Induced second-class currents from isospin and chiral-symmetry breaking

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(Received 22 January 1979)

Isospin and chiral-symmetry breaking due to an $\epsilon_3 u_3$ Hamiltonian are studied in connection with induced second-class vector currents. In the case of neutron β decay it is found that the induced second-class coefficient is an order of magnitude larger than expected on naive grounds. Predictions for induced effects in $\Sigma^- \to \Lambda$ β decay, $\tau \to \delta \nu_\tau$, and $\pi \to e \nu \gamma$ are also presented.

I. INTRODUCTION

An early generation of experiments on the mass-12 and mass-19 systems[1,2] had suggested the presence of second-class axial-vector currents[3] provoking a great deal of theoretical speculation.[4,5] The concern was amply justified since the large values of the reported induced pseudotensor form factors stood as a threat to our present understanding of the weak interactions.[5] The situation became even more confusing after the experimental result for the $A=12$ system was challenged on very general theoretical grounds.[6] Fortunately it has turned out that later experiments[7] are now in agreement with theoretical expectations[6] and therefore second-class axial-vector currents should not be the subject of further major concern. Small second-class coefficients may be understood, in principle, as arising from chiral-symmetry breaking, i.e., from induced rather than from genuine second-class currents. Therefore, in order to settle this issue it would be important to decide on how large are these induced effects.

It is known that in the framework of the quark model unequal up- and down-quark masses can induce second-class vector and axial-vector coefficients. In fact, Halprin, Lee, and Sorba[8] have shown that in a light-quark model with $m_u \neq m_d$, the effect of gluon vertex corrections induces large second-class form factors if confinement is ignored. On the other hand, taking confinement into account reduces the effect well below the first-class counterpart. The problem is, however, that the connection between the quark model and the more conventional ideas of chiral symmetries realized in the manner of Nambu-Goldstone[9] has yet to emerge from a deeper understanding of quantum chromodynamics.[10] Therefore it would be important to decide if analogous conclusions about induced effects may be drawn from a more standard approach based on the inclusion of an $\epsilon_3 u_3$ term in a chiral Hamiltonian.

The presence of an $\epsilon_3 u_3$ in the Hamiltonian was suggested a long time ago[11] in an attempt to explain the $\eta \to 3\pi$ decay. Early current-algebra calculations,[12] however, predicted a rate an order of magnitude too small. As shown later by Langacker and Pagels[13] the source of the discrepancy was an incorrect derivation of the appropriate Ward-Takahashi identity. Thus, when properly derived the $\eta \to 3\pi$ amplitude, in the chiral-SU(3) $\times$SU(3)-symmetry limit, agrees with the experimental one at the (20–30)% level. This result is reasonable since SU(3)$\times$SU(3) is known to be broken by roughly that amount.[9] The problem is then how to estimate chiral-symmetry-breaking corrections.

It has been shown by Langacker and Pagels[13] that chiral perturbation theory predicts a correction to the $\eta \to 3\pi$ amplitude of about 34% but in the wrong direction. On the other hand, the $\eta \to 3\pi$ problem has been reanalyzed recently[14] in the framework of the extended partially conserved axial-vector current (EPCAC) hypothesis[15,16] and a correction of $\approx$20% in the right direction has been obtained. This EPCAC approach also predicts correct $\Delta I = 1$ baryon mass differences.[14,17] A systematic study of these $\epsilon_3 u_3$ induced mass differences in the framework of quantum chromodynamics has been performed recently by Langacker and Pagels,[18] who find results consistent with those obtained from EPCAC.

Motivated by this successful development in our understanding of the consequences of the $\epsilon_3 u_3$ Hamiltonian we plan to explore in this paper its effect in inducing second-class vector form factors in $\Delta S = 0$ β decays. We shall limit the discussion to matrix elements of the weak vector current between baryons in order to avoid model-dependent nuclear complications.

