$B-L$ as the fourth color within an $SU(2)_L \times U(1)_R \times U(1)$ model

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Quark-lepton correspondence and the corresponding interpretation of $B-L$ as the fourth color are combined in an $SU(2)_L \times U(1)_R \times U'(1)$ electroweak model. The right-handed neutrino interactions are automatically suppressed and the standard model is recovered up to a desirable first-order fine structure. A fundamental distinction is established between the real Weinberg angle and its effective (measured) values. For neutrino interactions, our predictions are in general of the Weinberg-Salam (WS) type, only with the effective replacement $\sin^2\theta_W \rightarrow \sin^2\theta_W^{\rm eff}$ such that (a) $\sin^2\theta_W^{\rm eff}$ (purely leptonic processes) $> \sin^2\theta_W^{\rm eff}$ (νq scattering), and (b) for $\sin^2\theta_W = 1/4$ one can still have a lower value for the weighted average of $\sin^2\theta_W^{\rm eff}$. Our expressions for the asymmetry parameter in electron-induced reactions agree with those of the WS model up to a second-order term. The additional neutral gauge boson is expected to be only 1 order of magnitude heavier than $W^{\pm}$. We argue that the present neutral-current experimental data are not capable of distinguishing between the standard and the present models. Finally we discuss the possibility of embedding our model in a unified gauge theory, showing that only spinorial $SO(4k+2)$ theories are available for this purpose.

I. INTRODUCTION

The present neutral-current experimental data seem to be in excellent agreement[1] with the predictions of the standard left-handed Weinberg-Salam (WS) model.[2] Nevertheless, the door is still open for the effective low-energy electroweak symmetry to be of the form $SU(2)_L \times U(1) \times G$ where the extra group G is expected to provide a fine structure to the original WS theory. It has been recently proven[3] that at zero momentum transfer the neutral-current interactions of neutrinos in such as $SU(2)_L \times U(1) \times G$ gauge theory are exactly the same as in the standard model if the neutrinos stay neutral under G and if there are no Higgs fields which transform nontrivially under both $SU(2)_L$ and G. In this paper however, we would like to discuss the simplest model which does not fall into the category of the above theorem but is still capable of reproducing the standard WS model up to a desirable first-order substructure.

Our present model satisfies the so-called quark-lepton correspondence.[4] This means that the two fundamental families of fermions, i.e., quarks and leptons, have one and the same structure under the flavor group. Thus they can be distinguished only by an additional quantum number which plays the role of the fourth color of Pati-Salam.[5] We note that such a quark-lepton correspondence is intrinsically violated in the standard WS model. First of all, the WS U(1) generator has eigenvalues in two different sequences, namely $(\cdots 1, 0, \cdots)$ and $(\cdots \frac{1}{3}, -\frac{2}{3}, \cdots)$, for leptons and quarks, respectively. Second, the missing right-handed neutrino ν_R leaves us without a leptonic analog for u_R. However, having such a quark-lepton correspondence does not necessarily mean having a left-right symmetry as well. The door is still open for the low-energy electroweak symmetry to be of the form[6] $SU(2)_L \times U(1)_R \times U'(1)$, which is a reasonable compromise between $SU(2)_L \times SU(2)_R \times U(1)$ and the standard $SU(2)_L \times U(1)$. The present model is therefore the minimal extension of the Weinberg-Salam model where the principal Pati-Salam idea is expressed.

The quark-lepton correspondence forces us to deal with right-handed neutrinos as well. Consequently, there are three gross features which we would like to see within the $SU(2)_L \times U(1)_R \times U'(1)$ model:

(1) *The masslessness of neutrinos*. Such a desired property does not emerge naturally in our model, and hence must be imposed. The same situation occurs in various $SU(2)_L \times SU(2)_R \times U(1)$ models as well. However, the present model does not involve $W_R^{\pm}$-like gauge bosons, and therefore we expect the masslessness property of the neutrinos to stay stable against first-order loop corrections once the relevant Yukawa coupling is chosen to be zero.

(2) *The suppression of right-handed neutrino interactions*. Within our model, the right-handed fermions do not have charge-current interactions at all, but there is *a priori* no fundamental reason why some of the neutral-current interactions should be suppressed. However, it turns out that ν_R, as an $SU(2)_L$ singlet with zero electric charge, has quantum numbers (QN's) which are proportional to those of some Higgs singlet whose large vacuum expectation value (VEV) is responsible for the primary stage of spontaneous symmetry breaking (SSB). This automatically guarantees the suppression of the neutral-current interactions as-

sociated with right-handed neutrinos in the stage where the standard $SU(2)_L \times U(1)$ is recovered.

(3) *The emergence of an effective WS-type model.* The deviation of the present model from the WS one is best described by a positive dimensionless parameter ϵ, which is the ratio of the only two independent VEV's in the model. When ϵ approaches zero, the exact WS model is obtained in all respects. However, we argue that the present neutral-current data allow ϵ to be as large as 0.04, giving rise to a first-order deviation from the standard model. In fact, the corresponding fine structure is the major prediction of our model.

