# Charmed baryons in a quark model with hyperfine interactions

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We employ a quark model based on quantum chromodynamics and tested against noncharmed baryons to predict masses and decay rates of ground-state and excited baryons containing one charmed quark. Among other conclusions, the calculations indicate that the orbitally excited $\Lambda_c$ $1/2^-$ analogous to the $\Lambda(1405)$ is stable against strong decays.

## I. INTRODUCTION

The success of the "naive" quark model with the charmonium family and the advent of new ideas concerning quark-quark forces motivated by quantum chromodynamics have led to a revival of interest in both experimental and theoretical "old" hadron spectroscopy. Much of the recent theoretical work in this area supports the idea[1] that at least the main features of hadron spectroscopy may be understood in terms of approximately nonrelativistic, medium-mass quarks moving in a flavor-independent confinement potential perturbed by various short-range interactions anticipated from one gluon exchange.

Most prominent among these latter forces is the hyperfine or magnetic-dipole–magnetic-dipole interaction

$$H_{\text{hyp}}^{ij}=A^{ij}\left\{\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\delta^3(\vec{r}_{ij})+\frac{1}{r_{ij}^{\,3}}\left[\frac{3\vec{S}_i\cdot\vec{r}_{ij}\vec{S}_j\cdot\vec{r}_{ij}}{r_{ij}^{\,2}}-\vec{S}_i\cdot\vec{S}_j\right]\right\}, \tag{1a}$$

where

$$A^{ij}=\begin{cases}\dfrac{2}{3}\,\dfrac{\alpha_s}{m_i m_j} & \text{between quarks } i \text{ and } j \text{ in a baryon},\\[2ex] \dfrac{4}{3}\,\dfrac{\alpha_s}{m_i m_j} & \text{between quark } i \text{ and antiquark } j \text{ in a meson},\end{cases} \tag{1b}$$

though, as will be elaborated in Sec. II, there is also evidence for the expected $1/r$ potential and perhaps for a residue of the spin-orbit forces as well.

Several features of the anticipated hyperfine interaction now seem to be confirmed.[2] There is, first of all, evidence from spectroscopy that the $\vec{S}_i\cdot\vec{S}_j$ (i.e., "contact") piece of the hyperfine interaction is indeed of short range. There is also evidence from baryon mixing angles that the second ("tensor") piece of the hyperfine interaction is present with the correct relative strength compared to the contact term. Finally, there is fairly strong evidence for the $m^{-1}$ dependence of the color-magnetic dipole moment of quarks from the splittings of strange baryons and mesons [e.g., $(\Sigma-\Lambda)/(\Delta-N)$, $(K^*-K)/(\rho-\pi)$, etc.]. On the other hand, the supposed $m^{-1}$ dependence of the color and electromagnetic moments of quarks is very closely related to the $\eta_c$ problem of charmonium spectroscopy: The $\psi$-$\chi(2.83)$ splitting may indicate that the color-magnetic moments are larger than expected, while $\psi\to\chi(2.83)\gamma$ may indicate that the electromagnetic moments are smaller than expected. While other, perhaps more palatable, explanations of this problem have been presented,[3] it is clearly important to find independent tests of this $m^{-1}$ dependence for charmed quarks. Fortunately, there are already some: The $D^*$-$D$ and $F^*$-$F$ splittings are in reasonably good agreement[1,4] with Eqs. (1). The study of charmed-baryon spectroscopy will provide further checks of this important effect.

Charmed baryons also provide a very good and perhaps even dramatic way of testing for the approximate flavor independence of the confinement forces. In baryons containing two quarks of equal mass $m$ and a third quark of different mass $m'$, it has been very useful[5] to abandon the requirement of overall wave-function antisymmetry and to antisymmetrize only in the equal-mass quarks (say quarks 1 and 2). In this case with variables

$$\vec{\rho} \equiv \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2) \tag{2}$$

and

$$\vec{\lambda} \equiv \frac{1}{\sqrt{6}} (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \tag{3}$$

pairs of spatial wave functions of mixed symmetry which would normally have been degenerate in energy (as a consequence of the overall permutational symmetry of the system) can be seen to correspond to different energies: Excitations of the $\vec{\lambda}$ variable involve the vibrations of the "odd" quark (for our purposes "odd" = "strange" or "charmed") unlike excitations of the $\vec{\rho}$ variable. These variables have associated with them "reduced masses"

$$m_\rho = m\,, \tag{4}$$

$$m_\lambda = \frac{3mm'}{2m+m'}\,, \tag{5}$$

and in the harmonic-oscillator model this leads to a frequency shift between $\rho$ and $\lambda$ excitations (analogous to the isotope shift in molecular vibrations) of

$$\frac{\omega_\rho - \omega_\lambda}{\omega_\rho} = 1 - \left(\frac{1+2x}{3}\right)^{1/2}, \tag{6}$$

where

$$x \equiv m/m'\,.$$

In strange baryons where $x \simeq 0.6$ this leads to a frequency shift of about 15% and is responsible[5] for the observed "reversal" of the $\Sigma\frac{5}{2}^- - \Lambda\frac{5}{2}^-$ mass difference relative to $\Sigma\frac{1}{2}^+ - \Lambda\frac{1}{2}^+$ in the ground state. In charmed baryons with $x \simeq 0.2$ the model gives a frequency shift of about 30% which we expect to produce a very dramatic effect. The $\Lambda(1405)\frac{1}{2}^-$ is seen in this picture as an almost pure $\lambda$-type excitation, 290 MeV above the $\Lambda(1115)\frac{1}{2}^+$ ground state; with the further decrease in excitation energy in the charmed-baryon case, we expect the charmed analog of the $\Lambda(1405)$ to be only about 215 MeV above the $\Lambda_c(2.25)\frac{1}{2}^+$. This excitation energy is insufficient to allow decay to either $\Lambda_c\frac{1}{2}^+$ (one pion is forbidden by isospin and two by phase space space), or $\Sigma_c\frac{1}{2}^+$ and $\Sigma_c\frac{3}{2}^+$ which, because of the reduced chromomagnetic moment of the charmed quark, are expected to lie near each other roughly 200 MeV above the $\Lambda_c\frac{1}{2}^+$. We therefore find that the $\lambda$-type excitation of the $\Lambda_c\frac{1}{2}^+$ is stable against strong decay.

