has been suggested that non-neutrino experiments such as the measurements of parity violation in atomic hydrogen and deuterium might be able to contribute to this confirmation.[10]

For $SU(2)\times U(1)$ gauge models with the usual doublet assignments for the left-handed fermions, and arbitrary assignments for the right-handed ones, the coupling constant solution implies[11] that $\rho = 1.08 \pm 0.17$.[12] Here, ρ measures the relative overall strengths of NC and CC interactions, and is unity for the simplest Higgs mechanism. If we then assume $\rho = 1$, as its measured value suggests, we can use the accurate measurements of the deep-inelastic processes $\nu\ (\bar{\nu}) + \text{nucleon} \to \nu\ (\bar{\nu}) + \text{anything}$[13] to determine the Weinberg mixing angle for any $SU(2)\times U(1)$ model with the usual left-handed assignments, irrespective of the right-handed ones. The result is $\sin^2\theta_W = 0.24 \pm 0.02$.

(2) The data on ν_μ-e and $\bar{\nu}_\mu$-e interactions appear to be settling down towards consistency with the Weinberg-Salam model.[4] The existing measurements are not very accurate due to the very small cross sections involved.

(3) Parity-violating asymmetries have been observed in the deep-inelastic scattering of longitudinally polarized electrons from deuterium and hydrogen.[14] The results for the one kinematical point which has been measured agree with the Weinberg-Salam predictions.[15]

(4) The situation with regard to parity violation in heavy atoms is uncertain. For bismuth, questions have been raised[16] about the reliability of the atomic calculations[17] on which the predictions of specific weak-interaction models are based. The result of one bismuth experiment[18] agrees with the Weinberg-Salam prediction determined by these calculations, but those of two others[19] fall below it, if they do not vanish altogether. From the standpoint of atomic calculations, a simpler heavy atom is thallium, which has only one p-wave electron outside of closed shells, where bismuth has three. An experiment in progress at Berkeley has just reported a preliminary observation of parity violation in thallium which agrees to within errors with the Weinberg-Salam prediction.[20]

TABLE I. The agreement (Y) or disagreement (N) of various $SU(2)\times U(1)$ models with polarized-electron scattering from deuterium ($e_\uparrow$), the two bismuth experiments with small or vanishing results (small Bi), and the neutrino-quark coupling constant solutions. We consider both the solution favored by the analyses (Refs. 7–9) (νq-A), and the much less favored one (Refs. 7, 9) as solution B (νq-B).

Right-handed doublets	$e_\uparrow$	Small Bi	νq-A	νq-B
none (Weinberg-Salam model)	Y	N	Y	N
$\begin{pmatrix} E^0 \\ e^- \end{pmatrix}_R$	N	Y	Y	N
$\begin{pmatrix} u \\ b \end{pmatrix}_R$	N	N	N	Y
$\begin{pmatrix} u \\ b \end{pmatrix}_R$ and $\begin{pmatrix} E^0 \\ e^- \end{pmatrix}_R$	N	Y	N	Y
$\begin{pmatrix} t \\ d \end{pmatrix}_R$			N	N

Now what popular models survive this influx of information? First, it hardly needs to be said that the Weinberg-Salam model has been strikingly successful, in neutrino physics, in polarized-electron scattering, and now in thallium as well. The discrepancy between this model and some of the bismuth results may disappear as atomic theory and experiment are further refined. Secondly, if we set the bismuth results aside for the moment, then essentially no other gauge model based on the group $SU(2)\times U(1)$ survives. The situation is described in Table I,[21] where we consider $SU(2)\times U(1)$ models with the canonical left-handed fermion assignments, and with the right-handed fermions in singlets except as shown. In calculating the predictions of the various models for polarized electron scattering, we have used the facts that $\rho \simeq 1$ and $\sin^2\theta_W \simeq 0.24$ irrespective of the right-handed assignments. For the model with the electron, but no quarks, in a right-handed doublet, the disagreement with polarized electron scattering is about three times the experimental error.[14,15] This model will be tested further by the measurement of electron scattering at a second kinematical point (currently in progress[22]), and by the study of the front-back asymmetry in $e^+e^- \to \mu^+\mu^-$, for which it predicts a null result. For the models with the u quark in a right-handed doublet, if one recalculates the predictions for electron scattering by Cahn and Gilman[15] using the current value of $\sin^2\theta_W$, one finds already decisive disagreements.

In SU(2) × U(1), the NC-CC interference term in ν_e-e scattering is proportional to

$$+[I_3(e_L)+\sin^2\theta_W], \tag{2.1}$$

where $I_3(e_L)$ is the third component of weak isospin of the left-handed electron. Given that $\sin^2\theta_W = 0.24 \pm 0.02$, the interference will be destructive for the canonical assignment $I_3(e_L) = -\frac{1}{2}$, independent of the right-handed sector of the theory. However, in view of the dramatic successes of the Weinberg-Salam version, and only this version, of SU(2) × U(1), we shall use this theory not only to predict the sign of the interference but also to make detailed estimates of what one may expect to see in the ν_e-e beam-dump experiment.

There is an amusing variant[23] of SU(2) × U(1) which abandons the conventional assignment of e_L in favor of a triplet assignment

$$\begin{pmatrix} \nu_e \\ e^- \\ h^{--} \end{pmatrix}_L \tag{2.2}$$

involving a doubly charged lepton h^{--}. The right-handed electron is assigned to a singlet. This model would predict *constructive* interference in ν_e-e scattering, since $I_3(e_L) = 0$. However, it is already ruled out because it predicts an asymmetry of incorrect sign in polarized-electron scattering.

There are several models based on larger groups, such as $SU(2)_L \times SU(2)_R \times U(1)$, whose predictions for neutrino NC interactions at energies well below the weak boson masses are *identical* to those of the Weinberg-Salam model, or else very nearly so.[24] In some cases, the predictions are required to be identical by a theorem.[25] These models survive along with the Weinberg-Salam model if they also share with the latter common predictions for parity-violating effects such as those seen in polarized-electron scattering and in thallium. For the left-right-symmetric models based on $SU(2)_L \times SU(2)_R \times U(1)$, it has been shown that the parity-violating NC interaction between fermions is indeed the same as in the Weinberg-Salam model, apart from an overall scale factor.[26,27] This scale factor can be adjusted close enough to unity to give agreement with the polarized-electron and thallium data.[26] For ν_e-e scattering, these models share, of course, the Weinberg-Salam predictions.