The proposed $SU(2)_L \times U(1)_R \times U'(1)$ model furthermore establishes a fundamental difference between the real Weinberg angle and its effective values. In fact, we argue that those effective quantities are the ones which are actually measured. As far as neutrino interactions are concerned, the predictions of our model are in general of the WS type, only with the effective replacement $\sin^2\theta_W \to \sin^2\theta_W^{\text{eff}}$. It is to say that in order to derive cross sections and other physical quantities, one can simply use the corresponding WS expressions and replace $\sin^2\theta$ by its effective values. However, we show that our model differs from the standard one by predicting different such "effective Weinberg angles" for different neutrino interactions. For $\sin^2\theta_W = \frac{1}{4}$ and $\sin^2\zeta = \frac{2}{3}$ (ζ is an additional mixing angle), θ_W coincides with θ_W^{eff} for purely leptonic processes. However, in this case the door is still open for the average measured value of $\sin^2\theta_W^{\text{eff}}$ to be less than $\frac{1}{4}$ because $\sin^2\theta_W^{\text{eff}}$ acquires the values $\frac{1}{4} - \frac{1}{4}\epsilon$ and $\frac{1}{4} - \epsilon$ for νu and νd scattering processes, respectively. This gives us a good reason to try explaining the present neutral-current data with $\sin^2\theta_W$ having the exact "theoretical" value of $\frac{1}{4}$. The primary motivation is of course that the present weighted average 0.23 for $\sin^2\theta_W^{\text{eff}}$ actually contains some negative first-order contributions due to $\epsilon \neq 0$.

An interesting situation occurs when the asymmetry parameter is calculated for polarized electron-quark scattering processes.[7] It turns out that the first-order contributions ($\sim\epsilon$), due to Z and Z' (Z' is the additional neutral gauge boson) exchanges, cancel each other. Thus the corresponding predictions of our model agree with those of the WS model up to a second-order term. For small ϵ and at low momentum transfer, the measured asymmetry in $ed \to eX$ cannot be used to distinguish between the $SU(2)_L \times U(1)_R \times U'(1)$ and $SU(2)_L \times U(1)$ models.

In the last section we discuss the possibility of embedding our $SU(2)_L \times U(1)_R \times U'(1)$ model in a unified gauge theory. The idea of having quark-lepton correspondence turns out to be a very restrictive one. It requires the same flavor structure for all the colored components. In addition, we want the $U'(1)$ factor which distinguishes 1^c from 3^c to emerge simultaneously with $SU(3)^{\text{col}}$ in the same stage of SSB and by the same mechanism. Among all available simple Lie groups,[8] only the orthogonal ones are shown to be capable of giving rise to a unified theory which does not contradict any of those restrictions. If we furthermore insist on having an anomaly-free flavor-chiral theory, we are finally led to spinorial $SO(4k+2)$-based theories.[9]

II. BASIC CONSEQUENCES OF QUARK-LEPTON CORRESPONDENCE

Within the effective low-energy electroweak symmetry $SU(2)_L \times U(1) \times U'(1)$ we assign the following QN's to the fundamental fermions:

	ν_L	e_L	e_R	ν_R	u_L	d_L	d_R	u_R	
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	
Y	a	a	b	c	a	a	b	c	
Y'	l	l	l	l	q	q	q	q ,	(2.1)

where the explicit values for a, b, c, l, and q are to be determined. The same holds of course for fermions of higher generations. The above classification is the minimal way to express the idea of quark-lepton correspondence. Following such a viewpoint, the two fundamental families of fermions have one and the same substructure under $SU(2)_L \times U(1)$, and thus can be distinguished only by their different $U'(1)$ QN's. Furthermore, such a quark-lepton correspondence forces us to introduce the right-handed neutrino ν_R into the model as the leptonic analog of u_R.

The "standard charge assignment," where the quark charges are restricted to $\frac{2}{3}$ and $-\frac{1}{3}$ and the lepton charges to 0 and -1, is primarily responsible for fixing the QN's of the fundamental fermions up to a single leftover parameter. Using the conventional definition of the electric charge

$$Q = T_3 + \tfrac{1}{2}(Y + Y')\,, \tag{2.2}$$

we obtain

$$a = -1 - l, \quad b = -2 - l, \quad c = -l, \quad q = l + \tfrac{4}{3}\,. \tag{2.3}$$

Now we assume that $SU(2)_L \times U(1)$ is actually a subgroup of some flavor group G^{fl}. For the sake of the present paper, it does not really matter what this bigger group is as long as it gives rise to the restriction that the sum over all the flavor QN's of the leptons (and therefore of the quarks) vanishes. This simply tells us that

$$2a + b + c = 0\,, \tag{2.4}$$

and we are led to

$$l=-1,\quad q=\tfrac{1}{3},\quad a=0,\quad b=-1,\quad c=1. \tag{2.5}$$

Especially notice that we cannot allow the U′(1) factor to emerge from G^{fl} too, since this would lead to the undesirable result $q=l=0$. However, the door is open for this extra U′(1) subgroup to emerge from a modified color group G^{col}, such that $G^{col}\supset SU(n)^{col}\times U'(1)$. Thus the present model turns out to be the minimal extension of the WS model where the principal Pati-Salam idea[5] is expressed.