The details of this and our other conclusions are contained in Secs. III and IV. The model itself is reviewed in Sec. II. We discuss our results and their application to the experimental search for charmed baryons in Sec. V.

## II. THE MODEL

The model on which our conclusions are based was introduced and applied to noncharmed baryons in Refs. 5 and 6, and has been reviewed recently in Refs. 2, 9, and 10, so we simply sketch the model here.

We assume that the Hamiltonian for baryons is approximately

$$H = \sum_{i=1}^{3} m_i + H_0 + H_{\text{hyp}}\,, \tag{7}$$

where

$$H_0 = \sum_{i=1}^{3} \frac{p_i^2}{2m_i} + \sum_{i<j} V_{\text{conf}}^{ij}(r_{ij}) \tag{8}$$

and

$$H_{\text{hyp}} = \sum_{i<j} H_{\text{hyp}}^{ij}\,, \tag{9}$$

where $H_{\text{hyp}}^{ij}$ is given by (1), and where $V_{\text{conf}}^{ij}$ is a flavor-independent function of the relative $qq$ separation. In the case $m_1 = m_2 = m$, $m_3 = m'$, and $V_{\text{conf}}^{ij} = \frac{1}{2}Kr_{ij}^2$ the introduction of the variables (2) and (3) decouples $H_0$ into two harmonic oscillators in $\rho$ and $\lambda$ each with spring constant $3K$ and with effective masses given by (4) and (5). If $m' > m$, the frequency $\omega_\lambda$ associated with $\lambda$-type excitations will be smaller than the frequency $\omega_\rho$ by a factor $(m/m_\lambda)^{1/2}$ and wave functions in $\lambda$ will be of smaller spatial extent by a factor of $(m/m_\lambda)^{1/4}$. These last observations, as previously mentioned, lead us in the case $m = m_u \simeq m_d$ and $m' = m_c$ to abandon the generalized Pauli antisymmetrization principle which would apply if all three quarks had equal masses, and to replace it with the principle that baryon wave functions should be antisymmetric only in the variables of equal mass (i.e., $u$ and $d$) quarks. In the strange baryons we referred to the resulting set of states as the $uds$ basis[5] states; here we will naturally refer to the $udc$ basis. This basis, which maximally violates SU(4) symmetry, is convenient because it respects the fact that $\omega_\lambda \neq \omega_\rho$ if $m_\lambda \neq m_\rho$; i.e., the $udc$ basis states diagonalize $H_0$ while states composed of SU(4)-invariant wave functions do not. Thus, if we introduce the isospin wave functions

$$\phi_{\Lambda_c} = \frac{1}{\sqrt{2}} (ud - du)c\,, \tag{10}$$

$$\phi_{\Sigma_c} = \frac{1}{\sqrt{2}} (ud + du)c \tag{11}$$

and the usual spin wave functions

$$\chi^s_{3/2} = \uparrow\uparrow\uparrow\,,$$
$$\chi^s_{1/2} = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)\,,\ \text{etc.}, \tag{12}$$

$$\chi^\rho_+ = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow\,,\quad \chi^\rho_- = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow\,, \tag{13}$$

$$\chi^\lambda_+ = -\frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)\,,$$
$$\chi^\lambda_- = \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)\,, \tag{14}$$

then from the wave functions ($\rho_+ = \rho_x + i\rho_y$, etc.)

$$N=0 \quad \psi_{00} = \frac{\alpha_\rho^{3/2}\alpha_\lambda^{3/2}}{\pi^{3/2}} \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{15}$$

$$1 \quad \psi^\rho_{11} = \frac{\alpha_\rho^{5/2}\alpha_\lambda^{3/2}}{\pi^{3/2}}\rho_+ \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{16}$$

$$1 \quad \psi^\lambda_{11} = \frac{\alpha_\rho^{3/2}\alpha_\lambda^{5/2}}{\pi^{3/2}}\lambda_+ \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{17}$$

$$2 \quad \psi^{\lambda\lambda}_{00} = \left(\frac{2}{3}\right)^{1/2} \frac{\alpha_\rho^{3/2}\alpha_\lambda^{7/2}}{\pi^{3/2}}(\lambda^2 - \tfrac{3}{2}\alpha_\lambda^{-2}) \times \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{18}$$

$$2 \quad \psi^{\rho\lambda}_{00} = \frac{1}{\sqrt{3}} \frac{\alpha_\rho^{5/2}\alpha_\lambda^{5/2}}{\pi^{3/2}}(2\vec{\rho}\cdot\vec{\lambda}) \times \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{19}$$

$$2 \quad \psi^{\rho\rho}_{00} = \left(\frac{2}{3}\right)^{1/2} \frac{\alpha_\rho^{7/2}\alpha_\lambda^{3/2}}{\pi^{3/2}}(\rho^2 - \tfrac{3}{2}\alpha_\rho^{-2}) \times \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{20}$$