Bjorken,[28] and Hung and Sakurai,[29] have emphasized that the phenomenologically successful neutral-current Lagrangian of the Weinberg-Salam model is not unique to gauge theories, but can also be derived in some more elementary ways. Thus, there are even some nongauge theories whose predictions for ν_e-e scattering are the same as those of the Weinberg-Salam theory. However, Hung and Sakurai[29] have noted that in the Bjorken approach, certain contributions to the neutrino charge radius must be suppressed, or the value of "$\sin^2\theta_W$" inferred from ν_e scattering would not agree with that inferred from ν_μ scattering.

Now what if one should not disregard the contradiction between two of the bismuth results and the Weinberg-Salam predictions? What if, contrary to the recent finding in thallium, parity violation in heavy atoms is actually much smaller than in the Weinberg-Salam model, while at the same time the polarized-electron scattering result and all the neutrino results are correct? In that case, Table I shows that *no* version of SU(2) × U(1) survives. A larger group, involving more than one neutral weak boson, is required. Moreover, the $SU(2)_L \times SU(2)_R \times U(1)$ models we discussed will not do; if in these models one reduces the scale factor for parity violation to accommodate heavy atoms, one loses the agreement with the polarized-electron result.

It is easy to write down NC coupling schemes involving two neutral weak bosons, Z_1^0 and Z_2^0, which will accomplish what is required.[30] One may either suppress the contribution of each of the bosons to parity violation in heavy atoms, or else arrange that their two contributions cancel. A coupling scheme for each of these possibilities is shown in Table II. In scheme I, the coupling to the electron of Z_1^0, the boson which contributes to neutrino physics, has been switched from approximately pure axial vector, as in the Weinberg-Salam model, to pure vector. This will not affect $\sigma(\nu_\mu e)$ or $\sigma(\bar{\nu}_\mu e)$, but will suppress the contribution to heavy atoms.[31] The contribution of Z_2^0 has been suppressed by making it couple oppositely to neutrons and protons. A proper gauge model with precisely these features, based on SU(2) × U(1) × U(1), has been constructed by Ma, Pramudita, and Tuan.[32] Given that $\sin^2\theta_W \cong 0.25$, the predictions of this model for ν_e-e scattering are indistinguishable from those of the Weinberg-Salam model. In Scheme II, the predictions for ν_e-e scattering are obviously those of the Weinberg-Salam model as well.

We have been taking it for granted that μ-e universality holds, but the comparison of ν_e-e and ν_μ-e interactions is, of course, a test of this hypothesis. Recently, a gauge model based on SU(3) × U(1) appeared[33] which predicts small pari-

TABLE II. Couplings of two weak bosons to neutrinos, quarks, and electrons which result in small parity violation in heavy atoms. WS means the coupling is as in Weinberg-Salam theory, and $e_{\uparrow}$ stands for polarized-electron scattering. In Scheme II, the quark coupling structure for Z_2^0 is arranged so that this boson does not contribute to polarized-electron scattering from deuterium (Ref. 34).

Boson	ν	Couplings Up and down quarks	e
Scheme I			
Z_1^0	WS	WS	Couples to V only
Z_2^0	0	V coupling is $I=1$	Adjusted to fit present and future $e_{\uparrow}$.
Scheme II			
Z_1^0	WS	WS	WS
Z_2^0	0	(Coupling to d) $=2$ (Coupling to u). Strength adjusted to cancel Z_1^0 in heavy atoms	Various possibilities

ty violation in heavy atoms, and which manages to reproduce the Weinberg-Salam Lagrangian for ν_μ and $\bar{\nu}_\mu$ initiated NC processes at the cost of violating universality. Indeed, in this model when $\sin^2\theta_W = 0.25$, there is *no* neutral-current contribution to ν_e-e scattering. Unfortunately, this maverick model is already ruled out by the polarized electron scattering experiment, for which we find that it predicts an asymmetry at least an order of magnitude too small.[34]

In summary, every model which we have examined either predicts destructive NC-CC interference in ν_e-e scattering, or else is already ruled out. It is amusing to note, however, that one can invent a model with two vector bosons which reproduces almost all the results of the standard model but gives constructive NC-CC interference. In this model one boson couples to leptons and quarks exactly as in the standard model while the other couples only to leptons. One can then choose the coupling strength of the second boson so as to reverse the sign of the interference term amplitude without changing its magnitude.[35]

Ideally, the ν_e-e experiments will be able to go beyond the level of accuracy needed to verify the sign of the interference, and reach that necessary to demonstrate that interference is indeed present. We have discussed the basic assumptions that such a demonstration would vindicate. It is worth noting that the only existing direct, unambiguous evidence that the neutral weak interaction is of V,A rather than S,P,T character is the observation of parity nonconservation in polarized electron scattering and in thallium. In the electron scattering, the neutral-weak-electromagnetic interference implied by the observed asymmetry demonstrates V,A character of the neutral weak interaction for the same reason that observation of NC-CC interference in ν_e-e scattering would do so. Namely, in both cases the interaction with which the NC term interferes preserves electron helicity. There will be no interference unless the NC interaction does the same, which requires that it be V,A. For thallium, one may recall that S,P, and T neutral weak interactions cannot violate parity without violating time-reversal invariance as well.[36] This fact has been exploited to turn the experimental limits on electric dipole moments of various atoms and molecules into limits on parity-violation in any S,P,T neutral weak interactions.[37] Observed parity violation must then come from V,A couplings.

All of our data on muonic neutrino-induced neutral-current processes, while consistent with V,A structure, provide no positive evidence for it, due to a "confusion theorem."[38]

The complete body of neutral-current data does provide, to be sure, impressive circumstantial evidence for V,A structure through its striking accord with the Weinberg-Salam predictions. While these predictions have been reproduced without assuming gauge principles, nobody has reproduced them without assuming V,A. Nevertheless, it is clear that additional *direct* evidence would be welcome.