In neutron β decay one would naively expect $\epsilon_3 u_3$ to induce a second-class vector form factor of the order of the proton-neutron mass difference divided by the nucleon mass, i.e., $g_S/g_V \approx 10^{-3}$, where g_V and g_S are the first- and second-class vector form factors, respectively. Instead, we show here that $g_S/g_V \gtrsim 2\times 10^{-2}$, i.e., an order of magnitude larger than the naive expectation. The calculation has been performed (Sec. II) using two different methods, viz. (a) assuming $\delta(980)$-meson pole dominance in the matrix element of the divergence of the weak vector current at zero momentum transfer, and (b) the more rigorous Goldstone-boson pair mechanism of Li and Pagels.[19] Both methods yield roughly the same answer.

Finally, it is argued that such a value of g_S, though still small, may eventually be detected by future precision measurements. In Sec. III predictions are presented for induced effects in $\Sigma^- \to \Lambda\ \beta$ decay where both $g_V(0)$ and $g_S(0)$ are of order $O(\epsilon_3)$. Also, brief comments are made on the decays $\tau \to \delta\nu_\tau$ and $\pi \to e\nu\gamma$ which offer a unique testing ground for the results obtained here. It should be stressed that an experimental confirmation of such ϵ_3-induced effects would have important implications for our present understanding of hadronic symmetry breaking. In particular it would provide important additional verification of the overall consistency of recent $\eta \to 3\pi$ and $\Delta I = 1$ baryon mass-difference calculations.[14,17,18]

II. CALCULATION OF THE INDUCED SECOND-CLASS VECTOR FORM FACTOR

Let us start by defining the matrix elements of the weak vector current between proton and neutron as

$$\langle p\,|V_\mu^{1+i2}(0)\,|n\rangle = \bar{u}_p[\gamma_\mu g_V(q^2) + i\sigma_{\mu\nu}q^\nu g_M(q^2)/2M + q_\mu g_S(q^2)/2M]u_n\,, \tag{1}$$

where g_V, g_M, and g_S are the first-class vector, weak magnetism, and second-class vector form factors, respectively. Taking the divergence on both sides of Eq. (1) one finds

$$\begin{aligned}\langle p\,|i\partial^\mu V_\mu^{1+i2}|n\rangle &= \bar{u}_p D(q^2)u_n \\ &= \bar{u}_p[(m_p - m_n)g_V(q^2) + q^2 g_S(q^2)/2M]u_n\,.\end{aligned} \tag{2}$$

Since vacuum symmetry is not spontaneously broken at this level, one obtains at $q^2 = 0$

$$D(0) = (m_p - m_n), \tag{3}$$

where we have normalized the first-class vector form factor as $g_V(0) = 1$. It should be clear that normalizing $g_V(0)$ in such a way is probably a very good approximation because, according to the nonrenormalization theorem,[20] one expects $g_V(0) = 1 + O(\epsilon_3^2)$ while $g_S(0) = O(\epsilon_3)$. On the other hand, specifying the chiral Hamiltonian as

$$H = H_0 + \epsilon_3 u_3, \tag{4}$$

where H_0 commutes with the vector charges Q^α $(\alpha = 1, 2, 3)$, one finds

$$\begin{aligned}\bar{u}_p D(q^2)u_n &= \langle p\,|\,[(Q^1 + iQ^2), H]\,|\,n\rangle \\ &= -\sqrt{2}\ \epsilon_3\langle p\,|u_+|\,n\rangle.\end{aligned} \tag{5}$$

In Eq. (5) u_+ is defined in the usual way as $u_+ = (u_1 + iu_2)/\sqrt{2}$. The $\epsilon_3 u_3$ term in Eq. (4) induces a proton-neutron mass difference which can be related to the right-hand side of Eq. (5) with the result

$$D(q^2) = (m_p - m_n)_{u_3} d(q^2), \tag{6}$$

where $d(0) = 1$. Equation (6) is perfectly compatible with Eq. (3) because the spinors in Eq. (2) are to be understood as eigenstates of the Hamiltonian (4), and therefore the mass difference in Eq. (3) is actually that induced by the $\epsilon_3 u_3$ term. Hence at $q^2 = 0$ one simply obtains an identity.