$$2 \quad \psi^{\lambda\lambda}_{22} = \frac{1}{\sqrt{2}} \frac{\alpha_\rho^{3/2}\alpha_\lambda^{7/2}}{\pi^{3/2}}\lambda_+^2 \times \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{21}$$

$$2 \quad \psi^{\rho\lambda}_{22} = \frac{\alpha_\rho^{5/2}\alpha_\lambda^{5/2}}{\pi^{3/2}}\rho_+\lambda_+ \times \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{22}$$

$$2 \quad \psi^{\rho\rho}_{22} = \frac{1}{\sqrt{2}} \frac{\alpha_\rho^{7/2}\alpha_\lambda^{3/2}}{\pi^{3/2}}\rho_+^2 \times \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{23}$$

$$2 \quad \psi^{\rho\lambda}_{11} = \frac{\alpha_\rho^{5/2}\alpha_\lambda^{5/2}}{\pi^{3/2}}(\rho_+\lambda_z - \rho_z\lambda_+) \times \exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \tag{24}$$

where the notation is $\psi_{lm}$ and where

$$\alpha_i = (3Km_i)^{1/4}\,,$$

one can explicitly construct the *udc* basis states in the harmonic approximation. For example, one has (to choose some simple low-lying cases)

$$|\Lambda_c\,{}^2S\,\tfrac{1}{2}^+\rangle = \phi_{\Lambda_c}\chi^\rho_+\psi_{00}\,, \tag{25}$$
$$|\Sigma_c\,{}^2S\,\tfrac{1}{2}^+\rangle = \phi_{\Sigma_c}\chi^\lambda_+\psi_{00}\,, \tag{26}$$
$$|\Sigma_c\,{}^4S\,\tfrac{3}{2}^+\rangle = \phi_{\Sigma_c}\chi^s_{3/2}\psi_{00}\,, \tag{27}$$
$$|\Lambda_c\,{}^4P_\rho\,\tfrac{5}{2}^-\rangle = \phi_{\Lambda_c}\chi^s_{3/2}\psi^\rho_{11}\,, \tag{28}$$
$$|\Lambda_c\,{}^2P_\lambda\,\tfrac{3}{2}^-\rangle = \phi_{\Lambda_c}\chi^\rho_+\psi^\lambda_{11}\,, \tag{29}$$
$$|\Sigma_c\,{}^4P_\lambda\,\tfrac{5}{2}^-\rangle = \phi_{\Sigma_c}\chi^s_{3/2}\psi^\lambda_{11}\,, \tag{30}$$
$$|\Lambda_c\,{}^2S_{\lambda\lambda}\,\tfrac{1}{2}^+\rangle = \phi_{\Lambda_c}\chi^\rho_+\psi^{\lambda\lambda}_{00}\,, \tag{31}$$
$$|\Lambda_c\,{}^2D_{\lambda\lambda}\,\tfrac{5}{2}^+\rangle = \phi_{\Lambda_c}\chi^\rho_+\psi^{\lambda\lambda}_{22}\,, \tag{32}$$
$$|\Sigma_c\,{}^2S_{\lambda\lambda}\,\tfrac{1}{2}^+\rangle = \phi_{\Sigma_c}\chi^\lambda_+\psi^{\lambda\lambda}_{00}\,. \tag{33}$$

It is apparent that these states in the *udc* basis are much simpler than symmetrized SU(4) basis states. Also it should be noted that at this level of calculation, states such as $|\Lambda_c\,{}^4P_\rho\,\tfrac{5}{2}^-\rangle$ and $|\Sigma_c\,{}^4P_\lambda\,\tfrac{5}{2}^-\rangle$ have already developed a mass difference due to $\omega_\lambda < \omega_\rho$ which, as we shall see shortly, is of the order of 150 MeV.

Of course we do not expect $H_0$ to be of harmonic form. However, we proceed to approximate solutions of the true problem by writing

$$V^{ij}_{\text{conf}} = \tfrac{1}{2}Kr_{ij}^2 + U(r_{ij})\,, \tag{34}$$

where $U(r_{ij})$ is some unknown potential which we expect to incorporate a short-range attractive potential (the Coulomb-type potential) and deviations of the long-range part of the potential from the harmonic-oscillator form. We then note that all potentials $U(r_{ij})$ will in first order split the equal-mass harmonic-oscillator eigenstates up into the same pattern[2,6] so that the $H_0$ matrix elements of the ten wave functions (15) to (24) are given in the equal-mass limit by only three constants $E_0$, $\Omega$, and $\Delta$ known from our previous work on noncharmed baryons. We then take $m_\rho \neq m_\lambda$ into account by reducing the energy of $\lambda$ excitations by the harmonic-oscillator factor of $(m_\rho/m_\lambda)^{1/2}$. The full Hamiltonian matrix is finally obtained by the addition of the mass difference $\Delta m = m_c - m_u$ and the hyperfine perturbations.