Now we argue that the above discussion gives rise to the following far-reaching conclusions:

(1) *Exactly three colors are available for quarks*. Suppose that quarks appear in n different colors. Within G^{col} which contains $SU(n)^{col}\times U'(1)$ this means that the restriction

$$nq+l=0 \tag{2.6}$$

must be satisfied. Thus having $l=-1$ and $q=\frac{1}{3}$, we must also have $n=3$. This result forcefully demonstrates how a pure property of the color sector is determined by information which comes from the flavor sector.

(2) *$B-L$ can be identified as the fourth color. A priori* Y' could express any arbitrary linear combination of the baryon and the lepton numbers. However, the only combination which is capable of predicting the above values (2.5) for l and q turns out to be

$$Y'=B-L\,. \tag{2.7}$$

We note that a similar operator generates the trivial group in various $SU(2)_L\times SU(2)_R\times U(1)$ electroweak models.

(3) *The WS hypercharge contains both flavor as well as color pieces*. It can be easily verified that the symmetric combination $Y+Y'$ generates the trivial U(1) group of the WS model. Alternatively speaking, the present model separates the WS hypercharge into two different components such that it contains both flavor and color contributions. Thus we expect a typical breakdown of $U(1)\times U'(1)$ into WS U(1) at the primary stage of SSB.

III. THE HIGGS STRUCTURE

No general principles appear to be available at present for the choice of the Higgs system. Nevertheless, one would like it to be: (a) the minimal one needed for conformity with observed phenomena, and (b) the most general one in the Yukawa sector. As far as the present model is concerned, the Higgs structure satisfies the above criteria. It turns out that the conventional Higgs doublet is involved in the procedure by which the fermions acquire their masses, while an additional Higgs singlet assures the emergence of the photon as the only massless gauge boson in the model.

The traditional doublet

$$\phi=\phi(\pm\tfrac{1}{2},-1,0) \tag{3.1}$$

is as usual the only Higgs multiplet (there exists always the possibility of having several such doublets) which has Yukawa couplings with the fundamental fermions. Its interaction with leptons is given by

$$\mathcal{L}_Y=\gamma_1^e\bar{e}_R(\phi_1 e_L-\phi_2\nu_L^e)+\gamma_2^e\bar{\nu}_R^e(\bar{\phi}_2 e_L+\bar{\phi}_1\nu_L^e) + (e\to\mu,\tau,\ldots)+\text{H.c.} \tag{3.2}$$

The VEV

$$\begin{pmatrix}\langle\phi_1\rangle\\ \langle\phi_2\rangle\end{pmatrix}=\begin{pmatrix}\lambda\\ 0\end{pmatrix}, \tag{3.3}$$

consistent with electric-charge conservation, would give mass to the neutrinos unless we assume $\gamma_2=0$. Thus some desirable symmetry principle which prevents the neutrinos from acquiring a mass is still badly needed. This is almost the same situation as in various $SU(2)_L\times SU(2)_R\times U(1)$ models. However, in the present model we do not have $W_R^\pm$-like gauge bosons at all, and therefore we expect the masslessness property of the neutrinos to stay stable against first-order loop corrections once we choose γ_2 to be zero. With this choice we are left with

$$m(e,\mu,\tau,\ldots)=\lambda\gamma_1^{e,\mu,\tau,\ldots},\quad m(\nu_e,\nu_\mu,\nu_\tau)\equiv 0\,. \tag{3.4}$$

In order that the photon would emerge as the only massless gauge boson in the present model it is necessary to introduce an additional Higgs multiplet. Following the principle of minimality in the Higgs system, this extra scalar field has to be an $SU(2)_L$ singlet. This choice is also favored by the fact that the previous Higgs doublet by itself is capable of inducing the SSB of WS $SU(2)_L\times U(1)$ down to $U(1)_{em}$. Thus we expect the so-called Φ field to be responsible for the primary descent of $SU(2)_L\times U(1)\times U'(1)$ into WS $SU(2)_L\times U(1)$. The only restriction is of course that Φ must be electrically neutral, i.e., $\Phi(0,p,-p)$. Hence Φ cannot give mass to fermions nor couple to a quark-lepton vertex. Furthermore, we turn attention to the fact that *the* QN's *of Φ are proportional to those of ν_R*. This is the first indication of a possible natural suppression of all neutral-current interactions which involve right-handed neutrinos, provided that $\langle\Phi\rangle$ far exceeds $\langle\phi\rangle$. To summarize this section, the hierarchical chain of the SSB outlined above is given below:

$$\mathrm{SU}(2)_L\times\mathrm{U}(1)\times\mathrm{U}'(1)\xrightarrow{\langle\Phi\rangle}\mathrm{WS}\ \mathrm{SU}(2)\times\mathrm{U}(1)\xrightarrow{\langle\phi\rangle}\mathrm{U}(1)_{\mathrm{em}} \tag{3.5}$$

Our only remark is that the exact WS model is expected to be realized only in the limiting case where $\langle\Phi\rangle\to\infty$.