As a first approximation one may assume that as $q^2 \to 0$ $D(q^2)$ is dominated by the $\delta(980)$-meson pole, in which case

$$d(q^2) = \frac{m_\delta^2}{m_\delta^2 - q^2}. \tag{7}$$

Substituting Eqs. (6) and (7) in Eq. (2) and performing a series expansion around $q^2 = 0$, one finds[21]

$$g_S(0) = 2M(m_p - m_n)_{u_3}\left(\frac{1}{m_\delta^2} - \frac{1}{6}\langle r_V^2\rangle\right), \tag{8}$$

where $\langle r_V^2\rangle$ is the isovector radius of the nucleon,[22] i.e., $\langle r_V^2\rangle/6 = 0.066/\mu_\pi^2$. Using $(m_p - m_n)_{u_3} \approx -2.5$ MeV as determined in Refs. 14 and 18 together with the experimental δ-meson mass, it follows from Eq. (8) that

$$\frac{g_S(0)}{g_V(0)} \approx 10^{-2}. \tag{9}$$

In order to verify the approximation that led to Eq. (9) one may use the more rigorous Goldstone-boson pair mechanism of Li and Pagels[19] to calculate $d(q^2)$ in Eq. (6). If the divergence of the weak vector current is a gentle operator then $D(q^2)$ satisfies the following unsubtracted dispersion relation:

$$D(q^2) = \frac{1}{\pi}\int_{t_0}^{\infty} \frac{dt}{t - q^2}\,\mathrm{Im}D(t) \tag{10}$$

where $t_0 = (\mu_K + \mu_{\bar{K}})^2$, $(\mu_\pi + \mu_\eta)^2$ corresponding to the two-pseudoscalar-meson production threshold (in the chiral-symmetry limit $K\bar{K}$ and $\pi\eta$ are the

Goldstone-boson pairs coupling to $\partial^\mu V_\mu$). At this threshold one uses unitarity constraints together with the chiral-symmetry limit of the S-wave projection of the meson-meson $\to$ nucleon-antinucleon amplitude to find the exact two-pseudoscalar-meson contribution to $\mathrm{Im}D(q^2)$ near threshold. In this case, Eq. (10) becomes

$$D(q^2) = (m_p - m_n)_{u_3} \frac{1}{4\Lambda} \int_0^{4\Lambda^2} \frac{dt}{t - q^2} \sqrt{t} \,. \qquad (11)$$

The contribution from four mesons to $\mathrm{Im}D(t)$ behaves like $t^2\sqrt{t}$ and that from six mesons like $t^4\sqrt{t}$ and therefore the two-pseudoscalar-meson state (see Fig. 1) dominates the threshold region [note that the integrand in Eq. (11) diverges like $1/\sqrt{t}$ as $t \to 0$, although the integral is finite at $t=0$]. The cutoff in Eq. (11) has been introduced in order to separate the threshold from the high-energy contribution to $D(q^2)$. A numerical fit to all $\Delta I = 1$ baryon mass differences shows that $\Lambda \approx 250-300$ MeV, thus confirming threshold dominance in this instance.

Solving Eq. (11) and substituting it in Eq. (2) one finds

$$g_S(0) = -2M(m_p - m_n)_{u_3}\left(\frac{\langle r_V{}^2\rangle}{6} + \frac{1}{4\Lambda^2}\right), \qquad (12)$$

where a series expansion around $q^2 = 0$ has been performed. At first sight it would seem possible to set $\Lambda = \infty$ in Eq. (12) in which case $g_S(0) \approx 2\times 10^{-2}$. However, one should bear in mind that in this approach the u_3-induced mass difference is itself a function of Λ and as $\Lambda \to \infty$ it becomes linearly divergent.[19] This fact is of not much consequence here since even with Λ as small as 250 MeV one has

$$\frac{g_S(0)}{g_V(0)} \approx 3.5\times 10^{-2}\,. \qquad (13)$$

In summary, saturation of the divergence of the weak vector current by a $\delta(980)$-meson pole or by a Goldstone-boson pair leads to a rather large induced second-class vector form factor. Granting the validity of these dynamical approximations, a measurement of $g_S(0)$ can provide additional verification of the overall consistency of recent quark-mass calculations[14,17,18]. Examples of some feasible tests are presented in the next section.

