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Neutral currents and quark mass ratio $m_u/m_d = m_s/m_c$ in an SU(2) × U(1) × U(1) model

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It is proposed that the relationship $m_u/m_d = m_s/m_c$ is a zeroth-order relationship of the spontaneously broken Lagrangian. This relationship is achieved by a symmetry which exchanges fields of different charges, and SU(2) × U(1) × U(1) is the smallest extension of SU(2) × U(1) by which one can implement such an exchange symmetry. Extended to leptons, the scheme achieves $m_e/m_\mu \sim m_d/m_c = m_u/m_s$ in a natural way. Limited to light quarks and leptons and the Higgs particles to which they couple, the model considered has identical neutral-current parametrization to the standard Weinberg-Salam model. When heavy quarks and leptons are introduced, and an extra Higgs field is added to provide their masses, the neutral-current parametrization is modified. The limit to the Weinberg-Salam case is discussed. Various sets of parameter values can be chosen which provide an adequate description of neutrino scattering data, polarized electron scattering from deuterium, and any one of the results of the different atomic parity violation experiments in bismuth.

INTRODUCTION

Current-algebra estimates of quark mass values typically find that $m_u/m_d \lesssim \frac{1}{2}$ and $m_d/m_s \lesssim \frac{1}{20}$.[1] These "current quark" ratios suggest that the up and down quark masses are small, 10 MeV or so, and even the extreme case $m_u = 0$ has been investigated[2] because it would allow one to understand CP as a natural conservation law of the strong interactions.[3] Requiring that chiral SU(3) is an approximate symmetry of the strong interactions leads to the statement that $m_u \neq 0$, however.[1] A careful analysis by Langacker and Pagels[1] which includes corrections to chiral-symmetry breaking shows that a rather small ratio $m_u/m_d = 0.38 \pm 0.13$ is needed. The observation that this ratio is of the same order of magnitude as that of m_s/m_c (Ref. 4) leads us to suggest in this paper that $m_u/m_d = m_s/m_c$ is a zeroth-order relation of the Lagrangian. Our viewpoint is that these relations and the large difference in mass scales between m_c and m_s on the one hand and m_d and m_u on the other hand should be achieved without requiring *ad hoc* order-of-magnitude differences between various Yukawa couplings.

We start with the usual SU(2) × U(1) gauge theory[5] of left-handed doublet fermions to see what conditions are needed to generate mass relationships of the type discussed above. We work out in detail an example where an interchange symmetry is enforced between singlets: $u_R \leftrightarrow s_R$ and $d_R \leftrightarrow c_R$. The scheme requires two Higgs-particle doublets for implementation. The exchange symmetry between quarks of different charge can only be carried out with a more complicated electromagnetic current than in the SU(2) × U(1) theory, and at least another U(1) group whose gauge field is mixed with the photon must be appended to SU(2) × U(1) in order to satisfy the symmetry requirement on the quark kinetic energy terms.

The electron and muon doublets are handled in a fashion similar to that of the quarks, and the mass ratio m_e/m_μ is naturally of the same order of magnitude as the m_u/m_s and m_d/m_c ratios. As remarked above, our objective is to achieve the large differences among quark masses and among lepton masses with Yukawa couplings of the same order of magnitude.[6]

Neutral currents are considered, and we see that when only light fermions and the Higgs particles to which they are coupled are included, the analysis of Georgi and Weinberg[7] ensures that all of the neutrino neutral-current interactions are identical to those of the standard model.[5] The example we choose is shown to have the further property that parity-violating effects in the electron neutral-current interactions are also identical to the standard model, and the conditions that this be true are pointed out.

Finally, we consider the inclusion of heavy quarks and leptons in the scheme, and we point out that addition of a third Higgs particle provides masses for heavy quarks of charge $\frac{2}{3}$ and $-\frac{1}{3}$ and accommodates the τ lepton and its neutral partner. At this point we reconsider the neutrino and electron neutral-current interactions and we show that the parameters of our model allow agreement with neutrino and antineutrino scattering data, with polarized-electron scattering, and with any one of the bismuth atomic parity-violation results.

QUARK MASS RATIOS

We start with the proposal that $m_u/m_d \simeq m_s/m_c$ (Ref. 4) should be incorporated as a zeroth-order relationship of the Lagrangian after spontaneous breaking of the gauge symmetry. We can achieve this result if we couple the left-handed doublets and right-handed singlets of quarks to two different Higgs-particle doublets

$$\phi_{1,2}=\begin{pmatrix}\phi^0_{1,2}\\ \phi^-_{1,2}\end{pmatrix}$$

as follows:

$$\mathcal{L}_Y(q,\phi_1,\phi_2)=\frac{1}{\langle\phi^0_2\rangle_0}[(m_1\bar{u}_R\phi_1^\dagger D_{1L}+m_2\bar{u}_R\phi_1^\dagger D_{2L})+(m_1\bar{s}_R\phi_2^{c\dagger}D_{2L}+m_2\bar{s}_R\phi_2^{c\dagger}D_{1L})$$
$$+(M_1\bar{c}_R\phi_2^\dagger D_{1L}+M_2\bar{c}_R\phi_2^\dagger D_{2L})+(M_1\bar{d}_R\phi_1^{c\dagger}D_{2L}+M_2\bar{d}_R\phi_1^{c\dagger}D_{1L})]+\text{H.c.} \tag{1}$$