With the exception of the charmed quark mass, the parameters of this model are all determined by noncharmed baryons.[2] These parameters are $m_c = 1.75$ GeV, $m_d = 0.35$ GeV, $E_0 = 1.15$ GeV, $\Omega \simeq \Delta = 0.44$ GeV, and $\delta = 0.27$ GeV. The parameter $\delta$ is the "hyperfine constant" determined by the $\Delta$-$N$ mass difference; it differs slightly from the value 0.30 GeV quoted in Refs. 5 and 6 as a consequence of our subsequent inclusion of interband hyperfine mixing effects like those responsible for the charge radius of the neutron and other SU(6)-violating effects.[11] The charmed-quark mass quoted here must not be taken too literally as its precise value depends on the way one treats the zero-point energy of this system. The mass difference $m_\lambda \neq m_\rho$ will cause a shift in the zero-point energy of $\psi_{00}$ which we have taken into account perturbatively by taking matrix elements of

$$\Delta K = -\left(1 - \frac{m_\rho}{m_\lambda}\right)\frac{p_\lambda^2}{2m_\rho}$$

in an equal-mass harmonic-oscillator ground-state wave function that will give the observed proton charge radius. Since we fix the sum of these two effects to give correctly the observed $\Lambda_c \frac{1}{2}^+$ mass, our predictions are unaffected by this ambiguity, but the uncertainty in $m_c$ from this source may be as large as 0.10 GeV.

Before turning on the hyperfine interactions, the model outlined above leads to the following Hamiltonian in terms of the *udc* basis spatial wave functions corresponding to the $N=0$, 1, and 2 levels of the harmonic problem:

$$\begin{aligned} &E(S)=2.450\,, \quad E(S_{\rho\rho})=3.000\,,\\ &E(P_\lambda)=2.750\,, \quad E(D_{\lambda\lambda})=2.965\,,\\ &E(P_\rho)=2.890\,, \quad E(D_{\rho\lambda})=3.115\,,\\ &E(S_{\lambda\lambda})=2.825\,, \quad E(D_{\rho\rho})=3.200\,,\\ &E(S_{\rho\lambda})=3.005\,, \quad E(P_{\rho\lambda})=3.190\,, \end{aligned} \tag{35}$$

with

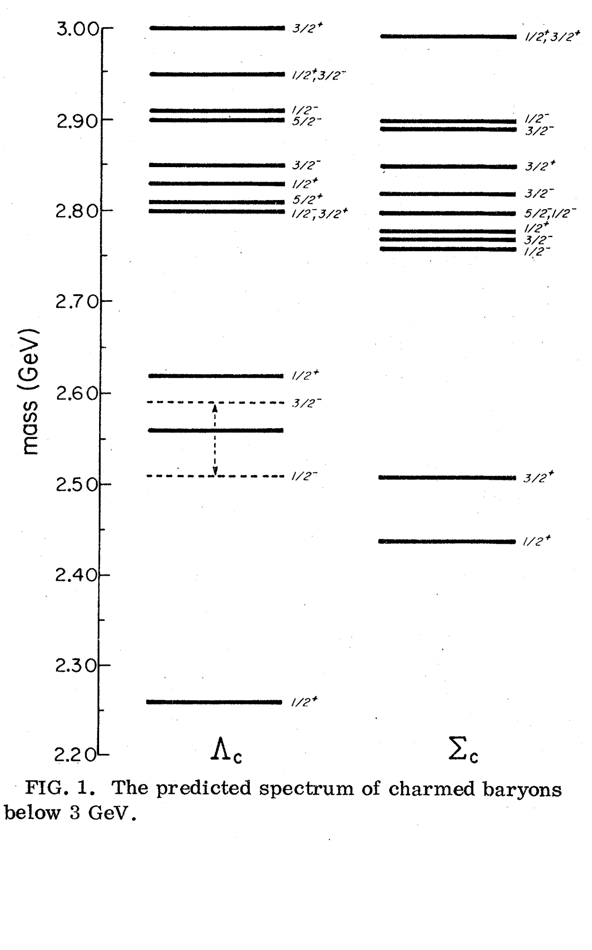


FIG. 1. The predicted spectrum of charmed baryons below 3 GeV.

TABLE I. The predicted masses and compositions of some low-lying $C=1$ baryons.

| State ($IJ^P$) | Mass (GeV) | Approximate composition |
|---|---|---|
| $\Lambda_c \frac{1}{2}^+$ | 2.26 | $0.95\,{}^2S + 0.30\,{}^2S_{\rho\rho}$ |
| $\Lambda_c \frac{1}{2}^-$ | 2.51 | ${}^2P_\lambda$ |
| $\Lambda_c \frac{3}{2}^-$ | 2.59 | ${}^2P_\lambda$ |
| $\Lambda_c' \frac{1}{2}^+$ | 2.67 | $0.86\,{}^2S_{\lambda\lambda} + 0.51\,{}^2S_{\rho\rho}$ |
| $\Sigma_c \frac{1}{2}^+$ | 2.44 | ${}^2S$ |
| $\Sigma_c \frac{3}{2}^+$ | 2.51 | ${}^4S$ |
| $\Lambda_c \frac{3}{2}^+$ | 2.80 | ${}^2D_{\lambda\lambda}$ |
| $\Lambda_c \frac{5}{2}^+$ | 2.81 | ${}^2D_{\lambda\lambda}$ |
| $\Lambda_c \frac{5}{2}^-$ | 2.90 | ${}^4P_\rho$ |
| $\Lambda_c \frac{7}{2}^+$ | 3.13 | ${}^4D_{\rho\lambda}$ |
| $\Sigma_c \frac{1}{2}^-$ | 2.76 | $-0.58\,{}^4P_\lambda + 0.79\,{}^2P_\lambda$ |
| $\Sigma_c \frac{3}{2}^-$ | 2.77 | ${}^2P_\lambda$ |
| $\Sigma_c \frac{5}{2}^-$ | 2.80 | ${}^4P_\lambda$ |
| $\Sigma_c \frac{1}{2}^+$ | 2.78 | $0.90\,{}^2S_{\lambda\lambda} + 0.43\,{}^2S_{\rho\rho}$ |
| $\Sigma_c \frac{3}{2}^+$ | 2.85 | $0.92\,{}^4S_{\lambda\lambda} + 0.39\,{}^4S_{\rho\rho}$ |
| $\Sigma_c \frac{5}{2}^+$ | 3.01 | ${}^2D_{\lambda\lambda}$ |
| $\Sigma_c \frac{7}{2}^+$ | 3.01 | ${}^4D_{\lambda\lambda}$ |

$$\langle S_{\rho\rho}|U|S_{\lambda\lambda}\rangle = -0.110\,,$$
$$\langle D_{\rho\rho}|U|D_{\lambda\lambda}\rangle = -0.045\,.$$