III. CROSS SECTIONS FOR NEUTRINO-ELECTRON SCATTERING

The possibility that the outgoing neutrino in a neutral-current interaction is of a different type from the incoming one has already been discussed in the Introduction. Should this be the case, then there will be no coherent interference between charged- and neutral-current contributions to $\nu_e e$ scattering. For the purposes of this paper we shall henceforth assume that the outgoing neutrino is of the same type as the incoming one: An incident ν_μ will give rise to an outgoing ν_μ, and an incident ν_e to an outgoing ν_e.

Cross sections for $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering are given in Ref. (2). For $\nu_e e$ and $\bar{\nu}_e e$ scattering, if μ-e universality is assumed, one simply replaces the neutral-current coupling constants $C_{V,A}$ and $D_{V,A}$ by $(C_{V,A}-1)$ and $(D_{V,A}-1)$. For reactor neutrinos, the resultant expressions must be corrected for the nonzero mass of the electron; the corrections may be obtained from an analysis of neutrino-proton elastic scattering[39] by setting the various form factors equal to unity or to zero, as appropriate for a point particle. The general expressions for neutrino-electron scattering have also been given in a talk by Gourdin.[40] They are summarized in Table III.[41]

In view of the experimental limits on electric dipole moments,[37] we will assume that time-reversal violating coefficients vanish, i.e.,

$$D_S = D_P = D_T = 0\,. \tag{3.1}$$

The differential cross section for ν_μ-electron scattering as a function of the incident energy E_e of the scattered electron is

$$\frac{d\sigma}{dE_e}(\nu_\mu e) = \frac{G^2 m_e}{2\pi}\left[A_\mu + 2B_\mu\left(1-\frac{E_e}{E_\nu}\right) + C_\mu\left(1-\frac{E_e}{E_\nu}\right)^2\right], \tag{3.2}$$

where we have neglected the electron mass. From Table III, we have

$$A_\mu = (g_V+g_A)^2 + \left(\frac{C_S+C_P}{4}\right)^2 + \left(\frac{C_S-C_P-4C_T}{4}\right)^2,$$

$$B_\mu = C_T^2 - \left(\frac{C_S+C_P}{4}\right)^2 - \left(\frac{C_S-C_P}{4}\right)^2, \tag{3.3}$$

$$C_\mu = (g_V-g_A)^2 + \left(\frac{C_S+C_P}{4}\right)^2 + \left(\frac{C_S-C_P+4C_T}{4}\right)^2.$$

The corresponding cross section for $\bar{\nu}_\mu$-electron scattering is obtained by interchanging the roles of A_μ and C_μ,

TABLE III. Center-of-mass-frame helicity amplitudes $\mathcal{A}_{fi}$ for $\nu_\mu(\bar{\nu}_\mu)$-e scattering. The amplitudes, in which the upper (lower) signs correspond to the $\nu(\bar{\nu})$ processes, are written in terms of the c.m. scattering angle θ, and the coupling constants C_i, D_i defined by the general local interaction

$$\mathcal{H}(x) = \frac{G}{\sqrt{2}}\sum_j (\bar{\nu}_\mu \Gamma_j \nu_\mu)[\bar{e}\Gamma_j(C_j + D_j\gamma_5)e]\,.$$

Here $\Gamma_j = 1$, $i\gamma_5$, $i\gamma_\lambda$, $i\gamma_\lambda\gamma_5$, and $\sigma_{\lambda\nu}$ for $j=S$, P, V, A, and T, respectively. For the V,A sector, we have used the familiar notation $g_{V,A} = -\frac{1}{2}(C_{V,A}+D_{A,V})$. For unpolarized electrons, the laboratory-frame cross sections are given by

$$\frac{d\sigma}{dE_e}[\nu_\mu(\bar{\nu}_\mu)-e] = \frac{G^2 m_e}{32\pi}\sum_{f,i}|\mathcal{A}_{fi}|^2\,,$$

and are quoted in the text in terms of the electron recoil energy E_e and incoming beam energy E_ν, using $\sin^2(\frac{1}{2}\theta) = E_e/E_\nu$.

ν process	$\bar{\nu}$ process	$\mathcal{A}_{fi}$
$\nu_L e_L \to \nu_L e_L$	$\bar{\nu}_R e_R \to \bar{\nu}_R e_R$	$-4(g_V \pm g_A)$
$\nu_L e_R \to \nu_L e_R$	$\bar{\nu}_R e_L \to \bar{\nu}_R e_L$	$4(g_V \mp g_A)\cos^2(\frac{1}{2}\theta)$
$\nu_L e_L \to \nu_R e_R$	$\bar{\nu}_R e_R \to \bar{\nu}_L e_L$	$[\pm(C_S - C_P) + (D_S - D_P)]\sin^2(\frac{1}{2}\theta) - 4(C_T \pm D_T)[1+\cos^2(\frac{1}{2}\theta)]$
$\nu_L e_R \to \nu_R e_L$	$\bar{\nu}_R e_L \to \bar{\nu}_L e_R$	$[\pm(C_S + C_P) - (D_S + D_P)]\sin^2(\frac{1}{2}\theta)$

$$\frac{d\sigma}{dE_e}(\bar{\nu}_\mu e) = \frac{G^2 m_e}{2\pi}\left[C_\mu + 2B_\mu\left(1 - \frac{E_e}{E_\nu}\right) + A_\mu\left(1 - \frac{E_e}{E_\nu}\right)^2\right]. \quad (3.4)$$

For ν_e-e scattering the interference between charged- and neutral-current interactions has the effect of replacing g_V and g_A by $(g_V + 1)$ and $(g_A + 1)$, respectively. Consequently, the differential cross sections for ν_e and $\bar{\nu}_e$ scattering are given by

$$\frac{d\sigma(\nu_e e)}{dE_e} = \frac{G^2 m_e}{2\pi}\left[A_e + 2B_\mu\left(1 - \frac{E_e}{E_\nu}\right) + C_\mu\left(1 - \frac{E_e}{E_\nu}\right)^2\right], \quad (3.5)$$

$$\frac{d\sigma(\bar{\nu}_e e)}{dE_e} = \frac{G^2 m_e}{2\pi}\left[C_\mu + 2B_\mu\left(1 - \frac{E_e}{E_\nu}\right) + A_e\left(1 - \frac{E_e}{E_\nu}\right)^2\right], \quad (3.6)$$

where

$$A_e = A_\mu + 4(g_V + g_A) + 4. \quad (3.7)$$

The integrated cross sections for monoenergetic neutrinos are

$$\sigma = \frac{G^2 m_e E_\nu}{2\pi}\,\Sigma, \quad (3.8)$$

where

$$\begin{aligned}
\Sigma(\nu_\mu e) &= A_\mu + B_\mu + \tfrac{1}{3}C_\mu, \\
\Sigma(\bar{\nu}_\mu e) &= \tfrac{1}{3}A_\mu + B_\mu + C_\mu, \\
\Sigma(\nu_e e) &= A_e + B_\mu + \tfrac{1}{3}C_\mu, \\
\Sigma(\bar{\nu}_e e) &= \tfrac{1}{3}A_e + B_\mu + C_\mu.
\end{aligned} \quad (3.9)$$

