PHYSICAL REVIEW D VOLUME 20, NUMBER 3 1 AUGUST 1979

Multiquark baryons and the MIT bag model

D. Strottman
Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545
(Received 4 December 1978)

The calculation of masses of $q^4\bar{q}$ and $q^5\bar{q}^2$ baryons is carried out within the framework of Jaffe's approximation to the MIT bag model. A general method for calculating the necessary $SU(6)\supset SU(3)\otimes SU(2)$ coupling coefficients is outlined and tables of the coefficients necessary for $q^4\bar{q}$ and $q^5\bar{q}^2$ calculations are given. An expression giving the decay amplitude of an arbitrary multiquark state to arbitrary two-body final states is given in terms of SU(3) Racah and $9\text{-}\lambda\mu$ recoupling coefficients. The decay probabilities for low-lying $1/2^-$ $q^4\bar{q}$ baryons are given and compared with experiment. All low-lying $1/2^-$ baryons are found to belong to the same SU(6) representation and all known $1/2^-$ resonances below 1900 MeV may be accounted for without the necessity of introducing *P*-wave states. The masses of many exotic states are predicted including a $1/2^-$ Z_0^* at 1650 MeV and $1/2^-$ hypercharge -2 and $+3$ states at 2.25 and 2.80 GeV, respectively. The agreement with experiment for the $3/2^-$ and $5/2^-$ baryons is less good. The lowest $q^5\bar{q}^2$ state is predicted to be a $1/2^+$ Λ^* at 1900 MeV.

I. INTRODUCTION

A dogma of contemporary particle physics is that hadrons are composed of quarks[1]; all known baryons and mesons may be constructed from quarks of differing flavors. To ensure that the Pauli principle is satisfied and to provide for confinement of quarks, there being no definitive evidence that free quarks have been observed, the additional quantum number of color was introduced with the proviso that all observable hadrons be color singlets.

The lowest states of mesons and baryons are assumed composed of $q\bar{q}$ and q^3, respectively, and those hadrons not belonging to such SU(6) representations as may be obtained from the states of two or three *S*-wave quarks are assumed in the usual quark model to be constructed from a configuration in which one or more of the quarks has a nonzero relative angular momentum. This approach has obtained a considerable degree of success with the $SU(6)\otimes O(3)$ (Ref. 2) and the $SU(6)_W$ (Ref. 3) versions of the model. Because of the large numbers of parameters involved in the calculation of the masses of the hadrons, the primary success of these models is in correlating the large amount of data, although a limited amount of predictive power is possible, particularly if some particles are missing from otherwise complete multiplets.

The MIT bag model[4] is relativistic—a feature from which will derive problems in calculating decay amplitudes—the quarks being either massless or nearly so. The four unknown parameters embodied in the model have been chosen to fit the masses of certain members of the meson octet and the baryon octet and decuplet. Thus, one has the ability to investigate the nature of other baryons without having to introduce additional parameters into the model, unlike for the usual quark models. In particular, the bag model permits an investigation of those hadrons containing more than the canonical two or three quarks. Jaffe[5] has taken an initial venture in this direction by calculating properties of mesons constructed from $q^2\bar{q}^2$ and was able to argue that the results are not necessarily in disastrous disagreement with experiment. Indeed, recent work[6] suggests the assignment[5] of the $\epsilon(700)$ as a $q^2\bar{q}^2$ meson may be more nearly correct than assuming it to be a *P*-wave meson as is done in the usual quark models.