After supplementing these matrix elements with those of the hyperfine interactions, the problem becomes one of matrix diagonalization. Most of the relevant hyperfine matrix elements may be found in Refs. 5 and 6. The few that may not be are listed in the Appendix.

## III. THE PREDICTED SPECTRUM AND COMPOSITION OF $C=1$ BARYONS

Diagonalization of the various $IJ^P$ sectors of $C=1$ baryons leads to the predicted low-lying states shown in Fig. 1; the masses and compositions of some of the more relevant states are also listed in Table I. The spectra shown have included the effects of interband hyperfine mixing on the energies of states in the $N=0$ and $N=1$ bands.

Most of the prominent features of the resulting spectrum can be understood qualitatively in terms of the limit $x = m_u/m_c \to 0$ in which extreme the hyperfine interactions of quark 3 would become negligible and $\omega_\lambda$ would equal $(1/3)^{1/2}\omega_\rho$ for harmonic forces. In the spin state $\chi^\rho$ the two noncharmed quarks are in $S=0$ and so contact interactions are proportional to $-\frac{3}{4}$, while in $\chi^\lambda$ and $\chi^s$ the first two quarks are in $S=1$ giving an interaction proportional to $+\frac{1}{4}$. Thus, apart from rather small second-order effects, in this limit we expect the $\Lambda_c\frac{1}{2}^+$ to drop by as much as the nucleon, while $\Sigma_c\frac{1}{2}^+$ and $\Sigma_c\frac{3}{2}^+$ would be expected to be degenerate at $\frac{2}{3}(\Delta-N) \simeq 200$ MeV above the $\Lambda_c\frac{1}{2}^+$. Those features are reflected in the predicted charm spectrum shown above even though $x=0.20$. It is gratifying that the preliminary experimental situation appears to support this prediction.

The positions of the $\Lambda_c\frac{1}{2}^-$ and $\Lambda_c\frac{3}{2}^-$ may also be checked by similar arguments. These states are approximately pure $\lambda$ excitations of the type $\phi_{\Lambda_c}\chi^\rho\psi^\lambda$ as were their strange counterparts, the $\Lambda\frac{1}{2}^-(1405)$ and $\Lambda\frac{3}{2}^-(1520)$; in both the $S=1$ and $C=1$ cases, as a result, the contact forces in these states are unaffected by $m'>m$. Consequently, for example, the $\Lambda_c\frac{1}{2}^- - \Lambda_c\frac{1}{2}^+$ gap and the $\Lambda\frac{1}{2}^- - \Lambda\frac{1}{2}^+$ gap should differ only by the change in $\omega_\lambda$ of about $-75$ MeV. In Ref. 4 we predict $\Lambda\frac{1}{2}^- - \Lambda\frac{1}{2}^+ \simeq \Lambda\frac{3}{2}^- - \Lambda\frac{1}{2}^+ \simeq 375$ MeV, so here we predict $\Lambda_c\frac{1}{2}^- - \Lambda_c\frac{1}{2}^+ \simeq \Lambda_c\frac{3}{2}^- - \Lambda_c\frac{1}{2}^+ \simeq 300$ MeV, corresponding to the solid bar at $\simeq 2.56$ GeV in Fig. 1. Let us recall, however, that experimentally one observes $\Lambda\frac{1}{2}^-(1405) - \Lambda\frac{1}{2}^+(1115) \simeq 290$ MeV and $\Lambda\frac{3}{2}^-(1520) - \Lambda\frac{1}{2}^+(1115) \simeq 405$ MeV; the failure of our simple model to give the $\Lambda\frac{3}{2}^-(1520) - \Lambda\frac{1}{2}^-(1405)$ splitting is in fact one of its most serious failures. In Refs. 5 and 8 it is shown that it may be possible to associate this splitting with a residue of the spin-orbit couplings which we have hitherto neglected. Numerically, it turns out that such effects would be nearly the same size in the $\Lambda_c\frac{3}{2}^- - \Lambda_c\frac{1}{2}^-$ case: They would lower the $\Lambda_c\frac{1}{2}^-$ by about 50 MeV and raise the $\Lambda_c\frac{3}{2}^-$ by about 25 MeV. This effect is indicated in Fig. 1 by dashed lines. Alternatively, we can simply rely on the charm-strange analogy to write, for example,

$$\Lambda_c\tfrac{1}{2}^- - \Lambda_c\tfrac{1}{2}^+ \simeq \Lambda\tfrac{1}{2}^-(1405) - \Lambda\tfrac{1}{2}^+(1115) - 75 \text{ MeV}$$
$$\simeq 215 \text{ MeV}.$$

Finally we note that the only other low-lying charmed baryon is a $\Lambda_c\frac{1}{2}^+$ state at about 2.62 GeV. This state is mostly a radial excitation of the $\lambda$ type of the ground state and so is predicted (since once again $\chi^\rho$ is the spin function) to be at an excitation energy related, apart from orbital mode splitting effects, to that of the Roper resonance above the nucleon.

