Comments and Addenda

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Comment on the application of current algebra and the bag model to $K \rightarrow 2\pi$ decays

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It is shown that applying current algebra and partial conservation of the axial-vector current to the real part instead of the magnitude of the $K \rightarrow 2\pi$ matrix element yields an enhancement factor equal to the secant of the final-state $\pi\pi$ scattering phase shift. This enhancement factor increases the ratio of $K_1^0 \rightarrow 2\pi^0$ to $\Lambda \rightarrow n\pi^0$ matrix elements calculated by Katz and Tatur in the MIT bag model to 1178 MeV which is to be compared with the experimental ratio of 1142 MeV.

The phase of the physical $K \rightarrow 2\pi$ decay matrix element $\langle\pi\pi|iH_w|K\rangle$ is Watson's phase if CP violation is neglected.[1] Since the phase shift δ_0^0 of the $I=0$ s-wave $\pi\pi\rightarrow\pi\pi$ scattering amplitude at a center-of-mass energy equal to the kaon mass is approximately[2] 45°, the imaginary part of the physical decay matrix element of the neutral kaon is approximately equal to the real part. On the other hand, current algebra and partial conservation of the axial-vector current (PCAC) have been used both (a) to relate[1] $\langle\pi\pi|iH_w|K\rangle$ to the $K\rightarrow\pi$ transition matrix element $\langle\pi|H_w|K\rangle$ which is *real* and (b) to construct[3] a momentum-dependent $K\rightarrow 2\pi$ matrix element which is *real*. This difference in phase has often been suppressed by implicity[4] assuming either (a) that the *magnitude* of $\langle\pi\pi|iH_w|K\rangle$ is related to $\langle\pi|H_w|K\rangle$ or (b) that current algebra and PCAC provide information on the momentum dependence of the *magnitude* of $\langle\pi\pi|iH_w|K\rangle$.

In this article we argue that current algebra and PCAC provide nontrivial information about the *real* part, not the magnitude, of $\langle\pi\pi|iH_w|K\rangle$ and suggest that current algebra and PCAC therefore be used (a) to relate the real part of $\langle\pi\pi|iH_w|K\rangle$ to $\langle\pi|H_w|K\rangle$ or (b) to provide information on the momentum dependence of the real part of $\langle\pi\pi|iH_w|K\rangle$. Furthermore, we point out that once the real part of $\langle\pi\pi|iH_w|K\rangle$ is evaluated either by (a) relating it to $\langle\pi|H_w|K\rangle$ or by (b) constructing a momentum-dependent amplitude, then the magnitude can be easily evaluated from the knowledge that the phase of $\langle\pi\pi|iH_w|K\rangle$ is Watson's phase. Finally, we comment on the implications of our results for the $K\rightarrow 2\pi$ matrix element evaluated by Katz and Tatur[5] in the MIT bag model.

It is easy to see that the imaginary part $\mathrm{Im}\langle\pi\pi|iH_w|K\rangle$ vanishes in the soft-pion limit. From CP invariance and unitarity on the mass shell

$$\mathrm{Im}\langle\pi^\alpha\pi^\beta;\,\mathrm{out}|iH_w(0)|K\rangle = T(\pi^\gamma\pi^\delta\rightarrow\pi^\alpha\pi^\beta)^*\langle\pi^\gamma\pi^\delta;\,\mathrm{out}|iH_w(0)|K\rangle\,, \quad (1)$$

where $T(\pi^\gamma\pi^\delta\rightarrow\pi^\alpha\pi^\beta)$ is the $\pi^\gamma\pi^\delta\rightarrow\pi^\alpha\pi^\beta$ scattering amplitude. But $T(\pi^\gamma\pi^\delta\rightarrow\pi^\alpha\pi^\beta)$ has an Adler zero[6]; that is, it vanishes when the momentum of any one of the pions is extrapolated to zero by use of PCAC. Therefore $\mathrm{Im}\langle\pi^\alpha\pi^\beta;\,\mathrm{out}|iH_w(0)|K\rangle$ vanishes when the momentum of any one of the final state pions is extrapolated to zero. Note that since commutation relations of charges with the weak Hamiltonian H_w have not been used, this result is independent of the form of H_w.

Since $\mathrm{Im}\langle\pi\pi|iH_w|K\rangle$ vanishes in the soft-pion limit, the usual[3] soft-pion limits obtained by using current algebra and PCAC are nontrivial limits for the real part $\mathrm{Re}\langle\pi\pi|iH_w|K\rangle$. For a $\Delta I=\frac{1}{2}$ weak Hamiltonian which satisfies the equal-time commutation relations $[Q_5^\alpha, H_w]=[Q^\alpha, H_w]$ these nontrivial soft-pion limits are now of the form

$$\begin{aligned} f_\pi^{-1}\langle\pi^0|H_w|K^0\rangle &= \lim_{\pi^+\rightarrow 0}\mathrm{Re}\langle\pi^+\pi^0|iH_w|K^+\rangle = -\lim_{\pi^0\rightarrow 0}\mathrm{Re}\langle\pi^+\pi^0|iH_w|K^+\rangle = \lim_{\pi^+\rightarrow 0}\mathrm{Re}\langle\pi^+\pi^-|H_w|K_1^0\rangle \\ &= \lim_{\pi^-\rightarrow 0}\mathrm{Re}\langle\pi^+\pi^-|H_w|K_1^0\rangle = \lim_{\pi^0\rightarrow 0}\mathrm{Re}\sqrt{2}\,\langle\pi^0\pi^0|H_w|K_1^0\rangle\,. \end{aligned} \quad (2)$$

For applications corresponding to case (a) we assume that $\mathrm{Re}\langle\pi\pi|H_w|K_1^0\rangle$ varies slowly as the pion momentum is extrapolated to zero and use, for example,

$$\mathrm{Re}\langle\pi^+\pi^-|H_w|K_1^0\rangle = f_\pi^{-1}\langle\pi^0|H_w|K^0\rangle . \quad (3)$$

Corresponding to case (b) we follow the method of, say, Ref. 4 to construct a momentum-dependent real part of the form

$$\mathrm{Re}\langle\pi^+\pi^-|H_w|K_1^0\rangle = A(2q^2 - k_+^2 - k_-^2) + B(k_+^2 + k_-^2 - q^2) , \quad (4)$$

where A and B are real constants and q, k_+, and k_- are the momenta of K, π^+, and π^-.