IV. FIRST-ORDER DEVIATION FROM THE WS MODEL

Recalling from the last section, the relevant VEV patterns are

$$\langle\phi(\pm\tfrac{1}{2},-1,0)\rangle=\begin{pmatrix}\lambda\\0\end{pmatrix},\quad \langle\Phi(0,p,-p)\rangle=\frac{1}{p}\lambda'. \tag{4.1}$$

Consequently the charged gauge bosons acquire the WS mass

$$m^2(W^\pm)=\tfrac{1}{4}g^2\lambda^2, \tag{4.2}$$

while for the neutral ones we obtain the following (mass)^2 matrix:

$$M^2=\tfrac{1}{4}\begin{pmatrix}g^2\lambda^2 & -gG\lambda^2 & 0\\ -gG\lambda^2 & G^2(\lambda^2+\lambda'^2) & -GG'\lambda'^2\\ 0 & -GG'\lambda'^2 & G'^2\lambda'^2\end{pmatrix}, \tag{4.3}$$

g, G, and G' are the coupling constants associated with $\mathrm{SU}(2)_L$, U(1), and U′(1), respectively. The photon is given by

$$A=\cos\theta(\cos\zeta\,B+\sin\zeta B')+\sin\theta\,W^3, \tag{4.4}$$

where $\vec{W}$, B, and B' are the original gauge bosons and the mixing angles are defined by

$$g\tan\theta=G\cos\zeta=G'\sin\zeta. \tag{4.5}$$

Thus the connection with the standard WS model can be established via

$$B_{\mathrm{WS}}=\cos\zeta\,B+\sin\zeta\,B',\qquad \frac{1}{g_{\mathrm{WS}}'^2}=\frac{1}{G^2}+\frac{1}{G'^2}. \tag{4.6}$$

The neutral gauge fields with definite nonvanishing masses are

$$Z=\cos\delta\,Z_0-\sin\delta\,Z_0',\qquad Z'=\sin\delta\,Z_0+\cos\delta\,Z_0', \tag{4.7}$$

where

$$Z_0\equiv-\sin\theta(\cos\zeta\,B+\sin\zeta\,B')+\cos\theta\,W^3,\qquad Z_0'\equiv-\sin\zeta\,B+\cos\zeta\,B'. \tag{4.8}$$

The additional mixing angle δ is the only one which depends on the λ^2/λ'^2 ratio. It turns out that $\sin\zeta$ becomes zero for $\lambda'^2\to\infty$. Moreover, Z' becomes superheavy in this limit, and thus $\sin\delta$ actually measures the deviation from the WS model. In this point it is worth noting that the secular equation associated with the mass matrix (4.3) gives rise to the following mass formula:

$$m^2(W^\pm)-\cos^2\theta\,m^2(Z)=\frac{m^2(W^\pm)}{m^2(Z')}\left[\left(1+\frac{\tan^2\theta}{\cos^2\zeta}\right)m^2(W^\pm)-m^2(Z)\right]. \tag{4.9}$$

This explicitly shows how the traditional WS mass relation emerges when $m^2(Z')$ becomes infinite.

The above expressions clearly indicate that the positive dimensionless parameter ϵ defined by

$$\epsilon\equiv\lambda^2/\lambda'^2, \tag{4.10}$$

plays a major role in the $\mathrm{SU}(2)_L\times\mathrm{U}(1)\times\mathrm{U}'(1)$ model. When ϵ approaches zero we expect the present model to coincide with the standard one. This suggests that ϵ should be small (in fact, we show later that $\epsilon\sim0.04$) leading to a first-order deviation from the WS model. To the first order of ϵ, the gauge eigenfields are

$$W^\pm=(1/\sqrt{2})(W^1\pm iW^2),\quad m^2(W^\pm)=\tfrac{1}{4}g^2\lambda^2,$$
$$A=\cos\theta(\cos\zeta\,B+\sin\zeta\,B')+\sin\theta\,W^3,\quad m^2(A)=0, \tag{4.11}$$

$$Z\simeq Z_0-\epsilon\frac{\cos\zeta\sin^3\zeta}{\sin\theta}Z_0',\quad m^2(Z)\simeq(1-\epsilon\sin^4\zeta)\frac{m^2(W^\pm)}{\cos^2\theta},$$

$$Z'\simeq Z_0'+\epsilon\frac{\cos\zeta\sin^3\zeta}{\sin\theta}Z_0,\quad m^2(Z')\simeq\left(\frac{2\tan\theta}{\sin2\zeta}\right)^2\frac{m^2(W^\pm)}{\epsilon}.$$

These gauge fields are supposed to couple to the fundamental fermions of the theory through the following interactions:

$$\begin{aligned}-4i\mathcal{L}_I'&=\bar{\nu}_e\gamma(1-\gamma_5)\nu_e(gW^3-G'B')+\bar{e}\gamma(1-\gamma_5)e(-gW^3-G'B')\\&\quad+\bar{e}\gamma(1+\gamma_5)e(-GB-G'B')+\bar{\nu}_e\gamma(1+\gamma_5)\nu_e(GB-G'B')+\sqrt{2}g\bar{e}\gamma(1-\gamma_5)\nu_eW^++\text{H.c.}+(e\to\mu,\tau)\\&\simeq g\bar{\nu}_e\gamma(1-\gamma_5)\nu_e\left[\frac{1+\frac{1}{4}\epsilon\sin^22\zeta}{\cos\theta}Z-\frac{\tan\theta}{\tan\zeta}Z'\right]\\&\quad+g\bar{e}\gamma(1-\gamma_5)e\left[-2\sin\theta A+\frac{-\cos2\theta+\frac{1}{4}\epsilon\sin^22\zeta}{\cos\theta}Z-\frac{\tan\theta}{\tan\zeta}Z'\right]\\&\quad+g\bar{e}\gamma(1+\gamma_5)e\left[-2\sin\theta A+\frac{2\sin^2\theta+\epsilon\sin^2\zeta\cos2\zeta}{\cos\theta}Z-2\frac{\tan\theta}{\tan2\zeta}Z'\right]\\&\quad+g\bar{\nu}_e\gamma(1+\gamma_5)\nu_e\left[\epsilon\frac{\sin^2\zeta}{\cos\theta}Z-2\frac{\tan\theta}{\sin2\zeta}Z'\right]+\sqrt{2}g\bar{e}\gamma(1-\gamma_5)\nu_eW^++\text{H.c.}+(e\to\mu,\tau),\end{aligned} \tag{4.12}$$