Calculations involving more than three quarks are invariably tedious; Jaffe[7] introduced a method which makes calculations more tractable albeit at the cost of ignoring the $\Lambda-\Sigma$ mass difference. Even with this simplification, one must still manipulate clusters of several quarks or antiquarks belonging to irreducible representations of SU(6), SU(3) as well as SU(2). Pedestrian techniques suffice when only two or three quarks are involved, but with a greater number of quarks, it is prudent to generalize to SU(6) and SU(3) the familiar Racah algebra of the SU(2) group. The SU(3) group has been of considerable use in the shell-model description of light nuclei,[8] and in recent years a considerable amount of technology for use in such calculations has been developed, including analytic expressions for certain $SU(3)\supset SU(2)$ coupling coefficients and Racah coefficients[9-11] as well as computer programs that allow the evaluation of essentially any $SU(3)\supset SU(2)$ Wigner or SU(3) Racah coefficient.[12] Similarly, a SU(3) $9\text{-}\lambda\mu$ coefficient which relates unitary transformations among nine SU(3) representations and is the generalization to SU(3) of the SU(2) $9\text{-}j$ symbol has been discussed.[13,14] All these devices will be necessary in treating problems involving an arbitrary number of quarks.

The SU(6) group is also familiar to nuclear spectroscopists as a transformation among the six degenerate levels in the nuclear 1s-0d shell, although in this case, the coupling coefficients are not useful as the relevant subgroup in the shell model is SU(3) rather than SU(3)⊗SU(2). However, much of the *formalism* is almost identical and may be borrowed with minimal change; the necessary SU(6)⊃SU(3)⊗SU(2) coupling coefficients have been either already calculated[15] or may be obtained in a straightforward manner.[16]

The purpose of this paper is twofold. First, more general techniques are developed to calculate the energies and decay amplitudes of multiquark states. This necessitates introducing recoupling coefficients connecting six and nine SU(3) representations and SU(6)⊃SU(3)⊗SU(2) coupling coefficients which may have outer multiplicity. This in turn requires a more precise notation to label the representations of SU(3) than its dimension. This will be done in Sec. II.

Secondly, the application of these techniques to the calculation of masses of baryons constructed from the configurations $q^4\bar{q}$ and $q^5\bar{q}^2$ and the decay amplitudes of the $q^4\bar{q}$ states and a comparison with experiment will be done in Secs. III, IV, and V.

II. MASSES AND DECAY RATES: METHOD OF CALCULATION

In this section the MIT bag model will be very briefly discussed to establish the notation for the SU(N) operators and for the basis states. The method of calculating the energy and the decay amplitudes of a several quark state will then be discussed.

In the MIT bag model[4] the mass of a hadron is assumed composed of the contributions from the rest and kinetic energies of the quarks, E_q, the contributions from quantum fluctuations (a volume term, E_v, and the zero-point energy, E_0) and the so-called color-magnetic energy arising from the exchange of colored gluons, E_g:

$$\begin{aligned} &E=E_0+E_v+E_q+E_g\,,\\ &E_q=\frac{1}{R}\sum_{i=1}[\chi(m_iR)^2+m_i{}^2R^2]^{1/2},\quad m_s=279\text{ MeV},\\ &E_v=\tfrac{4}{3}\pi R^3B,\quad B^{1/4}=146\text{ MeV},\\ &E_0=-\frac{1.84}{R}\,,\\ &E_g=-\frac{\alpha_c}{R}\sum_{i<j}\sum_a\vec{\sigma}_i\cdot\vec{\sigma}_j\lambda_i^a\lambda_j^a M(m_iR,m_jR)\,. \end{aligned}\tag{1}$$

In Eq. (1) R is the bag radius and $M(m_iR, m_jR)$ is the color-magnetic interaction energy obtained by integrating the quark wave functions over the bag. Jaffe[7] has shown that if $M(m_iR, m_jR)$ is replaced by

$$M\left(m_s\frac{n_s}{N}R,\, m_s\frac{n_s}{N}R\right),$$

where m_s is the mass of the s quark, n_s the number of s quarks, and N is the total number of quarks, the color-magnetic contribution to the energy of an n-quark–m-antiquark state may be expressed solely in terms of the quadratic group invariants (Casimir operators) of SU(2), SU(3), and SU(6). The kets are thus naturally labelled by the irreducible representations of SU(3)⊗SU(2) ⊃SU(6). Throughout this paper the subscripts f and c shall refer to the flavor and color degrees of freedom and SU(3)⊗SU(2) will be a group of transformations among color-spin variables. To ensure antisymmetry of the total wave function of identical particles, the $\text{SU}(3)_f$ representation must be the conjugate of the SU(6) representation.