This Yukawa Lagrangian is invariant under the exchange symmetry $\phi_1\leftrightarrow\phi_2^c$ (where ϕ_i^c means charge conjugate), $u_R\leftrightarrow s_R$, $c_R\leftrightarrow d_R$, and $D_{1L}\leftrightarrow D_{2L}$. There is an additional global phase symmetry $\phi_i\to e^{i\theta_i}\phi_i$ and $q_R\to e^{i\theta_q}q_R$ which is obeyed by this interaction Lagrangian, Eq. (1). The eigenvalues of the mass matrix are identified with physical masses according to the relationships

$$m_c^2=\tfrac{1}{2}\{\epsilon^2m^2+M^2+[(\epsilon^2m^2-M^2)^2+4\epsilon^2(m\cdot M)^2]^{1/2}\},$$
$$m_u^2=\tfrac{1}{2}\{\epsilon^2m^2+M^2-[(\epsilon^2m^2-M^2)^2+4\epsilon^2(m\cdot M)^2]^{1/2}\}, \tag{2}$$
$$m_s^2=\tfrac{1}{2}\{m^2+\epsilon^2M^2+[(m^2-\epsilon^2M^2)^2+4\epsilon^2(m\cdot M)^2]^{1/2}\},$$
$$m_d^2=\tfrac{1}{2}\{m^2+\epsilon^2M^2-[(m^2-\epsilon^2M^2)^2+4\epsilon^2(m\cdot M)^2]^{1/2}\},$$

where $\epsilon=\langle\phi_1^0\rangle/\langle\phi_2^0\rangle$ has been introduced and we defined $m^2=m_1^2+m_2^2$, $M^2=M_1^2+M_2^2$, $m\cdot M=m_1M_1+m_2M_2$. With the conditions that

$$\frac{\epsilon[m^2M^2-(m\cdot M)^2]^{1/2}}{\epsilon^2m^2+M^2}\ll 1,\quad \frac{\epsilon[m^2M^2-(m\cdot M)^2]^{1/2}}{m^2+\epsilon^2M^2}\ll 1 \tag{3}$$

we find the desired relationships

$$\frac{m_d}{m_c}=\frac{m_u}{m_s}\simeq\frac{[m^2M^2-(m\cdot M)^2]^{1/2}}{[(\epsilon^2m^2+M^2)(m^2+\epsilon^2M^2)]^{1/2}}\ll 1\,. \tag{4}$$

Special cases of the conditions of Eq. (3) are

$$\epsilon\ll 1\quad\text{or}\quad\frac{m^2M^2-(m\cdot M)^2}{(\epsilon^2m^2+M^2)(m^2+\epsilon^2M^2)}\ll 1\,.$$

The $\epsilon\ll 1$ case corresponds to an approximate symmetry under $\phi_1\to-\phi_1$ and $u_R\to-u_R$, $d_R\to-d_R$ in the Lagrangian which includes the spin-zero potential. The parameters m_1, m_2, M_1, and M_2 are all taken to be of the same order of magnitude in our estimates.

KINETIC-ENERGY SYMMETRY AND NEUTRAL CURRENTS

The W boson couples to the usual Cabibbo combination of quark fields. However, the exchange symmetry between quarks of different charge requires that at least one more neutral gauge boson mixed with the electromagnetic field be introduced to ensure that the quark kinetic energy terms are exchange symmetric. With an additional U(1) gauge group we can require that one gauge field, call it C, obeys $C\to-C$ under the exchange symmetry. We may write the covariant derivatives of the spin-zero fields as

$$D_\mu\phi_{1,2}=\left[\partial_\mu-ig\left(V_\mu-\frac{q_2}{\sqrt{2}}C_\mu\right)\right]\phi_{1,2}\,, \tag{5a}$$

while the quark singlet fields have covariant derivatives:

$$D_\mu u_R=\left[\partial_\mu-\frac{ig}{\sqrt{2}}\left(\frac{q_1}{3}B_\mu+q_2C_\mu\right)\right]u_R\,,$$
$$D_\mu c_R=\left[\partial_\mu-\frac{ig}{\sqrt{2}}\left(\frac{q_1}{3}B_\mu+q_2C_\mu\right)\right]c_R\,, \tag{5b}$$
$$D_\mu s_R=\left[\partial_\mu-\frac{ig}{\sqrt{2}}\left(\frac{q_1}{3}B_\mu-q_2C_\mu\right)\right]s_R\,,$$
$$D_\mu d_R=\left[\partial_\mu-\frac{ig}{\sqrt{2}}\left(\frac{q_1}{3}B_\mu-q_2C_\mu\right)\right]d_R\,.$$