$$\begin{aligned}
-4i\mathcal{L}'_q &= \bar{u}\gamma(1-\gamma_5)u(gW^3+\tfrac{1}{3}G'B') + \bar{d}\gamma(1-\gamma_5)d(-gW^3+\tfrac{1}{3}G'B') \\
&\quad + \bar{d}\gamma(1+\gamma_5)d(-GB+\tfrac{1}{3}G'B') + \bar{u}\gamma(1+\gamma_5)u(GB+\tfrac{1}{3}G'B') + \sqrt{2}g\bar{d}\gamma(1-\gamma_5)uW^+ + \text{H.c.} + \left[\begin{pmatrix}u\\d\end{pmatrix}\to\begin{pmatrix}c\\s\end{pmatrix},\begin{pmatrix}t\\b\end{pmatrix}\right] \\
&\simeq \tfrac{1}{3}g\bar{u}\gamma(1-\gamma_5)u\left(4\sin\theta A + \frac{3-4\sin^2\theta-\frac{1}{4}\epsilon\sin^2 2\zeta}{\cos\theta}Z + \frac{\tan\theta}{\tan\zeta}Z'\right) \\
&\quad + \tfrac{1}{3}g\bar{d}\gamma(1-\gamma_5)d\left(-2\sin\theta A + \frac{2\sin^2\theta-3-\frac{1}{4}\epsilon\sin^2 2\zeta}{\cos\theta}Z + \frac{\tan\theta}{\tan\zeta}Z'\right) \\
&\quad + \tfrac{1}{3}g\bar{d}\gamma(1+\gamma_5)d\left(-2\sin\theta A + \frac{2\sin^2\theta-\epsilon\sin^2\zeta(1+2\sin^2\zeta)}{\cos\theta}Z + \frac{\tan\theta}{\sin 2\zeta}(1+2\sin^2\zeta)Z'\right) \\
&\quad + \tfrac{1}{3}g\bar{u}\gamma(1+\gamma_5)u\left(4\sin\theta A - \frac{4\sin^2\theta+\epsilon\sin^2\zeta(1-4\sin^2\zeta)}{\cos\theta}Z + 2\frac{\tan\theta}{\sin 2\zeta}(1-4\sin^2\zeta)Z'\right) \\
&\quad + \sqrt{2}g\bar{d}\gamma(1-\gamma_5)uW^+ + \text{H.c.} + \left[\begin{pmatrix}u\\d\end{pmatrix}\to\begin{pmatrix}c\\s\end{pmatrix},\begin{pmatrix}t\\b\end{pmatrix}\right]. \qquad (4.13)
\end{aligned}$$

At this stage several remarks are to be made concerning the special structure of the above interactions:

(1) The neutral current associated with the right-handed neutrinos couples only to Z' and ϵZ. It is to say that the corresponding amplitudes are naturally suppressed to the level of ϵG_F. This is of course a most desirable result in a theory which contains right-handed neutrinos. In this model it arises automatically as a simple consequence of the fact that the QN's of the Higgs field Φ are proportional to those of ν_R.

(2) When ϵ approaches zero, the exact WS model is obtained in all respects. Z' acquires infinite mass, the interactions of the right-handed neutrinos are totally suppressed, and the coupling constants of Z take their conventional WS forms. Thus Eqs. (4.12) and (4.13) are of the form

$$\mathcal{L}' = \mathcal{L}'_{\text{WS}} + \epsilon\Delta\mathcal{L}', \qquad (4.14)$$

expressing a first-order deviation from the standard model.

(3) It is well known that the neutral current associated with electrons becomes a pure axial-vector current for the special case of $\sin^2\theta_W = \frac{1}{4}$ in the WS model. We note that the same can be achieved in the present model for both $\sin^2\theta = \frac{1}{4}$ and $\sin^2\zeta = \frac{2}{3}$. A few consequences of these special values of the mixing angles are mentioned later.

V. NEUTRINO INTERACTIONS

Our main task in this section is to show that as far as neutrino interactions are concerned, the predictions of our present model are in general of the WS type, only with the effective replacement $\sin^2\theta \to \sin^2\theta_{\text{eff}}$. However, we show that our model differs from the standard one by predicting a different such effective Weinberg angle for each neutrino reaction, separately. To see this fine-structure effect, first notice that laboratory neutrinos are produced by charged current interactions. Therefore, within the framework of the present model, those neutrinos must be purely left-handed. To proceed we separate our discussion into two parts.