## IV. THE DECAYS OF $C=1$ BARYONS

The most striking feature of this calculation is the observation that all of the decays

$$\Lambda_c\tfrac{1}{2}^- \to \Sigma_c\tfrac{3}{2}^+ + \pi, \tag{36}$$

$$\Lambda_c\tfrac{1}{2}^- \to \Sigma_c\tfrac{1}{2}^+ + \pi, \tag{37}$$

$$\Lambda_c\tfrac{1}{2}^- \to \Lambda_c\tfrac{1}{2}^+ + \pi \tag{38}$$

are forbidden: The first two by energy conservation and the third by isospin conservation. Moreover, if the true $\Lambda_c\frac{1}{2}^-$ mass is given by the dotted line in Fig. 1, and we think this the more likely possibility, then the decay $\Lambda_c\frac{1}{2}^- \to \Lambda_c\frac{1}{2}^+ + \pi\pi$ will also be forbidden and the $\Lambda_c\frac{1}{2}^-$ *will be stable against strong decay*. In this case, as we shall see below, the $\Lambda_c\frac{1}{2}^-$ will decay predominantly to $\Lambda_c\frac{1}{2}^+(2.26)+\gamma$ with a width of approximately 0.6 keV and should be readily observable in invariant-mass distributions in this channel.

To complete our discussion of $C=1$ baryons we turn our attention to the calculation of their expected decay widths. We shall explicitly consider only the six predicted low-lying states $\Lambda_c\frac{1}{2}^+(2.26)$, $\Lambda_c\frac{1}{2}^-(2.51)$, $\Lambda_c\frac{3}{2}^-(2.59)$, $\Lambda_c\frac{1}{2}^+(2.62)$, $\Sigma_c\frac{1}{2}^+(2.44)$, and $\Sigma_c\frac{3}{2}^+(2.51)$. Since the threshold for decay into $N(940)D(1870)$ is 2810 MeV, the $\Lambda_c\frac{1}{2}^+(2.26)$ is of course stable to all but weak decays while the remaining five states can decay only into a lower-lying charmed baryon and either a pion or a photon. Since in the single-quark-transition picture[12] pion emission must occur from a noncharmed quark, spectator-quark independence would imply that the primitive quark amplitudes relevant to $N^*$ decays should apply here as well. The calculation of such a decay from one *udc*-basis state to another will therefore be governed by the amplitude

$$A(B_{udc} \to B'_{udc}\pi^0(\hat{k})) = 2\langle B'_{udc}|(g\sigma_{1z}k + h\vec{\sigma}_1\cdot\vec{p}_1)e^{-ikr_{1z}}(\tau_1)_3|B_{udc}\rangle, \tag{39}$$

where $\hat{k}=k\hat{z}$, $g$ and $h$ are constants known from analysis of $N^*$ decays, and where

$$\Gamma(B_{udc}\to B'_{udc}\pi^0) = \frac{1}{2\pi}|A|^2 q, \tag{40}$$

while for photodecay we have the two amplitudes[13]

$$A_{3/2} = \sqrt{2}\,\mu_p\left\langle B'_{udc}(+\tfrac{1}{2})\left|2\frac{e_1}{e}[q(\tfrac{1}{2}\sigma_1)_- + p_{1-}]e^{-iqr_{1z}} + x\frac{e_3}{e}[q(\tfrac{1}{2}\sigma_3)_- + p_{3-}]e^{-iqr_{3z}}\right|B_{udc}(+\tfrac{3}{2})\right\rangle, \tag{41}$$

$$A_{1/2} = \sqrt{2}\,\mu_p\left\langle B'_{udc}(-\tfrac{1}{2})\left|2\frac{e_1}{e}[q(\tfrac{1}{2}\sigma_1)_- + p_{1-}]e^{-iqr_{1z}} + x\frac{e_3}{e}[q(\tfrac{1}{2}\sigma_3)_- + p_{3-}]e^{-iqr_{3z}}\right|B_{udc}(+\tfrac{1}{2})\right\rangle, \tag{42}$$

TABLE II. Some decays of low-lying $C=1$ baryons: $Z \equiv 3/(1+2x)$, $h' \equiv [(1+x)/2]h$, $h'' = (\frac{5}{4}Z^{1/2} - \frac{1}{2}Z + \frac{1}{4}Z^2)h$.