It is apparent from Eqs. (3.3) and (3.5)–(3.7) that the one helicity amplitude which is modified in going from $\nu_\mu e$ to $\nu_e e$ scattering contributes only to the A coefficient in the differential cross section. This amplitude corresponds to $\nu_L e_L \to \nu_L e_L$ (see Table III) and is the only one that receives coherent contributions from both charged and neutral currents. The expression for A_e in Eq. (3.7) can be recognized as consisting of a purely neutral-current term N_c, a purely charged-current term C_c, and an interference term I between charged and neutral currents:

$$N_c = A_\mu;\quad C_c = 4;\quad 2I = 4(g_V + g_A). \quad (3.10)$$

From Eqs. (3.3), (3.7), and (3.10) one can see that there is a general relation between these quantities:

$$|I| \le \sqrt{N_c}\sqrt{C_c}. \quad (3.11)$$

The equality holds only when the S, P, T contributions to A_μ vanish; therefore, if one can measure N_c sufficiently accurately in $\nu_\mu e$ scattering, and can extract I from $\nu_e e$ scattering, then one can test for the presence of S, P, T interactions by seeing whether the measured quantities yield an equality or inequality in Eq. (3.11).

The B and C coefficients in the differential cross sections arise solely from the neutral-current interaction, and they are the same for both $\nu_\mu e$ and $\nu_e e$ scattering because μ-e universality has been assumed. This assumption implies that total cross sections must satisfy

$$3\sigma(\bar{\nu}_\mu e) - \sigma(\nu_\mu e) = 3\sigma(\bar{\nu}_e e) - \sigma(\nu_e e), \quad (3.12)$$

a relation which, as pointed out by Gourdin[40] and Sehgal,[42] can be used to test μ-e universality. Detection of a B coefficient in the cross section will be direct evidence for the presence of S, P, T in the neutral current. The C coefficient receives contributions from the $V+A$ component of the neutral current and from S, P, T components, and it cannot be used to distinguish between these types of interaction by itself.

A. Beam-dump experiments

Up to now, the discussion applies to monoenergetic neutrinos. In an actual experiment, one works with a spectrum of neutrinos, and so one must integrate the previous cross sections over the spectrum. In a beam-dump experiment,[6] the neutrinos are generated by the decay sequence of stopping π^+ mesons, $\pi^+ \to \mu^+ \to e^+$, and as long as lepton number is conserved, there will be three distinct species emitted:

$$\begin{aligned}
\pi^+ \to\ &\mu^+ + \nu_\mu \\
&\searrow e^+ + \nu_e + \bar{\nu}_\mu.
\end{aligned} \quad (3.13)$$

The spectrum, therefore, consists of a monoenergetic muon-type neutrino ν_μ, and of an electron-type neutrino and muon-type antineutrino, ν_e and $\bar{\nu}_\mu$, each with an energy spectrum characteristic of muon decay at rest. Its three components can be written as

$$\begin{aligned}
\varphi(\nu_e) &= 2NE_{\nu_e}{}^2(W - E_{\nu_e}), \\
\varphi(\bar{\nu}_\mu) &= NE_{\bar{\nu}_\mu}{}^2(W - \tfrac{2}{3}E_{\bar{\nu}_\mu}), \\
\varphi(\nu_\mu) &= \tfrac{1}{6}NW^4\,\delta(E_{\nu_\mu} - E_0),
\end{aligned} \quad (3.14)$$

where

$$E_0 = \frac{m_\pi{}^2 - m_\mu{}^2}{2m_\pi} \approx 29.8\ \text{MeV}. \quad (3.15)$$

W is the maximum energy of the neutrinos from muon decay

$$W=\frac{m_\mu{}^2+m_e{}^2-(m_{\nu_1}+m_{\nu_2})^2}{2m_\mu}\approx\tfrac{1}{2}m_\mu\,, \qquad (3.16)$$

and N is an arbitrary normalization constant. These spectra are normalized so that there are equal numbers of each kind of neutrino:

$$\int\varphi(\nu_e)\,dE_{\nu_e}=\int\varphi(\bar\nu_\mu)\,dE_{\bar\nu_\mu}=\int\varphi(\nu_\mu)\,dE_{\nu_\mu}\,. \qquad (3.17)$$

The probability of finding an electron in the energy range (E_e, E_e+dE_e) produced by an incident neutrino ν_x is given by

$$\frac{dP(\nu_x)}{dE_e}=\int_{E_e}^{W}\frac{d\sigma(\nu_x)}{dE_e}\,\varphi(\nu_x)\,dE_{\nu_x}\,. \qquad (3.18)$$

From the spectra in Eq. (3.14) and the differential cross sections in Eqs. (3.2)–(3.7), the differential probabilities for each species of neutrino are

$$\frac{dP(\nu_e)}{dE_e}=\frac{2NG^2m_e}{2\pi}\,\frac{(W-E_e)^2}{12}\left[A_e(W^2+2WE_e+3E_e{}^2)+2B_\mu(W^2-E_e{}^2)+C_\mu(W-E_e)^2\right], \qquad (3.19)$$

$$\frac{dP(\bar\nu_\mu)}{dE_e}=\frac{NG^2m_e}{2\pi}\,\frac{(W-E_e)}{18}\big[A_\mu(W-E_e)^2(3W-E_e)+2B_\mu(W-E_e)(3W^2+WE_e-E_e{}^2)+C_\mu(3W^3+3W^2E_e+3WE_e{}^2-3E_e{}^3)\big]\,, \qquad (3.20)$$

$$\frac{dP(\nu_\mu)}{dE_e}=\frac{NG^2m_e}{2\pi}\,\frac{W^4}{6}\times\begin{cases}A_\mu+2B_\mu(1-E_e/E_0)+C_\mu(1-E_e/E_0)^2 & (E_e<E_0)\\ 0\quad (E_e>E_0)\,,\end{cases} \qquad (3.21)$$

where E_0 is given in Eq. (3.15). The total probability for finding an electron with energy E_e is given by the sum of these three probabilities.