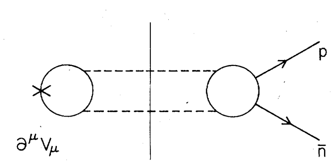

FIG. 1. Leading threshold singularity [see Eq. (10)]. Dotted lines represent $K\bar{K}$ and $\pi\eta$ Goldstone-boson pairs.

III. TESTS

Although the value just found for $g_S(0)$ is an order of magnitude larger than expected on naive grounds, it is perhaps still small to be detected by neutron β-decay measurements. Muon capture in principle offers a better testing ground, although present experimental accuracy would have to be considerably improved before such a test could become feasible. Similar considerations apply to the forward scattering of neutrinos off nucleons where a parity-violation effect is expected if $\epsilon_3 \neq 0$. In $\Sigma \to \Lambda\,\beta$ decay, however, a precision measurement of $g_V(0)$ would provide a rather clean test of ϵ_3-induced effects [$g_V(0)$ vanishes identically if $\epsilon_3 = 0$]. On the other hand, the higher momentum transfer involved in this decay might help to detect $g_S(0)$, which is expected to be comparable to $g_V(0)$, both being of $O(\epsilon_3)$. In view of this the predictions for both form factors are derived in part A of this section.

In a separate context, Leroy and Pestieau[23] have suggested looking for the heavy-lepton decay $\tau \to \delta\nu_\tau$ as a signal for second-class vector currents. In part B we calculate the branching ratio $\Gamma(\tau\to\delta\nu)/\Gamma(\tau\to\pi\nu)$ to be expected from ϵ_3-induced effects. Finally, Bernabeu, Tarrach, and Yndurain[24] and Montemayor and Moreno[25] have recently pointed out that the pion radiative decay $\pi \to e\nu\gamma$ might also offer the possibility of testing our ideas about isospin symmetry breaking. A brief discussion about this test is presented in the last part of this section.

A. $\Sigma \to \Lambda\,\beta$ decay

Defining the matrix elements of the weak vector current for this decay in analogy with Eq. (1) one finds after taking the divergence

$$\begin{aligned}\langle \Lambda \,|\, i\partial^\mu V_\mu^{1+i2} \,|\, \Sigma^-\rangle &= \overline{U}_\Lambda D(q^2) U_\Sigma \\ &= \overline{U}_\Lambda \left[(m_\Lambda - m_\Sigma) g_V^\Lambda(q^2) \right.\\ &\qquad \left. + \frac{q^2}{2M} g_S^\Lambda(q^2)\right] U_\Sigma .\end{aligned} \qquad (14)$$

Specifying the Hamiltonian as in Eq. (4), it follows that

$$\overline{U}_\Lambda D(q^2) U_\Sigma = -\sqrt{2}\,\epsilon_3 \langle \Lambda \,|\, U_+ \,|\, \Sigma^-\rangle. \qquad (15)$$

Relating the right-hand side of Eq. (15) to the $\Sigma^+ - \Sigma^-$ mass difference induced by the $\epsilon_3 u_3$ Hamiltonian and substituting it in Eq. (14), one has

$$\left(\frac{f}{d}\right)^{-1}\frac{1}{\sqrt{3}}(M_{\Sigma^+}-M_{\Sigma^-})_{u_3}\,d(q^2)=(M_\Lambda-M_\Sigma)g_V^\Lambda(q^2)+\frac{q^2}{2M}g_S^\Lambda(q^2), \quad (16)$$

where $d(0)=1$. Since $M_\Lambda - M_\Sigma = O(\epsilon_8)$, whereas $M_{\Sigma^+}-M_{\Sigma^-}=O(\epsilon_3)$, one can safely use experimental data for the former in which case one obtains from Eq. (16) at $q^2=0$

$$g_V^\Lambda(0)=0.016\pm0.001, \quad (17)$$

where $(M_{\Sigma^+}-M_{\Sigma^-})u_3=-7.2\pm0.6$ has been used.[14] In order to verify the consistency of the above procedure one could instead relate the Λ-Σ mass difference to the Σ^+-Σ^- mass difference in which case one finds

$$g_V^\Lambda(0)=\frac{\epsilon_3}{\epsilon_8}=0.017\pm0.002, \quad (18)$$

where $f/d=3.26$ from the medium-strong differences and[14] $\epsilon_3/\epsilon_8=0.017\pm0.002$ have been used.