A. Purely leptonic processes

Let us consider first the basic process $\nu_\mu e \to \nu_\mu e$. At low energy and at not too high momentum transfer, this reaction can be described by the following effective Lagrangian

$$\begin{aligned}
\mathcal{L}^{\text{eff}} &\sim g^2\frac{1+\frac{1}{4}\epsilon\sin^2 2\zeta}{\cos\theta}\bar{\nu}_\mu\gamma(1-\gamma_5)\nu_\mu\frac{1}{m^2(Z)}\left[\frac{-\cos 2\theta+\frac{1}{4}\epsilon\sin^2 2\zeta}{\cos\theta}\bar{e}\gamma(1-\gamma_5)e + \frac{2\sin^2\theta+\epsilon\sin^2\zeta\cos 2\zeta}{\cos\theta}\bar{e}\gamma(1+\gamma_5)e\right] \\
&\quad + g^2\frac{\tan\theta}{\tan\zeta}\bar{\nu}_\mu\gamma(1-\gamma_5)\nu_\mu\frac{1}{m^2(Z')}\left[\frac{\tan\theta}{\tan\zeta}\bar{e}\gamma(1-\gamma_5)e + 2\frac{\tan\theta}{\tan 2\zeta}\bar{e}\gamma(1+\gamma_5)e\right], \qquad (5.1)
\end{aligned}$$

which takes into account both Z and Z' exchange amplitudes. Substituting the explicit expressions for the gauge masses given by (4.11) into (5.1), using the familiar relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m^2(W^\pm)}, \qquad (5.2)$$

and keeping terms up to the first order of ϵ, the four-fermion effective Lagrangian takes the form

$$\mathcal{L}^{\text{eff}} = (G_F/\sqrt{2})\bar{\nu}_\mu\gamma(1-\gamma_5)\nu_\mu[g_V^e\bar{e}\gamma e - g_A^e\bar{e}\gamma\gamma_5 e], \qquad (5.3)$$

where

$$g_V^e = -\tfrac{1}{2} + 2\sin^2\theta + \epsilon(1-2\sin^2\zeta\cos^2\theta), \quad g_A^e = -\tfrac{1}{2}. \qquad (5.4)$$

We see that only the vector coupling gets affected by having $\epsilon \neq 0$, while the axial-vector one preserves its WS value. Moreover, the last expressions clearly indicate that the relevant quantity for the reaction $\nu_\mu e \to \nu_\mu e$ is not the original $\sin^2\theta$ but its effective version, i.e.,

$$\sin^2\theta_{eff} = \sin^2\theta + \tfrac{1}{2}\epsilon(1 - 2\sin^2\zeta\cos^2\theta). \tag{5.5}$$

In fact, this effective quantity is the one which is actually measured.

Now we compare the cross sections σ and σ' for the two processes $\nu_\mu e \to \nu_\mu e$ and $\bar\nu_\mu e \to \bar\nu_\mu e$, respectively. The ratio σ/σ' turns out to be

$$\frac{\sigma}{\sigma'} = \frac{16\sin^4\theta + 3(1 - 4\sin^2\theta)}{16\sin^4\theta + (1 - 4\sin^2\theta)} + \epsilon\frac{32\sin^2\theta(2\sin^2\theta - 1)}{[16\sin^4\theta + (1 - 4\sin^2\theta)]^2}(1 - 2\sin^2\zeta\cos^2\theta). \tag{5.6}$$

Thus for $\sin^2\theta = \frac{1}{4}$ one can still have $\sigma \neq \sigma'$. However, if it turns out that $\sigma = \sigma'$, this would imply $\sin^2\theta = \frac{1}{4}$ as well as $\sin^2\zeta = \frac{2}{3}$. Notice that these values are the ones for which the neutral current of electrons becomes a pure axial one. Furthermore, it is worth noting that for these special values of θ and ζ the effective Weinberg angle (5.5) becomes independent of ϵ.

At this point it is amusing to calculate the weak couplings $\bar g^e_{V,A}$ associated with a similar process which involves right-handed neutrinos rather than left-handed ones. Following similar calculations one finds that

$$\bar g^e_V = \epsilon(1 - 2\sin^2\zeta\cos^2\theta), \quad \bar g^e_A = 0. \tag{5.7}$$

Apart from suppressing the corresponding "right-handed" amplitude to a level of ϵG_F, these results unexpectedly prevent an effective current-current interaction of the form $J^\nu_R J^{e\dagger}_A$ from taking place.

B. νq elastic scattering

The weak effective coupling constants $g_{V,A}$ for the scattering processes $\nu u \to \nu u$ and $\nu d \to \nu d$ can be determined using the relevant coupling constants which are explicitly given by the interaction (4.13). It can be easily verified that these couplings acquire the following values:

$$g^u_V = \tfrac{1}{2} - \tfrac{4}{3}\sin^2\theta - \tfrac{1}{3}\epsilon(1 - 4\sin^2\zeta\cos^2\theta), \quad g^u_A = \tfrac{1}{2}$$
$$g^d_V = -\tfrac{1}{2} + \tfrac{2}{3}\sin^2\theta - \tfrac{1}{3}\epsilon(1 + 2\sin^2\zeta\cos\theta), \quad g^d_A = -\tfrac{1}{2}. \tag{5.8}$$