B. Integrated cross sections for a beam-dump experiment

Since it is not likely that there will soon be sufficient experimental information to study the differential cross sections in detail, we consider the integrated cross sections. We integrate the sum of the expressions in Eqs. (3.19), (3.20), (3.21) from some lower-energy cut E_c up to the maximum energy W, and obtain

$$\begin{aligned}P(E_c\le E_e\le W)&=\int_{E_c}^{W}\left(\frac{dP(\nu_e)}{dE_e}+\frac{dP(\bar\nu_\mu)}{dE_e}+\frac{dP(\nu_\mu)}{dE_e}\right)dE_e\\&=\left(\frac{NG^2m_e}{360\pi}\right)\{A_e(W-E_c)^3[60W^2-60W(W-E_c)+18(W-E_c)^2]\\&\quad+B_\mu(W-E_c)^3[20W^2+35W(W-E_c)-16(W-E_c)^2]+A_\mu(W-E_c)^4[5W+2(W-E_c)]\\&\quad+C_\mu(W-E_c)^2[30W^3-15W(W-E_c)^2+12(W-E_c)^3]\\&\quad+\theta(E_0-E_c)\frac{W^4}{E_0^2}(E_0-E_c)[30A_\mu E_0{}^2+30B_\mu E_0(E_0-E_c)+10C_\mu(E_0-E_c)^2]\}\,,\end{aligned} \qquad (3.22)$$

where the θ function is equal to one when $E_c<E_0$ and vanishes when $E_c>E_0$. The numerical values of the coefficients of A_e, A_μ, B_μ, C_μ are displayed in Table IV for various choices of the cutoff energy E_c as a fraction of the maximum energy W.

An important qualitative feature of Table IV is that for cutoff energies greater than $E_0=0.56W$, the coefficients of A_e and C_μ are considerably larger than those of A_μ. The reason for this is that in the energy range ($E_0<E_c<W$) the neutrino beam consists solely of the ν_e and $\bar\nu_\mu$ from μ de-

TABLE IV. Coefficient functions for Eq. (3.22) in arbitrary units.

Coef. of \ E_c/W	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
A_e	7.2	4.3	2.1	0.72	0.10
A_μ	6.6	2.3	0.11	0.022	0.0013
B_μ	10.6	4.4	1.6	0.43	0.047
C_μ	10.8	6.9	4.0	1.8	0.47

cay. Now in $d\sigma(\bar{\nu}_\mu)/dE$ of Eq. (3.4), A_μ is multiplied by a factor $(1-E_e/E_\nu)^2$ which suppresses its contribution to the integrated cross section; by contrast the A_e term in the $\nu_e - e$ cross section of Eq. (3.5) is independent of energy, and hence its coefficient is not suppressed. The C_μ term is independent of energy in Eq. (3.4), and is multiplied by $(1-E_e/E_\nu)^2$ in Eq. (3.5). Thus most of its coefficient in the integrated cross section comes from the $\bar{\nu}_\mu$ component of the beam; moreover, the coefficient of C_μ tends to be larger than that of A_e because the $\bar{\nu}_\mu$ spectrum is harder than the ν_e spectrum in μ decay [see Eq. (3.14)].

As for B_μ, it is multiplied by a factor $(1-E_e/E_\nu)$ in the spectra of Eqs. (3.4), (3.5), (3.6), and so its coefficient in the integrated cross section tends to be smaller than those of A_e and C_μ, but not as small as that of A_μ. For cutoff energies below E_0, the dominant contribution to A_μ comes from the monoenergetic ν_μ emitted in π^+ decay.

Within the next two years, it is quite likely that the coefficients A_μ, B_μ, and C_μ will have been measured directly in high-energy ν_μ-e scattering experiments at CERN and Fermilab. Thus, we shall eventually be able to use these measurements in conjunction with future beam-dump experiments to extract the value of A_e, and use them again to extract from A_e the interference term between charged and neutral currents. For the moment, however, the Weinberg-Salam model appears to be working well,[4,7-9] and so we shall use its predicted values of A_μ and C_μ in place of measured ones for the remainder of our analysis. (An earlier version of this work[10] made use of the Aachen-Padova results for $\nu_\mu e$ scattering.[43])

In the Weinberg-Salam model the neutral-current coupling constants are

$$\begin{aligned} &C_S = C_P = C_T = 0\,, \\ &g_V = -\tfrac{1}{2} + 2\sin^2\theta_W\,, \\ &g_A = -\tfrac{1}{2}\,, \end{aligned} \tag{3.23}$$

and hence from Eqs. (3.3) and (3.7):

$$\begin{aligned} &C_\mu = (g_V - g_A)^2 = 4\sin^4\theta_W\,, \\ &A_\mu = (g_V + g_A)^2 = (1-2\sin^2\theta_W)^2\,, \\ &A_e = (1-2\sin^2\theta_W)^2 + 4 - 4(1-2\sin^2\theta)\,. \end{aligned} \tag{3.24}$$

Notice that the expression for A_e displays the NC contribution, the CC contribution, and the interference term explicitly. For the canonical value of $\sin^2\theta_W = \frac{1}{4}$, the values of the purely neutral-current terms are

$$C_\mu = \tfrac{1}{4}; \quad A_\mu = \tfrac{1}{4}\,. \tag{3.25}$$

For the *negative* interference predicted by the standard model, the value of A_e is predicted to be

$$A_e(-) = \tfrac{1}{4} + 4 - 2 = \tfrac{9}{4}\,. \tag{3.26a}$$

Were the neutral current to be exactly as in the Weinberg-Salam model except for its sign, then its interference with the charged current would be *positive*, and the value of A_e would be almost three times larger:

$$A_e(+) = \tfrac{1}{4} + 4 + 2 = \tfrac{25}{4}\,. \tag{3.26b}$$

Should it happen that there is no interference between charged and neutral currents (as would happen if the outgoing neutrino in the neutral-current interaction were different from the incoming one), then the value of A_e would fall midway between $A_e(-)$ and $A_e(+)$:

$$A_e(0) = \tfrac{1}{4} + 4 = \tfrac{17}{4}\,. \tag{3.26c}$$

In all three cases, the value of A_e is much larger than C_μ and A_μ, and hence the ν_e component of the spectrum will give the dominant contribution to a beam-dump experiment.