The magnitude (or the imaginary part) of $\langle\pi\pi|iH_w|K\rangle$ can be obtained from the real part and Watson's phase. From

$$\langle\pi^+\pi^-|H_w|K_1^0\rangle = |\langle\pi^+\pi^-|H_w|K_1^0\rangle|\exp(i\delta_0^0) \quad (5)$$

we get

$$|\langle\pi^+\pi^-|H_w|K_1^0\rangle| = \mathrm{Re}\langle\pi^+\pi^-|H_w|K_1^0\rangle\sec\delta_0^0 . \quad (6)$$

Hence, corresponding to case (a) we get

$$|\langle\pi^+\pi^-|H_w|K_1^0\rangle| = f_\pi^{-1}\langle\pi^0|H_w|K^0\rangle\sec\delta_0^0 \quad (7)$$

from Eqs. (3) and (6), and corresponding to case (b) we get

$$|\langle\pi^+\pi^-|H_w|K_1^0\rangle| = [A(2q^2 - k_+^2 - k_-^2) + B(k_+^2 + k_-^2 - q^2)]\sec\delta_0^0 \quad (8)$$

from Eqs. (4) and (6).

Equations (7) and (8) show that the effect of considering the soft-pion limits to be limits on the real part instead of on the magnitude of $\langle\pi\pi|iH_w|K\rangle$ is to introduce the enhancement factor $\sec\delta_0^0$ on the magnitude of the calculated $\Delta I=\frac{1}{2}$ $K\to 2\pi$ decay matrix element which would be compared with the magnitude of the experimental decay amplitude. Since $\delta_0^0 \simeq 45°$ at an energy equal to the kaon mass, this enhancement factor is rather large for the $\Delta I=\frac{1}{2}$ matrix element. Note that similar considerations apply to the small $\Delta I=\frac{3}{2}$ $K\to 2\pi$ decay matrix element and to other decays where the final-state rescattering amplitude has an Adler zero. However, the enhancement factor $\sec\delta_0^2$ for the $\Delta I=\frac{3}{2}$ $K\to 2\pi$ decay matrix element is near unity because[2] $\delta_0^2 \simeq -10°$ at an energy equal to the kaon mass.

Now we apply our results to the parity-violating nonleptonic weak decay matrix elements evaluated by Katz and Tatur in the MIT bag model. Katz and Tatur evaluated[5] $\langle\pi^0|H_w|K^0\rangle$ and[7] $\langle n|H_w|\Lambda\rangle$ in the MIT bag model and then found the ratio

$$|\langle\pi^0\pi^0|H_w|K_1^0\rangle| / |\langle n\pi^0|iH_w|\Lambda\rangle| = 833 \text{ MeV} \quad (9)$$

by assuming that the *magnitude* of $\langle\pi\pi|iH_w|K\rangle$ varies slowly as the pion momentum is extrapolated to zero. However, if we assume that the *real* part of $\langle\pi\pi|iH_w|K\rangle$ varies slowly and neglect the small $\Delta I=\frac{3}{2}$ amplitude, the enhancement factor $\sec\delta_0^0 \simeq \sec 45°$ evident in Eq. (6) increases this ratio to

$$|\langle\pi^0\pi^0|H_w|K_1^0\rangle| / |\langle n\pi^0|iH_w|\Lambda\rangle| = 1178 \text{ MeV} \quad (10)$$

and therefore brings the result of Katz and Tatur into remarkable agreement with the experimental value of 1142 MeV.

Despite this remarkable agreement, some caution is warranted. In their earlier work Donoghue *et al.*[8] have emphasized that the calculated nonleptonic decay amplitudes are sensitive to the precise bag parameters used and that the calculated $K\to 2\pi$ decay amplitudes are sensitive to the form chosen for the momentum dependence of the off-mass-shell $K\to 2\pi$ amplitudes. Furthermore, the mechanism proposed in this article does not provide the required suppression[7,8] of the $\langle\pi^+|H_w|K^+\rangle$ amplitude which is too large in this bag model.

[1]See, e.g., R. E. Marshak, Riazzuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969).

[2]B. R. Martin, D. Morgan, and G. Shaw, *Pion-Pion Interactions in Particle Physics* (Academic, New York, 1976).

[3]See, e.g., Y. Hara and Y. Nambu, Phys. Rev. Lett. 16, 875 (1966).

[4]One exception is provided by B. R. Holstein [Phys. Rev. 171, 1668 (1968)], who *explicitly* assumes that current algebra and PCAC provide information on the momentum dependence of the magnitude.

[5]J. Katz and S. Tatur, Phys. Rev. D 16, 3281 (1977).

[6]S. L. Adler, Phys. Rev. 137, B1022 (1965).

[7]J. Katz and S. Tatur, Phys. Rev. D 14, 2247 (1976).

[8]J. F. Donoghue and E. Golowich, Phys. Rev. D 14, 1386 (1976); J. F. Donoghue, E. Golowich, and B. R. Holstein, *ibid.* 12, 2875 (1975).