Finally, we have

$$D_\mu D_{1,2}=\left[\partial_\mu-ig\left(V_\mu+\frac{q_1}{3\sqrt{2}}B_\mu\right)\right]D_{1,2} \tag{5c}$$

for quark doublets. V is the usual 3×3 representation of the SU(2) gauge fields:

$$V=\sum_{i=1}^{3}V_i\tau_i/\sqrt{2}\,.$$

Because the ϕ_1 and ϕ_2 fields couple to only one neutral gauge field combination, there are two massless gauge fields unless additional spin-zero fields are included. The simplest choice is a pair of singlets, a neutral η and a charged χ, which interchange ($\eta\leftrightarrow\chi$) under the discrete symmetry $C\to-C$, $\phi_1\leftrightarrow\phi_2^c$, $u_R\leftrightarrow s_R$, $d_R\leftrightarrow c_R$, $D_1\leftrightarrow D_2$. The covariant derivatives of η and χ are then

$$D_\mu\eta=\left[\partial_\mu-\frac{ig}{\sqrt{2}}(q_1B_\mu-q_2C_\mu)\right]\eta\,,$$
$$D_\mu\chi=\left[\partial_\mu-\frac{ig}{\sqrt{2}}(q_1B_\mu+q_2C_\mu)\right]\chi\,, \tag{6}$$

respectively.

The photon field A is the linear combination

$$A=\left(\frac{q_2}{(q_1^2+q_2^2+q_1^2q_2^2)^{1/2}}\right)B+\left(\frac{q_1}{(q_1^2+q_2^2+q_1^2q_2^2)^{1/2}}\right)C+\left(\frac{q_1q_2}{(q_1^2+q_2^2+q_1^2q_2^2)^{1/2}}\right)V_3\,, \qquad (7)$$

involving all three neutral gauge fields, B, C, and V_3, where the Lorentz indices have been suppressed.

LEPTONS AND NEUTRAL-CURRENT INTERACTIONS

We fashion the lepton couplings after those of the standard model, and we introduce right-handed charged singlets e_R and μ_R and left-handed doublets E_L and M_L. Let us focus on the gauge field couplings and the consequent neutral-current interactions. We write

$$\begin{aligned} D_\mu e_R &= (\partial_\mu + i\sqrt{2}gq_2C_\mu)e_R\,,\\ D_\mu \mu_R &= (\partial_\mu + i\sqrt{2}gq_2C_\mu)\mu_R\,,\\ D_\mu E_L &= \left[\partial_\mu - ig\left(V_\mu - \frac{q_2}{\sqrt{2}}C_\mu\right)\right]E_L\,,\\ D_\mu M_L &= \left[\partial_\mu - ig\left(V_\mu - \frac{q_2}{\sqrt{2}}C_\mu\right)\right]M_L\,, \end{aligned} \qquad (8)$$

where

$$E_L=\begin{pmatrix}\nu_L\\ E_L^-\end{pmatrix}$$

and similarly for the doublet M_L. The interchange symmetry for leptons is then required to be $e_R \leftrightarrow \mu_R^{CP}$ and $E_L \leftrightarrow M_L^{CP}$, where CP stands for the charge-conjugation and parity transformed fields, along with the previous $\phi_1^c \leftrightarrow \phi_2$ symmetry.

A Yukawa interaction which obeys all of the symmetries of our model and which has global phase assignments such that

$$\bar{E}_L e_R \to e^{i\theta_1}\bar{E}_L e_R \quad\text{and}\quad \bar{M}_L\mu_R \to e^{i\theta_2}\bar{M}_L\mu_R$$

[see discussion following Eq. (1)] reads

$$\begin{aligned}\mathcal{L}_Y(\text{leptons}) &= A(\bar{E}_L\phi_1^c e_R + \bar{M}_R^c\phi_2\mu_L^c) + \text{H.c.}\\ &\quad + B(\bar{M}_L\phi_1^c e_R + \bar{E}_R^c\phi_2\mu_L^c) + \text{H.c.}\end{aligned} \qquad (9)$$

in a form which explicitly shows the $\phi_1^c \leftrightarrow \phi_2$ exchange symmetry. The L,R on the charge-conjugated fermion fields indicate $(1-\gamma_5)/2$ helicity projections, respectively, applied after charge conjugation. The ratio of electron mass to muon mass which follows from this Lagrangian after spontaneous symmetry breaking is

$$\frac{m_e}{m_\mu}=\epsilon\left(\frac{A^2-B^2}{A^2+B^2}\right)+\text{higher order in }\epsilon\,.$$

The case $\epsilon=0$ corresponds to $\phi_1 \to -\phi_1$ and $e_R \to -e_R$ symmetry, while $A=B$ corresponds to $E_L \leftrightarrow M_L$ symmetry.

This result is compatible with our quark model expressions when $\epsilon \ll 1$, and so $m_d/m_c = m_u/m_s \sim m_e/m_\mu$. The decay $\mu^- \to e^-e^+e^-$ can proceed by neutral-Higgs-boson exchange, but the branching ratio is several orders of magnitude less than the 10^{-9} experimental value when $M_H \geq M_W$.