To gain an idea of how accurate an experiment need be in order to determine whether a presumed interference is either negative or positive, or whether there is indeed an interference in the first place, we compute the magnitudes of the corresponding integrated cross sections. The results are shown in Table V, where the separate contributions of the C_μ, A_μ, and A_e terms are displayed as well as the total. For each cutoff energy we show the two cases of negative interference and positive interference; the case of zero interference lies halfway between these two. The units in Table V are arbitrary.

It is apparent from the table that anywhere from 70 to 90% of the total event rate comes from the A_e term, as is to be expected from Table IV and the values of A_e, C_μ, and A_μ in Eqs. (3.25) and (3.26). The magnitude of the event rate for constructive interference is roughly 2.4 times that for destructive interference for all values of E_c. From this we conclude that a 25% measurement of the event rate will be good enough to determine whether, if we assume that there is indeed interference between charged and neutral currents, it is destructive or constructive. A much more accurate measurement is needed to answer the question whether there is interference in the first place.

To illustrate the point, let us consider the case in which the cutoff energy is $\frac{1}{2}$ the maximum energy, i.e., $E_c \approx 27$ MeV. From Table V, the expected event rates are 12 for destructive interference and 29 for constructive interference. Then if, for example, we measure the event rate to be 16 ± 4, we can definitely conclude that if there is interference, it cannot be of the constructive type; how-

TABLE V. Event rates from E_c to W in arbitrary units for destructive, and constructive interference.

E_c/W	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
C_μ term	2.7	1.7	1.0	0.46	0.12
A_μ term	1.6	0.57	0.03	0.005	0.0003
$A_e(-)$ [$A_e(+)$]	16.2 [45.1]	9.7 [26.9]	4.8 [13.2]	1.6 [4.5]	0.23 [0.64]
Total (−) [Total (+)]	20.6 [49.5]	12.0 [29.2]	5.8 [14.2]	2.1 [5.0]	0.35 [0.76]

ever, we cannot exclude the possibility that there is no interference at all. To do this we would have to improve the accuracy to 15% at least.

C. Reactor experiments

One can analyze reactor experiments[5] in much the same way as the beam-dump experiment. The neutrinos are now of a single type, namely $\bar{\nu}_e$, and their energies are much lower (≤ 5 MeV) than in the beam dump. The cross section for elastic $\bar{\nu}_e$-e scattering integrated over the spectrum φ of reactor antineutrinos is[44]

$$\int \sigma\varphi dT dE_{\bar{\nu}} = \frac{G^2 m_e}{2\pi}[A_e G_2(T_1, T_2) + C_\mu G_1(T_1, T_2) + (|C_A|^2 - |C_V|^2) G_3(T_1, T_2)], \tag{3.27}$$

where T_1 and T_2 are, respectively, the lower and the upper bounds on the kinetic energy of the scattered electron. $G_1(T_1, T_2)$, $G_2(T_1 T_2)$, and $G_3(T_1, T_2)$ are integrals over the antineutrino spectrum of the functions 1, $(1 - E_e/E_{\bar{\nu}})^2$, and $(m_e E_e / E_{\bar{\nu}}^2)$, respec-

TABLE VI. Theoretical event rates for the reactor neutrino experiment. The contributions from each of the terms in Eq. (3.27) are shown for the cases of destructive, constructive, and no coherent interference, and for a pure charged-current interaction. The appropriate values of the coupling constants are given in Eqs. (3.25), (3.26), (3.28), (3.29). Two energy bins are used for the scattered electrons, and the units are arbitrary.

Term in event rate \ Electron kinetic energy	$1.5 \leq T \leq 3$ MeV	$3 \leq T \leq 4.5$ MeV
$A_e(-)$ contribution	11.9	1.6
C_μ contribution	8.5	2.5
$\|C_A\|^2 - \|C_V\|^2(-)$	−2.8	−0.6
Total, destructive interference	17.6	3.5
$A_e(+)$ contribution	33.1	4.5
C_μ contribution	8.5	2.5
$\|C_A\|^2 - \|C_V\|^2(+)$	4.7	1.1
Total, constructive interference	46.3	8.1
$A_e(0)$ contribution	22.6	3.0
C_μ contribution	8.5	2.5
$\|C_A\|^2 - \|C_V\|^2(0)$	0.9	0.2
Total, no coherent interference	32.0	5.7
Total, pure charged current	21.2	2.9

TABLE VII. Theoretical and experimental reactor neutrino cross sections as fractions of σ_{V-A}, the cross section for a pure charged-current interaction.

Case \ Electron energy	$1.5 \leq T \leq 3$ MeV	$3.0 \leq T \leq 4.5$ MeV
Destructive	0.83	1.20
Constructive	2.2	2.8
No coherent	1.5	2.0
Experimental (Ref. 5)	0.87 ± 0.25	1.7 ± 0.44

tively, and their numerical values are tabulated in Ref. 44. Notice that because of the lower energies of the neutrinos, one must now retain the term proportional to the electron mass, i.e., $G_3(T_1T_2)$.

Theoretical cross sections for the scattered electron in the energy bins $1.5 \leq T \leq 3$ MeV and $3 \leq T \leq 4.5$ MeV are shown in Table VI. As in the case of the beam-dump experiment, so here one considers the cases of destructive, constructive, and no coherent interference; all with $\sin^2\theta_W = \frac{1}{4}$. The values of A_e for these cases are given in Eq. (3.26), and the corresponding values of $|C_A|^2 - |C_V|^2$ are

$$|C_A|^2 - |C_V|^2 = \begin{cases} -\frac{3}{4}, & \text{destructive } (-), \\ \frac{5}{4}, & \text{constructive } (+), \\ \frac{1}{4}, & \text{no } (0). \end{cases} \quad (3.28)$$

In order to compare these calculations with the experimental data we also calculate the pure charged-current cross section, for which

$$A_e = 4, \quad C_\mu = |C_A|^2 - |C_V|^2 = 0. \quad (3.29)$$

It is apparent from Table VI that the situation for the reactor experiment is similar to that for the beam-dump one. The A_e term still contributes some 50–70% of the total event rate, and the signal with constructive interference is about $2\frac{1}{2}$ times larger than the signal with destructive interference. In contrast to the beam dump, however, the electron-mass-dependent term $G_3(T_1, T_2)$ can now make a significant contribution to the cross section; for example, in the case of destructive interference it reduces the cross section by some 20%.