The induced second-class vector form factor $g_S^\Lambda(0)$ may be determined by expanding Eq. (16) in series around $q^2=0$, the result being

$$g_S(0)\approx0.03, \quad (18')$$

if $\delta(980)$-meson pole dominance is assumed for $d(q^2)$, and

$$g_S(0)\approx0.05, \quad (19)$$

if the Goldstone-boson pair mechanism is used as in Sec. II. In both cases above an isovector transition root-mean-squared radius equal to that of the nucleon has been used.

A considerable improvement of the present experimental measurement $g_V^\Lambda=0.24\pm0.23$ is obviously needed before these predictions could be tested.[26]

An important issue in $\Sigma\to\Lambda$ decay, which has not been taken into account in the above calculation, is that of Σ^0-Λ mixing due to electromagnetic effects. The calculation of the mixing angle and the charge asymmetry produced by the electromagnetic Hamiltonian was done by Eimerl[26] some time ago. In the following we calculate these parameters using the chiral Hamiltonian Eq. (4):

The Σ^0-Λ mixing angle is defined by means of

$$\Lambda_{\rm phys}=\Lambda\cos\beta+\Sigma^0\sin\beta, \quad (20)$$

$$\Sigma^0_{\rm phys}=\Sigma^0\cos\beta-\Lambda\sin\beta, \quad (21)$$

where $\Lambda_{\rm phys}$ and $\Sigma_{\rm phys}$ stand for the physical Λ and Σ^0 states. The mixing angle can then be obtained after sandwiching the Hamiltonian Eq. (4) between the physical Λ and Σ^0 states, in which case one has

$$\tan2\beta=-2\frac{\langle\Lambda|\epsilon_3u_3|\Sigma^0\rangle}{M_{\Sigma^0}-M_\Lambda}. \quad (22)$$

Using the analysis of Ref. 14 to calculate $\langle\Lambda|\epsilon_3u_3|\Sigma^0\rangle$, one finds

$$\beta\approx-0.01, \quad (23)$$

which agrees in sign and magnitude with the electromagnetic contribution.[26] Having determined the mixing angle one can then compute the charge-asymmetry parameter defined as

$$\delta=\tfrac{1}{2}(R^{(+)}/R_0^{(+)}-R^{(-)}/R_0^{(-)}), \quad (24)$$

where $R_0^{(\pm)}$ are the rates calculated in the absence of electromagnetism and with $\epsilon_3=0$. Using Eq. (23), one finds

$$\delta\approx0.02, \quad (25)$$

which points to an enhancement of Σ^+ decay over Σ^- decay. It was shown by Eimerl that conventional radiative corrections produce a very small asymmetry while the mixing due to the electromagnetic Hamiltonian gives an asymmetry parameter of the same sign and magnitude as Eq. (25). Present experimental accuracy does not allow for a confirmation of these predictions, but it is hoped that the future availability of hyperon-beam facilities might settle this issue.

B. Heavy-lepton decay

Leroy and Pestieau[23] have pointed out that due to the negative G parity of the $\delta(980)$ meson the decay $\tau\to\delta\nu_\tau$ would, if detected, be a signal for the existence of second-class vector currents. These authors find the branching ratio

$$\frac{\Gamma(\tau\to\delta\nu_\tau)}{\Gamma(\tau\to\pi\nu_\tau)}=0.28\frac{f_\sigma^{\,2}}{f_\pi^{\,2}}, \quad (26)$$

where $f_\pi=92$ MeV is the pion decay constant and f_δ is defined by

$$\langle0|V_\mu^-|\delta^+(p)\rangle=f_\delta p_\mu. \quad (27)$$

In order to make a more specific prediction one would have to know f_δ from other considerations. A crude estimate of its value may be obtained by noting that, since $\delta(980)$ is not a Goldstone boson, then f_δ should vanish in the chiral-symmetry limit. Specifying the chiral Hamiltonian as in Eq. (4) one would expect then $f_\delta=O(\epsilon_3)$, or more specifically, $f_\delta\approx\epsilon_3/m_\delta^{\,2}$. These considerations obviously apply only to the ϵ_3-induced contribution to f_δ, i.e., if genuine second-class currents exist then f_δ might be very different from the above estimate. Using the value of ϵ_3 found in Ref. 14, one has