An $SU(2)\times U_B(1)\times U_C(1)$ (Ref. 8) scheme of weak and electromagnetic interactions of light quarks and leptons has emerged from our considerations of quark and lepton mass ratios. It has the same neutrino neutral-current interactions as the standard model. This is guaranteed by the theorem of Georgi and Weinberg, which states that "at zero momentum transfer the neutral-current interactions of any fermion f^0 (ν_L in our case) which is both electrically neutral and neutral under G_2 [$U_B(1)$ in our case] will be precisely the same as if the gauge group were just $G_1\times U(1)$ [$SU(2)\times U_C(1)$ in our case] and broken only by $\langle\phi_1\rangle$" (a role played by both $\langle\phi_1\rangle$ and $\langle\phi_2\rangle$ in the present case).[9]

Our model has the further property that the axial-vector current of the quarks and leptons does not contain $U(1)_B$. The axial-vector current is not mixed with the photon either, so the parity-violating $V\cdot A$ neutral-current interference terms of all fermion fields, charged and neutral, satisfy the Georgi-Weinberg conditions, and parity-violation effects in atoms and in polarized-electron–deuteron scattering are also the same as in the standard model. This feature of the $q^2=0$ neutral-current phenomenology is identical to that of a class of $SU(2)\times U(1)\times U(1)$ models discussed by Deshpande and Iskandar, designated case a in their paper.[10] The parameter which plays the role of the Weinberg-Salam-model parameter $\sin^2\theta_W$ is

$$\sin^2\theta = q_1^2q_2^2/(q_1^2+q_2^2+q_1^2q_2^2) \leftrightarrow \sin^2\theta_W\,. \qquad (10)$$

This quantity is the square of the mixing coefficient between the photon field A and the V_3 field of the gauge triplet, Eq. (7).

To summarize the foregoing analysis, we see that the condition

$$\frac{m_d}{m_c}\simeq\frac{m_u}{m_s}\ll 1\,,$$

a relationship between ratios of quarks of different charge, requires an extra U(1) gauge group which is coupled to electromagnetism. We find that the simplest choice of scalar fields and their vacuum expectation values which provides masses for all gauge fields except the photon then leaves the neutral-current results of the $SU(2)\times U_C(1)\times U_B(1)$ model completely identical to those of the standard model. Next we discuss the effects due to heavy quarks and leptons in the above scheme.

HEAVY QUARKS, THE τ, AND NEUTRAL-CURRENT PARAMETERS

The τ lepton doublet[11] and heavy quarks, whose presence is signaled by the Υ and Υ',[12] can be incorporated with no substantial change of our previous discussion if a separate Higgs-particle doublet

$$\phi_3=\begin{pmatrix}\phi_3^0\\ \phi_3^-\end{pmatrix}$$

is introduced. It is coupled only to the SU(2) gauge bosons and to the $U_B(1)$ gauge boson

$$D_\mu\phi_3=\left[\partial_\mu - ig\left(V_\mu-\frac{q_1}{\sqrt{2}}B_\mu\right)\right]\phi_3\,,$$

and therefore it cannot interact with the light quarks and leptons. A heavy quark doublet $(D_3)_L$ and quark singlets t_R and b_R of charge $\frac{2}{3}$ and $-\frac{1}{3}$, respectively, can then be coupled to ϕ_3. Similarly, a lepton doublet T_L and a singlet τ_R can accommodate the τ and its neutrino.[13] The Yukawa couplings of ϕ_3 generate the masses of the heavy quarks and the τ after spontaneous breakdown. One can naturally obtain the hierarchy $\langle\eta\rangle_0>\langle\phi_3\rangle_0>\langle\phi_2\rangle_0 \gg\langle\phi_1\rangle_0$ in the spin-zero-meson potential.

The modifications to the neutral-current interactions of the light quarks due to the presence of ϕ_3 are best presented by showing the explicit forms for the neutrino and electron neutral-current interaction coefficients. We expect deviations from the results of the standard model, since ϕ_3 transforms nontrivially under both the SU(2) group and the $U_B(1)$ group.[7] The group $U_C(1)$, under which the ϕ_1 and ϕ_2 fields coupled to light quarks transform, plays the role of the U(1) discussed in Ref. 7, while $U_B(1)$ plays the role of G_2.

We adopt one of several commonly used notations for the neutral-current interactions to write

$$\begin{aligned}\mathcal{L}_{\rm NC}(\nu)=\frac{G}{\sqrt{2}}\bar\nu\gamma_\mu(1-\gamma_5)\nu\\ \times[u_L\bar u\gamma^\mu(1-\gamma_5)u+u_R\bar u\gamma^\mu(1+\gamma_5)u\\ +d_L\bar d\gamma^\mu(1-\gamma_5)d+d_R\bar d\gamma^\mu(1+\gamma_5)d]\end{aligned}$$

for the neutrino interactions with quarks,[14] and

$$\begin{aligned}\mathcal{L}_{\rm NC}^{\rm PV}(e)=\frac{G}{\sqrt{2}}(c_{1_u}\bar e\gamma_\mu\gamma_5 e\bar u\gamma^\mu u+c_{2_u}\bar e\gamma_\mu e\bar u\gamma^\mu\gamma_5 u\\ +c_{1_d}\bar e\gamma_\mu\gamma_5 e\bar d\gamma^\mu d+c_{2_d}\bar e\gamma_\mu e\bar d\gamma^\mu\gamma_5 d)\end{aligned}$$

for the $V\cdot A$ interference which produces parity violation in electron interaction with quarks.[15] For the model expanded to include heavy fermions and the field ϕ_3, we find for neutrino parameters

