Production of free quarks in the early universe

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(Received 23 April 1979)

A model of the q-$\bar{q}$ interaction which avoids complete confinement is employed to calculate the number of free quarks which survive from the early universe. For a free-quark mass $M \lesssim 10$ GeV, our results are similar to those of previous calculations. But for $M \gtrsim 10$ GeV, the number per nucleon is given by $\ln(q/N) = \ln(\bar{q}/N) = -M/kT_* + 2\ln(M/kT_*) + C$, where the constant $C \simeq 20$. Quarks of such mass freeze out of equilibrium during the quark-hadron transition at a temperature $0.2 \lesssim T_* \lesssim 0.4$ GeV/k, which for instance predicts $15 \lesssim M \lesssim 30$ GeV if $(q+\bar{q})/N \sim 10^{-20}$.

At the present time, there is conflicting evidence regarding the existence of free quarks with charges $\pm e/3$ and $\pm 2e/3$. The results of LaRue *et al.*[1,2] imply an abundance $(q+\bar{q})/N \gtrsim 10^{-20}$ quarks per nucleon in niobium. The most sensitive of the other levitometer experiments produced an upper limit $(q+\bar{q})/N < 3\times 10^{-21}$ in iron.[3] However, very recent results of this group (Gallinaro *et al.*) are now inconclusive.[4] Smaller upper limits have been claimed in other types of searches, but their interpretation is not straightforward.[5] We will discuss in particular the constraints imposed by accelerator experiments later in this paper.

It is not the purpose of this paper to argue for or against the existence of free quarks. Rather, we shall derive a relationship between the present abundance of relic quarks produced in the early universe and their mass. For the most part, we shall adopt the standard assumptions regarding the nature of quarks, as embodied in quantum chromodynamics (QCD).[6] The crucial modification is the assumption that the effective potential between quarks in a color-singlet state reaches a limiting value at a finite quark separation, leading to incomplete confinement. Such behavior could be due to a finite gluon mass.

The results that we will obtain differ fundamentally from those of previous calculations,[7] if the free quark mass $M \gtrsim 10$ GeV. This is because such quarks "freeze out" of statistical equilibrium during the quark-hadron transition. Only the very energetic thermal quarks, which never feel the confining effects of the gluons, remain free. In the previous calculations,[7] the abundance of relic quarks was determined by the competition between the cosmological expansion rate and the free-quark reaction rate. The result of this competition (in the "standard model" of the universe) is to leave an abundance of relic quarks far in excess of the observational upper limit.

We shall also adopt the standard model to describe the early history of the universe.[8] The main justification (other than simplicity) for this choice is the fact that an arbitrary violation of any of the assumptions which define this model typically results in a production of helium much different from the observed value.[9]

During the epoch of interest, when the temperature was dropping from $kT \sim 1$ GeV to ~ 0.1 GeV, the universe was dominated by relativistic particles in statistical equilibrium. They formed a perfect fluid of total mass density ρ and pressure $p = \frac{1}{3}\rho$. With $g \equiv g_b + \frac{7}{8} g_f$, where g_b and g_f are the number of boson and fermion degrees of freedom,

$$\rho = (g/2)\rho_\gamma = (\pi^2 g/30\hbar^3)(kT)^4 .$$

The general-relativistic expansion rate then provides the formula

$$t \cong 2.42\times 10^{-6} g^{-1/2}(kT)^{-2} \text{ sec} \tag{1}$$

for the time since infinite density. (Unless indicated otherwise, energies will be expressed in GeV and distances in fermis. In addition, we take $c = 1$.)