In Table VII the theoretical cross sections are expressed as fractions of σ_{V-A}, the pure charged-current cross section for the appropriate energy bin, and they are compared with the presently available experimental results.[5] It is apparent that in the lower-energy bin, the data are consistent with the theoretical value for destructive interference; however it is also within 1 standard deviation of pure $V-A$, and within 3 standard deviations of the prediction for no interference. In the upper energy bin, the data are consistent with no coherent interference, but they are also within 2 standard deviations of the other theoretical possibilities. Thus, it can be concluded that reactor experiments to date provide some evidence for destructive interference; however, they do not strongly exclude the possibility of constructive interference, nor do they prove that there is any coherent interference in the first place. Experiments with greater precision are needed to settle these issues.

IV. CONCLUSION

We have seen that measurement of the sign of NC-CC interference in ν_e-e scattering will simultaneously test all of the popular surviving models of the weak interaction, and some of the less popular ones as well. This sign can be determined by a beam-dump or reactor experiment that measures the flux-normalized event rate to an accuracy of approximately 25%. With a more precise measurement, one can verify that an interference term is indeed present, thereby demonstrating that at least a part of the NC interaction is of V,A character, and produces outgoing neutrinos identical to the incoming ones.

The gain from an even more precise measurement would be somewhat limited. Hopefully, *muonic*-neutrino experiments will soon yield fairly accurate values for the constants A_μ and C_μ, and even a limit on the purely S,P,T- produced coefficient B_μ. Of course, precise electronic neutrino experiments could help to confirm these results, while simultaneously testing μ-e universality.[40] In addition, an accurate determination of A_e by such experiments, combined with one of A_μ by ν_μ-e measurements, would yield an upper limit on any S, P, and T contribution to A_μ. Such a contribution would make the interference term in A_e smaller, in magnitude, than the pure V,A value of $4\sqrt{A_\mu}$.

However, the most important thing to be learned from $\nu_e(\bar{\nu}_e)$-e scattering is the sign of the NC-CC interference in this reaction. This sign is predicted, with considerable agreement, by the various theories, and it cannot be inferred from any other measurements.

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[1]Interference and other tests of outgoing neutrino identity have been proposed by L. Sehgal, Phys. Lett. 55B, 205 (1975); and by L. Wolfenstein, Nucl. Phys. B91, 95 (1975).

[2]Study of ν_e-e scattering to demonstrate the V,A character of the NC interaction was suggested by B. Kayser, G. Garvey, E. Fischbach, and S. P. Rosen, Phys. Lett. 52B, 385 (1974). The possibility of S,P,T covariants in NC interactions was also considered by R. Kingsley, F. Wilczek, and A. Zee, Phys. Rev. D 10, 2216 (1974); and by D. Bardin, S. Bilenky, and B. Pontecorvo, Phys. Lett. 32B, 68 (1970).

[3]The equality between the couplings of ν_e and ν_μ to the neutral weak boson(s) will be tested experimentally by comparing the cross sections for the processes ν_e + nucleon $\rightarrow \nu_e$ + anything and ν_μ + nucleon $\rightarrow \nu_\mu$ + anything.

[4]For a review of existing results see C. Baltay, in *Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978,* edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo, 1979), p. 882.

[5]F. Reines, H. Gurr, and H. Sobel, Phys. Rev. Lett. 37, 315 (1976).

[6]H. H. Chen, F. Reines, R. Burman *et al.*, LAMPF Experiment No. 225, 1975 (unpublished); G. A. Brooks, H. H. Chen, and J. F. Lathrop in *Neutrino '77*, Proceedings of the International Conference on Neutrino Physics and Neutrino Astrophysics, Baksan Valley, 1977 (Nauka, Moscow, 1978), Vol. 2, p. 376.

[7]For a review of this subject see J. J. Sakurai, in *Proceedings of the Topical Conference on Neutrino Physics at Accelerators, Oxford, 1978,* edited by D. Perkins (Rutherford Laboratory, Chilton, Didcot, Oxfordshire, England, 1978), p. 333.

[8]L. Sehgal, Phys. Lett. 71B, 99 (1977).

[9]P. Hung and J. J. Sakurai, Phys. Lett. 72B, 208 (1977); G. Ecker, *ibid.* 72B, 450 (1978); P. Langacker and D. Sidhu, *ibid.* 74B, 233 (1978); L. Abbott and R. M. Barnett, Phys. Rev. Lett. 40, 1303 (1978); Phys. Rev. D 18, 3214 (1978); E. Monsay, *ibid.* 18, 2277 (1978); Phys. Rev. Lett. 41, 728 (1978); D. Sidhu and P. Langacker, *ibid.* 41, 732 (1978); R. Hendrick and L. F. Li, Phys. Rev. D 19, 779 (1979); E. Paschos, *ibid.* 19, 83 (1979); J. J. Sakurai, Ref. 7; H. H. Williams *et.al.*, in *Proceedings of the Topical Conference on Neutrino Physics at Accelerators, Oxford, 1978,* edited by D. Perkins (Rutherford Laboratory, Chilton, Didcot, Oxfordshire, England, 1978), p. 315.

[10]B. Kayser, *Neutrinos—78*, Proceedings of the International Conference on Neutrino Physics and Astrophysics, Purdue Univ., edited by E. C. Fowler (Purdue, W. Lafayette, Indiana, 1978), p. 979.

[11]J. Bernabéu and C. Jarlskog, Phys. Lett. 69B, 71 (1977).

[12]J. J. Sakurai, Ref. 7.

[13]M. Holder *et al.*, Phys. Lett. 72B, 254 (1977).