$$\begin{aligned}u_R&=-a-2b=\begin{cases}-0.19\pm0.06\\ -0.235\pm0.046,\end{cases}\\ u_L&=c-2b=\begin{cases}0.35\pm0.07\\ 0.286\pm0.069,\end{cases}\\ d_R&=a+b=\begin{cases}0.0\pm0.11\\ 0.0\pm0.169,\end{cases}\\ d_L&=-c+b=\begin{cases}0.40\pm0.07\\ -0.397\pm0.04,\end{cases}\end{aligned}\tag{11}$$

where the top value in each brace is that of Abbott and Barnett,[14] the bottom that of Langacker and Sidhu.[14] We find for the electron parity-violation parameters the expressions

$$\begin{aligned}c_{1u}&=c-a-4b,\quad c_{2u}=c-3a-5b-\tfrac{2}{3}s_\theta^{\,2}c\,,\\ c_{1d}&=-c+a+2b,\quad c_{2d}=-c_{2u}\,.\end{aligned}\tag{12}$$

Detailed expressions for a, b, c, and $s_\theta^{\,2}=\sin^2\theta$ are given in the Appendix. The parametrization of the standard model is recaptured by the limit $\langle\phi_3\rangle_0/\langle\eta\rangle_0=0$, which implies that $a=0$ and $b=\frac{1}{3}\sin^2\theta$, while $c=\frac{1}{2}$. When $\langle\phi_3\rangle_0/\langle\eta\rangle_0\to0$, the conditions are met which ensure the same neutrino-scattering parametrization as in the standard model.[5,7] The replication of the electron parity violation parametrization follows as a corollary, since the axial-vector current of the fermions is neutral under $U_B(1)$, which is the G_2 of Georgi and Weinberg.[7]

As is clear from the above discussion in reference to Eqs. (11) and (12), we can closely approximate the neutral-current results of the standard model by choosing a to be small. It is of interest, however, to examine the neutral-current parameters for a variety of values of a, b, c, and $s_\theta^{\,2}$. Values of u_R, d_R, u_L, and d_L have recently been extracted from hadronic neutral-current data.[14] Parity violation in electron-deuteron scattering and in optical transitions in bismuth depends upon the parameters

$$\begin{aligned}C&\equiv(c_{1u}-\tfrac{1}{2}c_{1d})+(c_{2u}-\tfrac{1}{2}c_{2d})(1-y)^2/(1+y)^2\\ &=1.845c-2.535a-6.725b-0.23cs_\theta^{\,2}\end{aligned}\tag{13}$$

and

$$\begin{aligned}Q_w&=584(c_{1u}+1.15c_{1d})\\ &=-584[0.15(c-a)+1.70b]\,,\end{aligned}\tag{14}$$

where $(1-y)^2/(1+y)^2=0.23$ at the value $y=0.21$.[16] For the parameter C, the experiment at SLAC[16] determines

$$C=0.445\pm0.075\,,\tag{13$'$}$$

while three different values of Q_w are found by

TABLE I. The experimental values of weak-neutral-current quantities and the Weinberg-Salam-model values for various choices of $\sin^2\theta_W$. This corresponds to the case $a=0$, $b=(\sin^2\theta)/3$, $c=\frac{1}{2}$ of our model.

				Experiment				
u_R	d_R	u_L	d_L	$C(y=0.21)$	Q_w	C_V	C_A	
					0 ± 20 [c]			
-0.19 ± 0.06 [a]	0.0 ± 0.11	0.35 ± 0.07	-0.40 ± 0.07	0.445 ± 0.075	-39 ± 7 [d]	0.20 ± 0.15	-0.50 ± 0.08	
-0.235 ± 0.046 [b]	0.0 ± 0.169	0.286 ± 0.069	-0.397 ± 0.046		-120 ± 40 [e]			
				Theory				
$\sin^2\theta=3b$	u_R	d_R	u_L	d_L	$C(y=0.21)$	Q_w	C_V	C_A
0.15	−0.10	0.05	0.40	−0.45	0.57	−93	−0.20	−0.5
0.18	−0.12	0.06	0.38	−0.38	0.499	−103	−0.14	−0.5
0.21	−0.14	0.07	0.36	−0.43	0.428	−113	−0.08	−0.5
0.24	−0.16	0.08	0.34	−0.42	0.358	−123	−0.02	−0.5
0.30	−0.20	0.10	0.30	−0.40	0.216	−143	+0.10	−0.5

[a] Values of Abbott and Barnett, Ref. 13.
[b] Values of Langacker and Sidhu, Ref. 13.
[c] Values of Washington experiment, Ref. 16.
[d] Value of Oxford experiment, Ref. 17.
[e] Value of Novosibirsk experiment, Ref. 18.

three different experiments:

$$Q_w=0\pm20,\quad \text{U. Washington}^{17} \tag{14'}$$

$$Q_w=-34\pm7,\quad \text{Oxford U.}^{18} \tag{14''}$$

$$Q_w=-120\pm40,\quad \text{Novosibirsk}^{19}\,. \tag{14'''}$$

Finally, the scattering of muon neutrino and antineutrino off electrons depends upon the parameters

$$c_V=3a+6b-c\,, \tag{15a}$$

$$c_A=-a-c\,. \tag{16a}$$

Recent experimental values are[20]

$$c_V=0.20\pm0.15\,, \tag{15b}$$

$$c_A=-0.5\pm0.08\,, \tag{16b}$$

and, by folding in $\bar{\nu}_e e$ scattering results,[21] the values

$$c_V=0.09\pm0.07\,,$$

$$c_A=-0.51\pm0.07$$

are quoted by the authors of Ref. 20.