The second basic formula follows from conservation of total entropy per (net) baryon (at temperatures $T \gtrsim 1$ MeV/k):

$$g n_\gamma / n_B \cong \text{constant}, \tag{2}$$

where n_B is the baryon number density, and the photon number density

$$n_\gamma = 31.8(kT)^3 \text{ fm}^{-3} . \tag{3}$$

We must next specify (at least approximately) the interaction between quarks, in particular a quark and antiquark of the same color. Consider the effective potential $U(r)$, which is the total energy of the $q-\overline{q}$ pair when fixed at a separation r. The lattice formulation of QCD (Ref. 10) predicts that $U=\eta r$ for $r \gtrsim 1$ fm, due to the energy in the gluon string between the quarks.[11] In addition, the Regge behavior of hadrons indicates that $\eta \sim 1$ GeV fm^{-1}.[12] Indeed, most of the features of the charmonium spectrum are consistent with the potential[13]

$$U=-0.026r^{-1}+1.18r \text{ GeV}. \tag{4}$$

(The coefficient of r^{-1} agrees with calculations of the asymptotic freedom of QCD.) We shall employ this potential for separations $r \lesssim 1$ fm. At $r \cong r_c$ ($r_c \gg 1$ fm), the potential flattens to a limiting value $U_0=2M \gg 1$ GeV, where M is the mass of a free quark, as shown in Fig. 1. This cutoff at r_c could be produced by a finite gluon mass. In the bound state, the effective quark "masses" $m \ll M$. Many effects may complicate the use of the effective potential at separations 1 fm $\lesssim r \lesssim r_c$. But fortunately, we do not need to know the details of the interaction at such distances. In particular, we only employ the $r_c(M)$ relation $r_c=U_0/\eta \cong 2M$ (GeV) fm for definiteness.

We can now begin our analysis by estimating the maximum temperature T_2 at which the quark gas became nonideal due to formation of bound systems (hadrons) as the universe cooled. We adopt the relation $r'dU/dr'=kT$ as an approximate criterion, where r' is the radius of a sphere around a quark for which the probability is $\frac{1}{2}$ that there will be no antiquark of the same color within that sphere. Since $n_B \ll n_\gamma$, $n_q \cong n_{\overline{q}}$, and we also assume that the universe is color neutral, so that there will be equal numbers of quarks of each

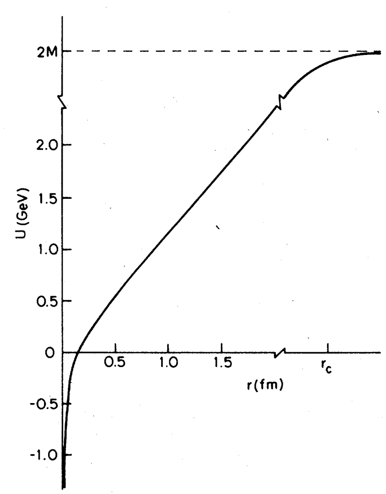

FIG. 1. The potential energy $U(r)$ of a quark and antiquark separated by a distance r.

color. In addition, we shall take the effective potential between three quarks in a color-singlet state to be similar to the q-$\overline{q}$ potential, so that the formation of baryons should proceed in a manner analogous to that of mesons.

Since we are estimating an upper limit for T_2, we can neglect the effects of gluon color screening[14] on the potential. However, the quark color screening should cut off the potential at distances $r \gtrsim r'$, so in this respect the quark gas should behave somewhat like a Coulomb gas. Now at temperatures above T_2, the ideal quark gas will contain mostly those flavors whose bound mass $m \lesssim kT$. Since T_2 will turn out to be ~0.4 GeV/k, we need only include ~3 flavors (u,d,s), giving $g_q=g_{\overline{q}}=18$ degrees of freedom. Including two massive leptons (e,μ) and three massless neutrinos (e,μ,τ) then gives $g_f=50$. Neglecting the longitudinal degrees of freedom present if the gluons are massive, the photons plus eight gluons give $g_b=18$, so the ideal gas of relativistic particles had $g=247/4$ effective degrees of freedom.

Since the number density of quarks of each color is $(g_q/8)n_\gamma$, we obtain

$$r'=0.132(kT)^{-1} \text{ fm}. \tag{5}$$

Using Eq. (4) for the potential, we find that $r'\,dU/dr'=kT$ when $r'=0.30$ fm, giving $kT_2=0.44$ GeV.