[14]C. Prescott *et al.*, Phys. Lett. 77B, 347 (1978).

[15]R. Cahn and F. Gilman, Phys. Rev. D 17, 1313 (1978); W. Marciano and A. Sanda, *ibid.* 18, 4341 (1978). See also J. Bjorken, *ibid.* 18, 3239 (1978).

[16]R. Cowan, S. Meshkov, and S. P. Rosen, Purdue University report (unpublished); S. Carter and H. Kelly, Phys. Rev. Lett. 42, 966 (1979); E. Henley, in *Transparencies from the VI International Workshop on Weak Interactions With Very High Energy Beams*, 1978, compiled by K. E. Lassila and B. L. Young (Iowa State University, Ames, Iowa, 1978), p. 857; S. P. Rosen, *ibid.*, p. 1043.

[17]See, for example, E. Henley and L. Wilets, Phys. Rev. A 14, 1411 (1976); P. Sandars, in *The Unification of Elementary Forces and Gauge Theories*, Proceedings of the Ben Lee Memorial Conference on Parity Nonconservation, Weak Neutral Currents, and Gauge Theories, Fermilab, 1977, edited by D. B. Cline and F. E. Mills (Harwood Academic Publishers, London, 1977), p. 153; V. Novikov, O. Sushkov, and I. Khriplovich, Zh. Eksp. Teor. Fiz. 71, 1665 (1976).

[18]L. Barkov and M. Zolotorev, Institute of Nuclear Physics report, Novosibirsk, (unpublished).

[19]N. Fortson, in *Weak Interactions—Present and Future*, proceedings of the SLAC Summer Institute on Particle Physics, 1978, edited by Martha Zipf (SLAC, Stanford, 1978), (unpublished), p. 305; P. Sandars, Ref. 17.

[20]R. Conti, P. Bucksbaum, S. Chu, E. Commins, and L. Hunter, Phys. Rev. Lett. 42, 343 (1979).

[21]B. Kayser, in *Transparencies from the VI International Workshop on Weak Interactions With Very High Energy Beams, 1978,* compiled by K. E. Lassila and B. L. Young (Iowa State University, Ames, Iowa, 1978), p. 871.

[22]C. Prescott, private communication.

[23]F. Wilczek and A. Zee, Phys. Rev. Lett. 38, 531 (1977).

[24]H. Fritzsch and P. Minkowski, Nucl. Phys. B103, 61 (1976); R. Mohapatra and D. Sidhu, Phys. Rev. D 16, 2843 (1977); J. Pati and A. Salam, *ibid.* 10, 275 (1974).

[25]J. Pati, S. Rajpoot, and A. Salam, Phys. Rev. D 17, 131 (1978); H. Georgi and S. Weinberg, *ibid.* 17, 275 (1978).

[26]J. Pati and S. Rajpoot, Phys. Lett. 79B, 65 (1978).

[27]G. Costa, M. D'Anna, and P. Marcolungo, Phys. Lett. 79B, 287 (1978).

[28]J. Bjorken, Phys. Rev. D 18, 3239 (1978).

[29]P. Hung and J. J. Sakurai, Nucl. Phys. B143, 81 (1978).

[30]One of us (B. K.) wishes to thank Ernest Henley for goading us into doing this.

[31]See, for example, Henley and Wilets, Ref. 17.

[32]E. Ma, A. Pramudita, and S. F. Tuan, Phys. Lett. 80B, 79 (1978).

[33]F. Buccella, M. Lusignoli, and A. Pugliese, Phys. Rev. Lett. 40, 1475 (1978).

[34]See Cahn and Gilman, Ref. 15.

[35]One simple realization of this model is obtained by requiring that the "effective coupling constants" for $\nu_\mu e$ scattering via the second boson Z_2^0 be (-2) times the "effective coupling constants" for $\nu_\mu e$ scattering via the first boson Z_1^0 (which couples to leptons and to quarks). If ν_μ couples with equal strength to Z_1 and Z_2, then this model will yield a forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$ that is 5 times as large as that predicted by the Weinberg-Salam model. Other realizations which leave $\sigma(\nu_\mu e)$ unchanged give factors greater than one but smaller than 5 for the forward-backward asymmetry.

[36]See, for example, B. Kayser, S. P. Rosen, and E. Fischbach, Phys. Rev. D 11, 2547 (1975).

[37]C. Bouchiat, Phys. Lett. 57B, 284 (1975); and E. Hinds, C. Loving, and P. Sandars, *ibid.* 62B, 97 (1976).

[38]The theorem states that in some of these processes, anything V,A can do, S,P,T can reproduce exactly. In the other processes, it is found that S,P,T can mimic V,A to within reasonable errors. See Kayser, Garvey, Fischbach, and Rosen, Ref. 2, and E. Fischbach, J. T. Gruenwald, S. P. Rosen, H. Spivack, and B. Kayser, Phys. Rev. D 15, 97 (1977).

[39]E. Fischbach, J. T. Gruenwald, S. P. Rosen, H. Spivack, and B. Kayser, Phys. Rev. Lett. 37, 582 (1976); and Phys. Rev. D 15, 97 (1977). For reactor neutrino cross sections, especially those integrated over the spectrum of $\bar{\nu}_e$ from a reactor see F. T. Avignone, III, and Z. D. Greenwood, Phys. Rev. D 16, 2383 (1977). A general, pre-gauge-theory analysis of the reactor experiment was given by D. Bardin, S. Bilenky, and B. Pontecorvo, Phys. Lett. 32B, 68 (1970).

[40]M. Gourdin, in *Proceedings of the International Neutrino Conference, Aachen, 1976,* edited by H. Faissner, H. Reithler, and P. Zerwas (Vieweg, Braunschweig, W. Germany, 1977), p. 234. A more specialized, very clear, discussion has also been given by H. Harari, Phys. Rep. 42C, 235 (1978).

[41]B. Kayser, invited talk delivered at the Annual Meeting of the American Physical Society, 1975, National Science Foundation report (unpublished).

[42]L. M. Sehgal, Phys. Lett. 48B, 60 (1974); Nucl. Phys. B70, 61 (1974).

[43]H. Reithler, in *Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies,* edited by F. Gutbrod (DESY, Hamburg, 1977), p. 343.

[44]See Avignone and Greenwood, Ref. 39.