In Table I we show values for the various quantities in the Weinberg-Salam model case $a=0$, $b=(\sin^2\theta)/3$, $c=\frac{1}{2}$ for a set of b values. In Table II selected values of a, b, and c and the corresponding values of the experimentally accessible quantities u_R, d_R, u_L, d_L, Q_w, C, c_V, and c_A are shown. The last column shows the fractional change in C as y changes from 0 to 1. The variations are comparable to those of the standard model for the range $0.15<\sin^2\theta_W<0.30$.

TABLE II. The values of experimentally accessible quantities and values for some choices of a, b, and c in the model when ϕ_3 and heavy fermions are introduced. $a=0$ corresponds to the standard model parametrization and to our SU(2) ×U(1)×U(1) model with no ϕ_3 Higgs doublet. Note that all (a,b,c) entries except the seventh (0.17, 0, 0.35) are within one standard deviation of the SLAC value for C, the electron-scattering parameter, for an allowed range of $\sin^2\theta$. The expression for C in terms of a, b, c, and $\sin^2\theta$ is given in Eq. (A7), and the $C_{\min}$ and $C_{\max}$ values are explained there.

a	b	c	u_R	d_R	u_L	d_L	$C(y=0.21)$ min	max	Q_W	C_V	C_A	$\frac{[C(y=1)-C(y=0)]_{\max}}{C(y=0.21)_{\max}}$
0.02	0.05	0.49	−0.12	0.07	0.39	−0.42	0.497	0.500	−91	−0.13	−0.51	0.39
0.02	0.07	0.49	−0.16	0.09	0.35	−0.42	0.356	0.359	−110	−0.05	−0.51	0.04
0.04	0.04	0.47	−0.12	0.08	0.39	−0.43	0.477	0.484	−77	−0.09	−0.51	0.34
0.08	0.03	0.43	−0.14	0.11	0.37	−0.40	0.364	0.379	−60	−0.01	−0.51	0.04
0.11	0.01	0.42	−0.13	0.12	0.40	−0.39	0.404	0.426	−37	−0.03	−0.53	0.11
0.14	0	0.40	−0.14	0.14	0.40	−0.40	0.357	0.383	−23	+0.02	−0.54	−0.08
0.17	0	0.35	−0.17	0.17	0.35	−0.35	0.213	0.246	−15	+0.16	−0.52	−0.97
0.17	−0.02	0.36	−0.13	0.15	0.40	−0.38	0.340	0.363	+3	+0.03	−0.53	−0.12

By comparing entries in Table II with those in Table I we notice that there are choices of the values of a, b, c and a range of values of $\sin^2\theta$ which give agreement with experiment comparable to that of the standard model with $0.18 < \sin^2\theta_W < 0.24$, but which yield Q_w values which are small in magnitude, close to the values of the Washington[17] and/or Oxford[18] measurements. The standard Weinberg-Salam model agrees, of course, only with the Novosibirsk[19] value. The values in Table II also indicate that, just as in the one-parameter model, an increase in the value of c_V into the range 0.1 to 0.2 tends to reduce the value of C, the electron-asymmetry parameter, to unacceptably low values. Not surprisingly, the three-parameter fit allows more leeway.

As the foregoing discussion shows, a parametrization of neutral-current effects which has a natural limit to the one-parameter description of the standard model can nonetheless yield a wide range of values of Q_w while still being in agreement with the electron-scattering asymmetry result. With the present status of the data, we feel that the one-parameter description is still tentative—perhaps an approximate picture with substantial corrections. Improvement in all neutral-current data is needed to narrow the range of possibilities.

CONCLUSIONS AND SUMMARY

We have proposed that the interesting phenomenological relationship obtained in several current-algebra analyses

$$\frac{m_u}{m_d} \lesssim \tfrac{1}{2}, \quad \frac{m_d}{m_s} \lesssim \tfrac{1}{20}$$

suggests a pattern:

$$\frac{m_u}{m_s} \approx \frac{m_d}{m_c} \ll 1 .$$

Implementation of this unorthodox relationship between ratios of masses of quarks which have different charges led us to an $SU(2)\times U(1)\times U(1)$ extension of the standard model. When only light quarks are considered, the simplest scheme which provides masses for all gauge bosons except the photon leads to neutral-current effects identical to those of the standard model because the conditions of the Georgi-Weinberg theorem[7] are satisfied for the neutrino, and a corollary to the theorem applies to the axial couplings of electron and quarks.

A simple extension of the model to include heavy quarks and leptons leads to a modification of the neutral-current results. The change is small if the vacuum expectation value of the spin-zero field ϕ_3, which couples only to heavy quarks and leptons, is small compared to the vacuum expectation value of a singlet scalar field η. The neutral current depends upon four correlated parameters when ϕ_3 is included. As displayed in Table II, there are different choices of parameters which give satisfactory agreement with neutrino scattering and electron scattering and with any of the three parity-violation results in bismuth.[17-19] The parametrization satisfies

$$u_R + d_R = u_L + d_L = \tfrac{1}{2}(c_{1u} + c_{1d})$$

as in the standard Weinberg-Salam case and it has a natural limit (when $\langle\phi_3\rangle/\langle\eta\rangle \to 0$) to the standard-model one-parameter description. One might expect that agreement with neutrino and electron scattering would determine values of parameters which produce agreement only with the Serphukov value. This is not the case.