As the temperature drops through T_2, the quarks will begin to bind. The crucial fact, however, is that any quark or antiquark of total energy $E>U_0/2=M$ is potentially free. During the quark-hadron transition, the average potential acting on these quarks is likely to remain less than U_0 due to partial screening. Thus we take them to be relativistic, even though $M \gg kT$, but this assumption is not critical. Their number density is then

$$n_q(\text{free})=n_{\overline{q}}(\text{free}) \cong (g_q/2\pi^2\hbar^3)\int_M^\infty e^{-E/kT}E^2dE$$
$$\cong \frac{g_q}{2\pi^2}\left(\frac{kT}{\hbar}\right)^3\left(\frac{M}{kT}\right)^2 e^{-M/kT} \tag{6}$$

while they are in statistical equilibrium with photons, etc. Note that these quarks still form an ideal gas. Note also that in this equation we should take $g_q \geqslant 30$, since at least five flavors (u,d,s,c,b) will be excited at such energies.

As the temperature drops further, the newly formed hadrons will approach an ideal gas. We estimate that this will occur at that temperature T_1 when the hadrons cease overlapping. If we use the criterion $\frac{4}{3}\pi r_\pi{}^3 n_\pi=1$, with a pion radius $\langle r_\pi{}^2\rangle^{1/2}=0.56$ fm (Ref. 15) and $n_\pi \cong \frac{3}{2}n_\gamma$, we obtain $kT_1=0.31$ GeV. But at this temperature, some higher mass hadrons will still be abundant. In an iteration to account for their presence we obtain kT_1

$\cong 0.2$ GeV.

At temperatures less than T_1 one can consider the reactions

$$q+\bar{q} \rightleftharpoons \gamma, \text{ mesons}, \ldots, \tag{7a}$$

$$q+q \rightleftharpoons N+\bar{q}, \ldots, \tag{7b}$$

$$\bar{q}+\bar{q} \rightleftharpoons \bar{N}+q, \ldots \tag{7c}$$

in determining the final abundance of relic quarks. Assuming that the cross section $\sigma(q\bar{q}) \sim \sigma(qq) = \sigma(\bar{q}\bar{q})$, the quark abundance will freeze out at approximately its equilibrium value at that temperature T_* when the expansion rate $t^{-1} = C_0(n_q+n_{\bar{q}})\langle\sigma v\rangle$, $C_0 \sim kT_*/M$. Using $\langle\sigma v\rangle \cong \pi r_c^2 c \cong 4 \times 10^{24} M^2\,\text{fm}^3\,\text{sec}^{-1}$, Eq. (1) for t (with $g=\frac{43}{4}$ for $0.1 > kT > 0.001$ GeV), and a more accurate value for C_0 obtained by full integration of the abundance evolution equations,[16] we obtain $kT_* \cong M/45$, which then yields $n_q+n_{\bar{q}} \cong 4\times 10^{-19} M^{-3} n_\gamma$ after freeze-out.

The number of photons per baryon at these temperatures is related to the corresponding number today (γ/N) by $n_\gamma/n_B = (4/11)(\gamma/N)$. Since the number of free quarks per baryon is strictly conserved after they have frozen out, we obtain for their present abundance

$$(q+\bar{q})/N \cong 1\times 10^{-19} M^{-3}\,(\gamma/N)\,. \tag{8}$$

From present-day observations, $10^8 \lesssim \gamma/N \lesssim 10^{10}$, which gives the same abundance of quarks found by previous studies[7] if $M \sim 1$ GeV.

It was found above that the freeze-out temperature $T_* \cong M/(45k)$. Therefore $kT_* \gtrsim 0.2$ GeV if $M \gtrsim 10$ GeV. Since Eq. (8) depended critically on the assumption that the quarks and hadrons formed an ideal gas, it is not valid unless freeze-out occurred below 0.2 GeV. Thus we reach the important conclusion that the previously accepted abundance given by Eq. (8) is only valid if $M \lesssim 10$ GeV.