| Amplitude | Nominal width [a] |
|---|---|
| $A(\Sigma_c \frac{1}{2}^+ \to \Lambda_c \frac{1}{2}^+ + \pi^0) = -\frac{2}{\sqrt{3}} k(g - \frac{1}{3}h')$ | $\sim$5 MeV [b] |
| $A(\Sigma_c \frac{3}{2}^+ \to \Lambda_c \frac{1}{2}^+ + \pi^0) = +\frac{2\sqrt{2}}{\sqrt{3}} k(g - \frac{1}{3}h')$ | 60 MeV |
| $A_{1/2}(\Lambda_c \frac{1}{2}^- \to \Lambda_c \frac{1}{2}^+ + \gamma) = \frac{i\sqrt{2}}{3} \mu_p \alpha Z^{1/4} \left( \frac{4x-1}{3} + \frac{2x^2 Z^{1/2}}{3} \frac{q^2}{\alpha^2} \right)$ | $\sim$0.6 keV |
| $A_{1/2}(\Lambda_c \frac{1}{2}^- \to \Sigma_c \frac{1}{2}^+ + \gamma) = \frac{\sqrt{6}}{18} i \mu_p \alpha Z^{3/4} \frac{q^2}{\alpha^2}$ | $\sim$15 eV [b] |
| $A_{3/2}(\Lambda_c \frac{1}{2}^- \to \Sigma_c \frac{3}{2}^+ + \gamma) = i \frac{1}{6} \mu_p \alpha Z^{3/4} \frac{q^2}{\alpha^2}$ | Negligible |
| $A_{1/2}(\Lambda_c \frac{1}{2}^- \to \Sigma_c \frac{3}{2}^+ + \gamma) = -i \frac{\sqrt{3}}{18} \mu_p \alpha Z^{3/4} \frac{q^2}{\alpha^2}$ | |
| $A(\Lambda_c \frac{3}{2}^- \to \Sigma_c \frac{1}{2}^+ + \pi^0) = -i \frac{\sqrt{6}}{9} \alpha \frac{k^2}{\alpha^2} Z^{3/4}(g + \frac{1}{2}h')$ | $\sim$10 keV [b] |
| $A_{3/2}(\Lambda_c \frac{3}{2}^- \to \Lambda_c \frac{1}{2}^+ + \gamma) = \frac{i}{\sqrt{3}} \mu_p \alpha Z^{1/4} \left( \frac{4x-1}{3} \right)$ | 8 keV |
| $A_{1/2}(\Lambda_c \frac{3}{2}^- \to \Lambda_c \frac{1}{2}^+ + \gamma) = +\frac{1}{3} i \mu_p \alpha Z^{1/4} \left( \frac{4x-1}{3} - \frac{4x^2 Z^{1/2}}{3} \frac{q^2}{\alpha^2} \right)$ | |
| $A(\Lambda_c {}^2S_{\lambda\lambda} \frac{1}{2}^+ \to \Sigma_c \frac{1}{2}^+ + \pi^0) = \frac{\sqrt{2}kZ}{18} (h + \frac{1}{2} g Z^{1/2} k^2/\alpha^2)$ | $\sim$3 MeV |
| $A(\Lambda_c {}^2S_{\rho\rho} \frac{1}{2}^+ \to \Sigma_c \frac{1}{2}^+ + \pi^0) = \frac{\sqrt{2}k}{6} (h + \frac{1}{2} g k^2/\alpha^2)$ | |

[a] Using theoretical values of the energy release. The widths are summed over final-state charges.
[b] These decays are especially sensitive to the available phase space.

where $B(M)$ is a baryon with $J_z = M$, and where

$$\Gamma(B_{udc} \to B'_{udc} + \gamma) = \frac{q}{(2J+1)\pi} [|A_{3/2}|^2 + |A_{1/2}|^2] . \quad (43)$$

Using these formulas, wave functions such as (25) to (33), and the results that

$$\vec{r}_1 - \vec{R}_{\text{c.m.}} = \frac{1}{\sqrt{6}} \vec{\lambda} \left( \frac{3}{1+2x} \right) + \frac{1}{\sqrt{2}} \vec{\rho} , \quad (44)$$

$$\vec{r}_2 - \vec{R}_{\text{c.m.}} = \frac{1}{\sqrt{6}} \vec{\lambda} \left( \frac{3}{1+2x} \right) - \frac{1}{\sqrt{2}} \vec{\rho} , \quad (45)$$

$$\vec{r}_3 - \vec{R}_{\text{c.m.}} = -\frac{2}{\sqrt{6}} \lambda \left( \frac{3x}{1+2x} \right) , \quad (46)$$

the relevant decay amplitudes and widths may readily be calculated. The results are displayed in Table II. It is noteworthy that even the strongly decaying states $\Sigma_c \frac{1}{2}^+$, $\Sigma_c \frac{3}{2}^+$, $\Lambda_c \frac{3}{2}^-$, and $\Lambda_c' \frac{1}{2}^+$ are quite narrow to the extreme that in the $\Lambda_c \frac{3}{2}^-$ case the decay to $\Lambda_c \frac{1}{2}^+ + \gamma$ may be significant. It is also interesting to note that if our conclusion about the strong stability of the $\Lambda_c \frac{1}{2}^-$ is correct, its width will, regrettably, not be measured. In that case the only real check on our decay calculations for this state will be to measure its very small branching ratio to $\Sigma_c \frac{1}{2}^+ + \gamma$.

## V. CONCLUSIONS

There are several issues at stake in the comparison of these predictions with experiment and with other calculations.[14] Of most immediate concern will be the opportunity to further check the notion that the color-magnetic moments of quarks are inversely proportional to their masses: If the pattern and size of splittings in $\Lambda_c \frac{1}{2}^+$, $\Sigma_c \frac{1}{2}^+$, and $\Sigma_c \frac{3}{2}^+$ is correct then we may consider this hypothesis as verified. Certainly the $D^*$-$D$ splitting indicates that we may expect success in this regard, but the nagging $\psi$-$\eta_c$ puzzle requires this additional evidence.

Other features of these calculations that it would be interesting to check revolve around the notion of orbital mode-splitting. Tests of this splitting constitute tests of the hypothesis of the flavor independence of the quark-confinement potential; however, the quantitative size of the effect is model dependent. That $\omega_\lambda < \omega_\rho$ requires only that the kinetic energy of a quark when excited be greater than its kinetic energy when it is in the ground state—

certainly a reasonable supposition. If we then take at face value the empirical success of this mode-splitting in the $S=-1$ sector, our quantitative estimates in the $C=1$ sector ought to be reliable. Thus, if the center of gravity of $\Lambda_c\frac{1}{2}^-$ and $\Lambda_c\frac{3}{2}^-$ is where we expect, then (in spite of the uncertainties) this would constitute a valuable piece of evidence in favor of the idea that the strong interactions pay attention to the color, and not the flavor, of the quarks. Similar conclusions would follow from the discovery of a low-massed $\Lambda'\frac{1}{2}^+$. Of course measuring the $\Lambda_c\frac{5}{2}^- - \Sigma_c\frac{5}{2}^-$ mass difference would be the clearest test of this effect; such measurements may be feasible since simple models indicate that excited states of charmed baryons will be produced as readily as ground states in neutrino interactions.