Again, in order to derive the relevant cross sections and other physical quantities, one can simply use the corresponding WS expressions and replace $\sin^2\theta$ by its effective values. For neutrino interactions those effective Weinberg angles are given explicitly by

$$\sin^2\theta_{eff} = \sin^2\theta + \begin{cases} \tfrac{1}{2}\epsilon(1 - 2\sin^2\zeta\cos^2\theta) & \text{for } \nu_\mu e \to \nu_\mu e \\ \tfrac{1}{4}\epsilon(1 - 4\sin^2\zeta\cos^2\theta) & \text{for } \nu u \to \nu u \\ -\tfrac{1}{2}\epsilon(1 + 2\sin^2\zeta\cos^2\theta) & \text{for } \nu d \to \nu d. \end{cases} \tag{5.9}$$

The differences between these effective values, namely

$$4(\sin^2\theta^{\nu e}_{eff} - \sin^2\theta^{\nu u}_{eff}) = \sin^2\theta^{\nu e}_{eff} - \sin^2\theta^{\nu d}_{eff} = \epsilon, \tag{5.10}$$

do not depend on the mixing angles and hence can be used for determining ϵ once the experimental data is improved. *In fact, having such a fine structure is the major prediction of our $SU(2)_L \times U(1) \times U'(1)$ model.* We note that the present neutral-current data, namely the weighted average value 0.23 ± 0.02 for $\sin^2\theta_{eff}$, allow ϵ to be as large as 0.04. Unfortunately, the fine structure which is predicted by such an ϵ is too small to be confirmed or rejected by the present data.

Once again we turn attention to the fact that for the special values $\sin^2\theta = \frac{1}{4}$ and $\sin^2\zeta = \frac{2}{3}$ (which have been discussed earlier and are the analog of $\sin^2\theta = \frac{1}{4}$ in the standard model) θ_W coincides with θ^{eff}_W for purely leptonic processes. However in this case, the door is still open for the average measured value of $\sin^2\theta_{eff}$ to be less than $\frac{1}{4}$. This is because $\sin^2\theta_{eff}$ acquires the values $\frac{1}{4} - \frac{1}{4}\epsilon$ and $\frac{1}{4} - \epsilon$ for νu and νd scattering processes, respectively. This gives us a very good reason to try explaining the present neutral-current data with $\sin^2\theta$ having the exact "theoretical" value of $\frac{1}{4}$. The primary motivation is of course that the present weighted average of 0.23 for $\sin^2\theta_{eff}$ actually contains some negative first-order contributions due to $\epsilon \neq 0$. Following this viewpoint and using the upper bound 0.04 for ϵ, the neutral gauge bosons acquire the masses

$$m^2(Z) \sim 0.98\tfrac{4}{3}m^2(W^\pm), \quad m(Z') \sim 6m(W^\pm). \tag{5.11}$$

The former is almost the WS mass relation, while the latter does not contradict general theorems concerning gauge hierarchies.[10]

VI. ELECTRON-QUARK SCATTERING

In this section we calculate the modified values of the asymmetry parameters for electron-quark scattering processes. Actually we show that as far as these asymmetry parameters are concerned the predictions of our model agree with those of the WS model up to a second-order term. This is due to the fact that the first-order contributions ($\sim\epsilon$) which follow from Z and Z' exchanges simply cancel each other.

To see this, we first notice that the asymmetry parameter A for eq scattering is given by[7]

$$A \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$
$$\sim g^2 Q_q \left[a_e \frac{1}{m^2(Z)} v_q + a'_e \frac{1}{m^2(Z')} v'_q + \left(v_e \frac{1}{m^2(Z)} a_q + v'_e \frac{1}{m^2(Z')} a'_q \right) \frac{1-(1-y)^2}{1+(1-y)^2} \right]. \tag{6.1}$$

The corresponding vector as well as the axial-vector weak couplings v_i and a_j can be directly calculated from the interactions (4.12) and (4.13), and one finds

$$\cos\theta\, v_e = 4\sin^2\theta - 1 + \epsilon \sin^2\zeta(2 - 3\sin^2\zeta),$$
$$3\cos\theta\, v_u = 3 - 8\sin^2\theta - \epsilon\sin^2\zeta(2 - 5\sin^2\zeta), \tag{6.2a}$$
$$3\cos\theta\, v_d = 4\sin^2\theta - 3 - \epsilon\sin^2\zeta(2 + \sin^2\zeta),$$

$$\sin\zeta\cos\zeta\, v'_e = -\tan\theta(2 - 3\sin^2\zeta),$$
$$3\sin\zeta\cos\zeta\, v'_u = \tan\theta(2 - 5\sin^2\zeta), \tag{6.2b}$$
$$3\sin\zeta\cos\zeta\, v'_d = \tan\theta(2 + \sin^2\zeta),$$

$$a_e = -a_u = a_d = \frac{1}{\cos\theta}(1 - \epsilon\sin^4\zeta),$$
$$a'_e = -a'_u = a'_d = \tan\theta\tan\zeta. \tag{6.2c}$$