At temperatures $T > T_2$, the quarks formed an ideal gas. Although $\sigma(q\bar{q})$ was much less than πr_c^2 at that time due to the screening, electromagnetic interactions were sufficient to guarantee that $n_q\langle\sigma v\rangle \gg t^{-1}$, so all the quarks were in statistical equilibrium. Thus we conclude that if the free-quark mass $M \gtrsim 10$ GeV, they must have frozen out at approximately the equilibrium value given by Eq. (6) at a temperature T_* in the range

$$0.2 \cong T_1 < T_* < T_2 \cong 0.4 \text{ GeV}/k\,. \tag{9}$$

It is certainly possible that the quark-hadron transition is sharp enough that $T_1 \cong T_2$. In any case. from Eqs. (2) and (6) we obtain a present abundance

$$(q+\bar{q})/N \cong (\gamma/N)(M/kT_*)^2 \exp(-M/kT_*)\,. \tag{10}$$

The coefficient depends on $g(T_*)$, where $\frac{69}{4} \leq g(T_*) \leq \frac{247}{4}$. This result is plotted in Fig. 2 for the allowed range of possible values for T_*, and the choice $\gamma/N = 10^9$. Also plotted is Eq. (8) in its region of validity.

The great difference in these two results [Eqs. (8) and (10)] can be understood in terms of the evolution of the effective quark mass. At temperatures above the quark-hadron transition, the screening of the potential kept the quark mass small, approximately equal to its "bound" value $m \ll M$. Since their mass was small enough to allow them to be relativistic, the quark number density was large (comparable to photons). During the quark-hadron transition the number of quarks able to interact with the potentially free high-energy quarks dropped suddenly as most of them became bound. As the hadrons formed in this way, the potentially free quarks acquired a large mass from their unscreened gluon field as they became actually free. The distance between unbound quarks then became very large: $r_q(\text{free}) \sim n_q^{-1/3}(\text{free}) \gtrsim 10^6$ fm $\gg r_c$ (their size).

If the mass M that the free quarks acquired was greater than ~10 GeV, their density $[\propto \exp(-M/kT_*)]$ was so low that they no longer interacted $[n_q(\text{free})\langle\sigma v\rangle_{q\bar{q}}t \ll 1]$ after the hadrons formed. Their abundance had frozen out during the quark-hadron transition at the value given by Eq. (10).

On the other hand, if their mass M was less than ~10 GeV, there were enough free quarks present to allow them to continue interacting as they emerged from the transition. Their freeze-out then occurred when their interactions with each

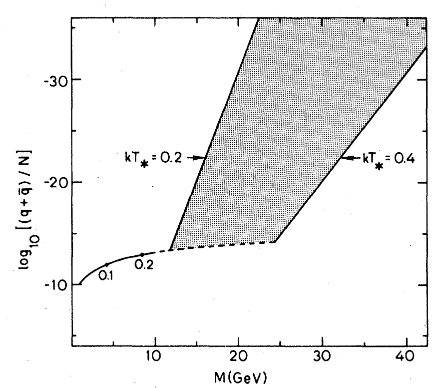

FIG. 2. The dependence of the present ratio of quarks plus antiquarks to nucleons on the mass of the free quark, assuming a present ratio of 10^9 photons per nucleon. For $M \lesssim 10$ GeV, the quark abundance freezes out after the quark-hadron transition. Two values of the corresponding freeze-out temperature T_* (GeV/k) are indicated in this region. For $M \gtrsim 10$ GeV, the quark abundance freezes out during the quark-hadron transition. The resulting quark abundance is shown for a range of possible values of such a freeze-out temperature.

other ceased, giving Eq. (8).

Since it is likely that $(q+\bar{q})/N \ll 10^{-14}$, it is seen from Fig. 2 that in fact, only the new result, Eq. (10), is relevant. Then for any specific abundance of quarks, a range of possible values of the free quark mass is implied. It is to be expected that this range (a factor of 2 in this analysis) will be decreased in the future as the nature of the quark-hadron transition is better understood.