Problems which we have not addressed are CP violation and the decay of hadrons which are composed of heavy quarks. It is possible that these questions are related, as exemplified by a model recently investigated by us.[22] Our focus in the present paper is the interesting connection between quark mass ratios, the extra U(1) gauge symmetry, and neutral-current phenomenology, and so we have not taken up at this point the detailed questions of CP nonconservation and heavy-hadron spectroscopy.

APPENDIX

The parameters a, b, c, and $s_\theta{}^2 = \sin^2\theta$ are related to gauge coupling constants and vacuum expectation values in the following way:

$$a = \frac{\sin^2\alpha}{\lambda} c , \tag{A1}$$

$$b = \tfrac{2}{3}c\left(\sin^2\theta - \frac{\sin^2\alpha}{\lambda}\,\frac{q_1{}^2}{(q_1{}^2 + q_2{}^2 + q_1{}^2 q_2{}^2)}\right), \tag{A2}$$

$$c = \tfrac{1}{2}\left(1 + \frac{\sin^2\alpha}{\lambda}\cos^2\alpha\right)^{-1} , \tag{A3}$$

where

$$\lambda = g^2\langle\eta\rangle_0^2/M_w{}^2 ,$$

$$\cos^2\alpha = g^2(\langle\phi_1\rangle_0^2 + \langle\phi_2\rangle_0^2)/M_w{}^2 , \tag{A4}$$

$$\sin^2\alpha = g^2\langle\phi_3\rangle_0^2/M_w{}^2 , \tag{A5}$$

$$\sin^2\theta = q_1{}^2 q_2{}^2/(q_1{}^2 + q_2{}^2 + q_1{}^2 q_2{}^2) . \tag{A6}$$

As remarked in the text, $a = 0$ reproduces the parametrization of the standard model. For the neutrino couplings, this is just the result of Ref. 6, since a $\alpha\langle\phi_3\rangle_0^2/\langle\eta\rangle_0^2$, and ϕ_3 breaks SU(2) and $U_B(1)$. The fact that the electron parity violation becomes identical to the standard model follows because the axial-vector electron and quark neutral currents couple only to C and not to B. This is a generalization of the result of Ref. 6.

Regarding parametrizations based on Eqs. (A1), (A2), (A3), and (A6), we note that values of the parameters are correlated, and that $a \geq 0$, $c \geq 0$, and $1 \geq \sin^2\theta \geq 0$ must be maintained.

Moreover, because $q_1{}^2$ and $q_1{}^2$ are positive, we have also that

$$3b/2c < \sin^2\theta < (a + \tfrac{3}{2}b)/(a+c)\ .$$

The minimum and maximum values of $C(y=0.21)$ in Table II then follow from this restriction since

$$C(y=0.21) = 1.845c - 6.725b - 2.535a - (0.23c)\sin^2\theta\ . \tag{A7}$$

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[1]J. Gasser and H. Leutwyler, Nucl. Phys. B94, 269 (1975); P. Langacker and H. Pagels, Phys. Rev. D 19, 2070 (1979); S. Weinberg, in *A Festschrift for I. I. Rabi*, edited by L. Motz (New York Academy of Sciences, New York, 1977).

[2]A. Zepeda, Phys. Rev. Lett. 41, 139 (1978); N. Deshpande and D. Soper, *ibid.* 41, 735 (1978). See, however, C. A. Dominguez, *ibid.* 41, 605 (1978), who argues that $m_u = 0$ is unsatisfactory because large violations of SU(3) and SU(2) would result from the analysis of Zepeda and of Deshpande and Soper, respectively.

[3]G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976); R. Jackiw and C. Rebbi, *ibid.* 37, 172 (1976); C. G. Callan, R. G. Dashen, and D. J. Gross, *ibid.* 63B, 334 (1976); R. Peccei and H. Quinn, Phys. Rev. D 16, 1791 (1977).

[4]Using chiral SU(4) ×SU(4), K. P. Das and N. G. Deshpande [Phys. Rev. D 19, 3387 (1979)] estimate $m_s/m_c \simeq 0.31$. Other estimates include $m_s/m_c \simeq 0.26$ by H. Georgi and H. D. Politzer [Phys. Rev. D 14, 1829 (1976)] and a similar value obtained by C. L. Ong [*ibid.* 16, 835 (1977)], and $m_s/m_c \simeq 0.13$ by S. Weinberg (Ref. 1), who remarks that "the question of the mass of the c quark appears at this time to be surrounded with confusion." Values of m_c have been found to be in the range 1 to 2 GeV. For example, a low value $m_c = 1.15$ GeV is quoted in the study of T. A. DeGrand, Y.J. Ng, and S. -H. H. Tye [Phys. Rev. D 16, 3251 (1977)] a classic value from $\Delta S = 2$ suppression studies of M. K. Gaillard and B. W. Lee [*ibid.* 10, 897 (1974)] is $m_c = 1.5$ GeV, and a larger value from mass fitting with a nonrelativistic potential is $m_c = 1.6$ by E. Eichten *et al.* [Phys. Rev. Lett. 34, 369 (1975)]. Relativistic treatments lower m_c estimates to ~ 1.1 GeV in potential models. Values of m_s from constituent quark estimates are generally in the neighborhood of 500 MeV, as in L. Schachinger *et al.* [Phys. Rev. Lett. 41, 1348 (1978)]. The extent of renormalization to achieve the bare (or current quark) value is not known, but is presumed to be significantly larger than for the m_c case.