$$\frac{f_\delta}{f_\pi} \approx -0.006, \tag{28}$$

which implies

$$\frac{\Gamma(\tau \to \delta\nu_\tau)}{\Gamma(\tau \to \pi\nu_\tau)} \approx 10^{-5}. \tag{29}$$

From the smallness of this branching ratio one may conclude that if $\tau \to \delta\nu_\tau$ decay is ever observed, it would indicate the presence of genuine second-class vector currents.

C. Radiative pion decay

It has been pointed out recently[24,25] that unequal up- and down-quark masses may induce large isospin-breaking effects in the physical amplitude for $\pi \to e\nu\gamma$. The main point here is that if $m_u \neq m_d$ then the ratio of the vector form factor in $\pi \to e\nu\gamma$, F_V, to the $\pi^0 \to \gamma\gamma$ decay amplitude, $F_{\pi 0}$, is no longer given by $F_{\pi 0}/F_V = \sqrt{2}$ but becomes a sensitive function of the quark-mass ratio. This function can be calculated exactly at the soft-pion point where $F_{\pi 0}$ is controlled entirely by the Adler-Bell-Jackiw anomaly[27] and F_V is computed from a triangle graph.

According to the calculation of Bernabeu, Tarrach, and Yndurain one should expect 50% isospin-breaking effects if $m_d/m_u = 0$ or ∞. However, it has been shown recently that such quark-mass ratios are inconsistent with the nonrenormalization theorem as well as with our present knowledge of hadronic symmetry breaking.[17,18] Nevertheless, one can verify that even with a quark-mass ratio consistent with the analyses of Refs. 14 and 18, one would expect between 6% and 20% effects in $\pi \to e\nu\gamma$ if the calculation of Ref. 24 is correct. In the course of that calculation the authors assume that the ratio of the coupling constants between quarks and pions is given by

$$\frac{g_{\pi uu}}{g_{\pi dd}} = \frac{m_u}{m_d},$$

an assumption which definitely is very hard to understand. Montemayor and Moreno[25] assume instead $g_{\pi uu} = g_{\pi dd}$ and find a much larger isospin-breaking effect in $\pi \to e\nu\gamma$ (up to 100% for a quark-mass ratio consistent with the value of ϵ_3 used here). A precision measurement of this amplitude could therefore provide clean additional evidence for the $\epsilon_3 u_3$-induced effects discussed here.

IV. SUMMARY

Motivated by recent successful developments in our understanding of the $\eta \to 3\pi$ decay and the $\Delta I = 1$ baryon mass differences,[14,17,18] a study has been made of second-class currents induced by an $\epsilon_3 u_3$ Hamiltonian. In neutron β decay it has been shown here that such a symmetry breaking induces a second-class vector form factor an order of magnitude larger than expected. Predictions for $\Sigma \to \Lambda\ \beta$ decay, $\tau \to \sigma\nu_\tau$, and $\pi \to e\nu\gamma$, which would provide more realistic tests, have also been presented. An experimental confirmation of these predictions is expected to have important implications for our present understanding of hadronic symmetry breaking. In particular, it would provide additional verification of the overall consistency of recent $\eta \to 3\pi$ and $\Delta I = 1$ baryon mass-difference calculations.[14,17,18]

ACKNOWLEDGMENTS

The author wishes to thank Paul Langacker, Heinz Pagels, and Alberto Sirlin for most enjoyable and illuminating conversations. A discussion with Augusto Garcia and the collaboration of Arnulfo Zepeda during an early stage of this work are also acknowledged. This work was supported in part by CONACYT (Mexico) under Contract No. 540-C.

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*On sabbatical leave from Centro de Investigacion y de Estudios Avanzados del I. P. N., Mexico.

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