Now, taking into account the mass relations

$$m^{-2}(Z) \sim \cos^2\theta(1 + \epsilon\sin^4\zeta) m^{-2}(W^\pm),$$
$$m^{-2}(Z') \sim \epsilon\cot^2\theta\sin^2\zeta\cos^2\zeta\, m^{-2}(W^\pm), \tag{6.3}$$

we find that

$$v_e \frac{1}{m^2(Z)} a_u \sim 1 - 4\sin^2\theta + \epsilon\sin^4\zeta,$$
$$v'_e \frac{1}{m^2(Z')} a'_u \sim -\epsilon\sin^4\zeta. \tag{6.4}$$

The same, i.e., the cancellation of the first-order terms, holds for all the other pairs. It is to say that, as far as the asymmetry parameters are concerned, we are back with the WS results. For the polarized process $ed \to eX$ we may therefore write

$$\frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{9 G_F q^2}{20\sqrt{2}\pi\alpha} \left[1 - \tfrac{20}{9}\sin^2\theta + (1 - 4\sin^2\theta)\frac{1-(1-y)^2}{1+(1-y)^2} + O(\epsilon^2) \right]. \tag{6.5}$$

Thus, for small ϵ and at low-momentum transfer, the above measured asymmetry cannot be used to distinguish between the $SU(2)_L \times U(1)$ and $SU(2)_L \times U(1) \times U'(1)$ models.

VII. UNIFYING THE $SU(2)_L \times U(1) \times U'(1)$ MODEL

In this last section we would like to discuss briefly the various possibilities of embedding our $SU(2)_L \times U(1) \times U'(1)$ model in a unified gauge theory. To start with, let us first list all the constraints which are supposed to be satisfied:

(1) The spin-$\frac{1}{2}$ fermion colors are actually restricted to 1^c, 3^c, and $\bar{3}^c$.

(2) The fermions obey the quark-lepton correspondence.

(3) The $U'(1)$ factor is generated by the fourth color.

(4) The theory should not be a vectorlike one.

The major color restriction has been studied by Gell-Mann, Ramond, and Slansky.[8] It turns out to permit only the alternatives listed in Table I. In Table I, G is a simple Lie group and f denotes the representation of the fermions. Now we come to the second restriction, namely the quark-lepton correspondence. This simply tells us that the fermions should be assigned to such a

TABLE I. Embeddings permitted by the major color restriction. f denotes the fermion representation.

Case	$G \supset G^{fl} \times G^{col}$	f
1	$SU(n) \supset SU(n_1) \times SU(n_3) \times U(1) \times SU(3)^c$	n
2	$SU(n) \supset SU(n-3) \times U(1) \times SU(3)^c$	$(n^k)_A$
3	$SU(n) \supset SU(n_1) \times SU(n_3) \times SU(\bar{n}_3) \times U(1) \times U(1) \times SU(3)^c$	n
4	$SO(n) \supset SO(n_1) \times SU(n_3) \times U(1) \times SU(3)^c$	n
5	$SO(n) \supset SO(n-6) \times U(1) \times SU(3)^c$	σ, σ'
6	$Sp(2n) \supset Sp(2n_1) \times SU(n_3) \times U(1) \times SU(3)^c$	$2n$
7	$F_4 \supset SU(3) \times SU(3)^c$	26
8	$E_6 \supset SU(3) \times SU(3) \times SU(3)^c$	27
9	$E_7 \supset SU(6) \times SU(3)^c$	56

representation f that decomposes into either

$$f=(n', 1^c)+(n', 3^c),$$

or (7.1)

$$f=(n', 1^c)+(n', 3^c)+(\overline{n}', 1^c)+(\overline{n}', \overline{3}^c),$$

under $G^{fl}\times G^{col}$. Although this can be arranged for several of the above cases, we would of course like this property to emerge naturally. It turns out that among all available simple Lie groups, only the orthogonal ones automatically possess such a property. Exactly the same conclusion follows independently from the third restriction. The U′(1) factor which distinguishes 1^c from 3^c must emerge simultaneously with $SU(3)^{col}$ in the same stage of SSB and by the same mechanism. Thus we are looking for a simple group G which cannot have an SU(3) subgroup unless this subgroup is accompanied by an extra U(1). Again this turns out to be a unique property of orthogonal groups. The algebra of SO(6) is equivalent to that of $SU(4)\supset SU(3)\times U(1)$, but there is no orthogonal group whose algebra is equivalent to that of SU(3) by itself. In fact, this equivalence is the only mechanism by which one can reach a color SU(3) group within a unified SO(n) gauge theory.

At this stage it appears that only two unification alternatives are capable of satisfying the first three constraints. These are the vectorial SO(n) and its spinorial version. If we furthermore insist on having an anomaly-free flavor-chiral theory (rather than a vectorlike one), the left-handed fermions should be assigned to a complex representation. It is well known that the vector representation n of SO(n) is self-conjugate, while the spinor representations are complex only for $n=4k+2$. This completes the deduction of the main result, which is the following: *The effective low-energy electroweak symmetry* $SU(2)_L\times U(1)\times U'(1)$, *which is based on the idea of quark-lepton correspondence and uses the combination* $B-L$ *as the fourth color, can be embedded only in unified theories which are based on* $SO(4k+2)$ *where the fermions are assigned to its spinorial representations*. Several unified gauge theories of this kind, namely theories based on SO(10) and SO(14), have already been discussed in the literature.

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