[5]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1971); A. Salam in *Elementary Particle Theory: Relativistic Groups and Analyticity* (*Nobel Symposium No. 8*), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367. The sequential six leptons and six quarks generalization with the $\Delta S = 1$ neutral-current cancellation mechanism of S. L. Glashow, J. Iliopoulos, and L. Maiani [Phys. Rev. D 2, 1285 (1970)] will be referred to as the standard model.

[6]There is a tradeoff in the sense that we lose the naturalness of $\Delta S = 1$ suppression in the neutral-Higgs-particle couplings to quarks. These couplings are small, however, and for Higgs particles whose masses are comparable to those of the gauge bosons there is no difficulty with $\Delta S = 1$ processes. Likewise, $\mu \rightarrow 3e$ is negligible in the lepton sector, where $M_H \gtrsim M_W$ is assumed.

[7]H. Georgi and S. Weinberg, Phys. Rev. D 17, 275 (1978).

[8]C and B designate the gauge fields associated with the respective U(1) symmetries.

[9]Our ϕ_1 and ϕ_2 couple only to SU(2) and $U_c(1)$, not to $U_B(1)$. These fields should not be confused with the ϕ_1 and ϕ_1 of Ref. 6, where ϕ_1 transforms under G_1 and ϕ_2 under G_2.

[10]N. Deshpande and D. Iskandar, Phys. Rev. Lett. 42, 20 (1979).

[11]M. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975); J. Kirby, in *Neutrinos—78*, proceedings of the International Conference for Neutrino Physics and Astrophysics, Purdue, 1978, edited by E. C. Fowler (Purdue Univ. Press. West Lafayette, Indiana, 1978), p. 361; G. J. Feldman, *ibid*, p. 647.

[12]S. W. Herb *et al.*, Phys. Rev. Lett. 39, 252 (1977); W. R. Innes *et al.*, *ibid* 39, 1240 (1977); C. W. Darden *et al.*, Phys. Lett. 76B, 246 (1978); Ch. Berger *et al.*, *ibid.* 76B, 243 (1978); J. K. Bienlein *et al.*, *ibid.* 78B, 360 (1978); C. W. Darden *et al.*, *ibid.* 78B, 364 (1978).

[13]Additional leptons and/or quarks beyond b, t, τ, and ν_τ should be introduced in order to cancel anomalies in vertices with three C fields or with two C fields and one B field. These anomalies arise in the light-quark and lepton currents. The B current anomalies owing to heavy quarks cancel against those owing to heavy leptons as in the standard model assignments for t, b, τ, and ν_τ. Therefore, additional heavy fermions coupled to C and B are needed to cancel the light-quark anomalies. We are interested only in the immediate consequences of light-quark mass ratios and neutral-current parameters, and we do not elaborate on this theme. The question of anomalies in SU(2) ×U(1) ×U(1) models has recently been discussed by K. Kang and J. E. Kim, Phys. Rev. D 18, 3467 (1978).

[14]P. Q. Hung and J. J. Sakurai, Phys. Lett. 72B, 208 (1977); L. M. Sehgal, *ibid.* 71B, 99 (1977); P. Scharbach, Nucl. Phys. B82, 155 (1974); L. F. Abbott and R. M. Barnett, Phys. Rev. Lett. 40, 1303 (1978) and Phys. Rev. D 18, 3214 (1978); P. Langacker and D. Sidhu, Phys. Rev. Lett. 41, 732 (1978).

[15]R. N. Cahn and F. J. Gilman, Phys. Rev. D 17, 1313 (1978).

[16]C. V. Prescott *et al.*, Phys. Lett. B 77B, 347 (1978).

[17]L. L. Lewis *et al.*, Phys. Rev. Lett. 39, 795 (1977);

N. Fortson, in *Neutrinos—78* (Ref. 11), p. 417.
[18]P. Baird *et al.*, Phys. Rev. Lett. 39, 798 (1977) and, More recently, P. G. H. Sandars, in *Proceedings of the XIX International Conference on High Energy Physics Tokyo, 1978* edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, 1979).
[19]L. M. Barkov and M. Zolotorev, Zh. Eksp. Teor. Fiz. Pisma Red. 27, 379 (1978); [JETP Lett. 27, 357 (1978)] and *Neutrinos–78* (Ref. 11), p. 423. A treatment of SU(2) × U(1) U(1) and atomic parity violation is given by Deshpande and Iskandar (Ref. 10).
[20]H. Faissner *et al.*, in *Neutrinos—78* (Ref. 11), p. 387.
[21]F. Reines *et al.*, Phys. Rev. Lett. 37, 315 (1976).
[22]D. W. McKay and H. J. Munczek, Phys. Rev. 19, 997 (1979).