The prediction that the $P$-wave baryon $\Lambda_c\frac{1}{2}^-$ will be stable against strong decay remains the most distinctive qualitative feature of our spectrum. In view of this possibility we would certainly suggest that charmed-baryon experiments have the capability of detecting photons so that $\Lambda\frac{1}{2}^+$-$\gamma$ invariant-mass plots can be made.

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## APPENDIX

Here we list some hyperfine matrix elements used in the calculations of the text but not quoted previously in the literature. They include both ground-state terms (in which we have included the effects of $\alpha_\rho \neq \alpha_\lambda$) and interband mixing terms. We have

$$\langle \Lambda_c\, {}^2S\, \tfrac{1}{2}^+ | H_{\rm hyp} | \Lambda_c\, {}^2S\, \tfrac{1}{2}^+ \rangle = -\tfrac{1}{2}\,\delta\,,$$

$$\langle \Lambda_c\, {}^2S_{\lambda\lambda}\, \tfrac{1}{2}^+ | H_{\rm hyp} | \Lambda_c\, {}^2S\, \tfrac{1}{2}^+ \rangle = 0\,,$$

$$\langle \Lambda_c\, {}^2S_{\rho\lambda}\, \tfrac{1}{2}^+ | H_{\rm hyp} | \Lambda_c\, {}^2S\, \tfrac{1}{2}^+ \rangle = \frac{\sqrt{3}}{4}\, x\delta\,,$$

$$\langle \Lambda_c\, {}^2S_{\rho\rho}\, \tfrac{1}{2}^+ | H_{\rm hyp} | \Lambda_c\, {}^2S\, \tfrac{1}{2}^+ \rangle = +\frac{\sqrt{6}}{4}\, \delta\,,$$

$$\langle \Sigma_c\, {}^2S\, \tfrac{1}{2}^+ | H_{\rm hyp} | \Sigma_c\, {}^2S\, \tfrac{1}{2}^+ \rangle = -\frac{1}{2}\left(\frac{4\hat{x}-1}{3}\right)\delta\,,$$

$$\langle \Sigma_c\, {}^2S_{\lambda\lambda}\, \tfrac{1}{2}^+ | H_{\rm hyp} | \Sigma_c\, {}^2S\, \tfrac{1}{2}^+ \rangle = \frac{\sqrt{6}}{4}\, x\delta\,,$$

$$\langle \Sigma_c\, {}^2S_{\rho\lambda}\, \tfrac{1}{2}^+ | H_{\rm hyp} | \Sigma_c\, {}^2S\, \tfrac{1}{2}^+ \rangle = \frac{\sqrt{3}}{4}\, x\delta\,,$$

$$\langle \Sigma_c\, {}^2S_{\rho\rho}\, \tfrac{1}{2}^+ | H_{\rm hyp} | \Sigma_c\, {}^2S\, \tfrac{1}{2}^+ \rangle = -\frac{\sqrt{6}}{12}\,(1-x)\delta\,,$$

$$\langle \Sigma_c\, {}^4S\, \tfrac{3}{2}^+ | H_{\rm hyp} | \Sigma_c\, {}^4S\, \tfrac{3}{2}^+ \rangle = +\frac{1}{2}\left(\frac{1+2\hat{x}}{3}\right)\delta\,,$$

$$\langle \Sigma_c\, {}^4S_{\lambda\lambda}\, \tfrac{3}{2}^+ | H_{\rm hyp} | \Sigma_c\, {}^4S\, \tfrac{3}{2}^+ \rangle = -\frac{\sqrt{6}}{8}\, x\delta\,,$$

$$\langle \Sigma_c\, {}^4S_{\rho\rho}\, \tfrac{3}{2}^+ | H_{\rm hyp} | \Sigma_c\, {}^4S\, \tfrac{3}{2}^+ \rangle = -\frac{\sqrt{6}}{8}\left(\frac{2+x}{3}\right)\delta\,,$$

where

$$\hat{x} = \frac{x}{(\frac{3}{4}y^2+\frac{1}{4})^{3/2}} \quad\text{and}\quad y^4 = \frac{2x+1}{3}\,,$$

and finally, relevant to the $\Lambda_c\frac{1}{2}^-$ and $\Lambda_c\frac{3}{2}^-$, we have

$$\langle \Lambda\, {}^2P_{\lambda(\rho\rho)} J^P | H_{\rm cont} | \Lambda\, {}^2P_\lambda J^P \rangle = \frac{\sqrt{6}}{4}\,\delta\,,$$

$$\langle \Lambda\, {}^2P_{\lambda(\lambda\lambda)} J^P | H_{\rm cont} | \Lambda\, {}^2P_\lambda J^P \rangle = 0\,,$$

where

$$P_{\lambda(\rho\rho)} \propto \lambda_+(\rho^2 - \tfrac{3}{2}\alpha_\rho^{-2})$$

and

$$P_{\lambda(\lambda\lambda)} \propto \lambda_+(\lambda^2 - \tfrac{3}{2}\alpha_\lambda^{-2})\,.$$


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