Let us compare this mass range with the lower limit on M implied by accelerator experiments. The maximum such lower limit is obtained by assuming that q-$\bar{q}$ pairs will be produced copiously when kinematically allowed. For colliding leptons, center-of-mass energies $E_{\text{c.m.}} > 2M (\gg 1$ GeV) are required. However, since each quark carries only $\sim\frac{1}{6}$ of the energy of a relativistic nucleon,[6] colliding nucleons require $E_{\text{c.m.}} \gtrsim 12\,M$. The most energetic quark searches have been carried out at the CERN ISR, at maximum energies $E_{\text{c.m.}} \cong 60$ GeV.[5] Their negative results thus imply that $M \gtrsim 5$ GeV. This is certainly consistent with the results in Fig. 2, which for instance predict that $15 \lesssim M \lesssim 30$ GeV if $(q+\bar{q})/N \sim 10^{-20}$. In addition, it should be noted that a relatively low quark mass does not necessarily mean that free quarks will be produced copiously in accelerators. As quarks attempt to separate, their energy can be lost to the production of mesons. The number of those kinematically favorable collisions which actually result in production of free quarks may therefore become appreciable only for $E_{\text{c.m.}} \gg 2\,M$.

A final question which must be addressed is the history of the free quarks up to the present. It is likely that each free quark emerged from freeze-out surrounded by a hadron cloud of radius r_c, the range of its gluon field. Subsequent collisions with nucleons after the antinucleons had disappeared at $kT \sim 20$ MeV might have increased the net baryon number and charge of this cloud. Such interactions, as well as those in Eq. (7), could begin anew in more recent epochs in the high density environment provided by stars, etc. Thus more recent q-$\bar{q}$ reactions might have significantly reduced the terrestrial abundance of free quarks, for instance. The presence of a quark will also affect the chemistry of an element. Thus any comparison of the results in this paper with abundances from quark searches will be plagued by large uncertainties. On the other hand, the logarithmic dependence of quark mass on abundance means that for any claimed present abundance, the uncertainties in our prediction of M are mainly due to our lack of precise knowledge of T_*.

It should be noted that the existence of fractionally charged particles would not necessarily be in conflict with color confinement. General considerations do, however, set severe constraints on such alternative explanations.

For example, Zee[17] has introduced the possibility of fractionally charged color-singlet "echo" quarks, the lightest of which is stable. The analysis of the evolution of the abundance of such quarks is a simple extension of the recent analysis of heavy quarks by Dover, Gaisser, and Steigman,[18] and parallels the original investigations of quark evolution by Zeldovich, Okun, and Pikelner.[7] First note that if the baryon-antibaryon asymmetry is present in echo quarks to the same extent as it is in ordinary quarks, there should be roughly the same number of relic echo quarks as ordinary nucleons.[16] This is clearly excluded. But even in the absence of asymmetry, the expected abundance of relic echo quarks (q') is large:[16]

$$(q'+\bar{q}')/N \sim 10^{-9}(M_{q'}\tilde{\beta})^{-1}. \tag{11}$$

Here $\tilde{\beta} = 10^{15}\langle\sigma v\rangle$ is expressed in $\text{cm}^3\,\text{sec}^{-1}$. If the interaction of these quarks with each other is strong, $\tilde{\beta} \sim 1$, which still allows too many to survive, similarly to Eq. (8).

Another possibility has been suggested by Susskind.[19] Suppose there is an electrically neutral, color-triplet quark. When they become confined (here assumed completely) into hadrons, these new quarks will sometimes form, with the ordinary quarks, fractionally charged color singlets. The surviving abundance of such objects is again at least as large as given by Eq. (11), however, in conflict with observations.

Of course, free quarks must also be present if the universe has any net color. But we may conjecture that the universe is color neutral for the same reason that it is electrically neutral.

We have benefited greatly by many discussions with Leonard Susskind. We also thank James Bjorken, William Fairbank, Stuart Freedman, Alan Litke, and Vahe Petrosian for their help and interest. One of us (R.V.W.) was supported by the John Simon Guggenheim Memorial Foundation through a fellowship and by the National Science Foundation through a grant (PHY 76-21454 A02). The other (G.S.) was supported in part by NASA grant No. NGR 05-020-668 while a Visiting Scholar at Stanford University.

*On leave from the Bartol Research Foundation of the Franklin Institute.

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