The basic SU(3)⊗SU(2) n-quark–m-antiquark product ket will be written as

$$|q^n[f](\lambda_f\mu_f)(\lambda_c\mu_c)\omega S_qI_qY_q\times\bar{q}^m[\bar{f}](\bar{\lambda}_f\bar{\mu}_f)(\bar{\lambda}_c\bar{\mu}_c)\bar{\omega}\bar{S}_q\bar{I}_q\bar{Y}_q;(00)_c\omega S\rho IY\rangle\,.\tag{2}$$

The quarks and antiquarks are all assumed to have angular momentum one-half which shall be loosely referred to as spin; the quantities S_q, $\bar{S}_q$, and S_t are the spins of the n-quark configuration, the m-antiquark configuration, and of the total state, respectively. In the following, as in the ket above, $[f]=[f_1f_2\cdots f_6]$ will denote the Young tableaux which labels an irreducible representation of SU(6) and—from the relation between the irreducible representations of SU(N) and S_k (Ref. 17)—also the representations of the symmetric group of k particles.

Young tableaux may also be used to label representations of SU(3); however, if it is recalled that a column of three boxes in a Young tableaux transforms as a scalar under transformations belonging to SU(3), it is apparent that two labels will suffice to label a representation of SU(3) rather than the three indices of a Young tableaux. If $[g_1g_2g_3]$ denotes a Young tableaux labelling a representation of SU(3), then the quantities[8]

$$\begin{aligned}&\lambda=g_1-g_2\,,\\ &\mu=g_2-g_3\,,\end{aligned}\tag{3}$$

TABLE I. Allowed SU(3) ⊗ SU(2) representations contained in all SU(6) representations for three, four, and five quarks. The notation for the SU(3) ⊗ SU(2) representation is $(\lambda\mu)S$, where λ and μ are given by Eq. (3).

[f]	$(\lambda\mu)S$
[3]	$(30)\frac{3}{2}$, $(11)\frac{1}{2}$
[21]	$(30)\frac{1}{2}$, $(11)\frac{1}{2}$, $(11)\frac{3}{2}$, $(00)\frac{1}{2}$
[111]	$(11)\frac{1}{2}$, $(00)\frac{3}{2}$
[4]	(40)2, (21)1, (02)0
[31]	(40)1, (21)0, (21)1, (21)2, (02)1, (10)0, (10)1
[22]	(40)0, (21)1, (02)0, (02)2, (10)1
[211]	(21)0, (21)1, (02)1, (10)0, (10)1, (10)2
[1111]	(02)0, (10)1
[5]	$(50)\frac{5}{2}$, $(31)\frac{3}{2}$, $(12)\frac{1}{2}$
[41]	$(50)\frac{3}{2}$, $(31)\frac{1}{2}$, $(31)\frac{3}{2}$, $(31)\frac{5}{2}$, $(12)\frac{1}{2}$, $(12)\frac{3}{2}$, $(20)\frac{1}{2}$, $(20)\frac{3}{2}$, $(01)\frac{1}{2}$
[32]	$(50)\frac{1}{2}$, $(31)\frac{1}{2}$, $(31)\frac{3}{2}$, $(12)\frac{1}{2}$, $(12)\frac{3}{2}$, $(12)\frac{5}{2}$, $(20)\frac{1}{2}$, $(20)\frac{3}{2}$, $(01)\frac{1}{2}$, $(01)\frac{3}{2}$
[311]	$(31)\frac{1}{2}$, $(31)\frac{3}{2}$, $(12)\frac{1}{2}$, $(12)\frac{3}{2}$, $(20)\frac{1}{2}$, $(20)\frac{1}{2}$, $(20)\frac{3}{2}$, $(20)\frac{5}{2}$, $(01)\frac{1}{2}$, $(01)\frac{3}{2}$
[221]	$(31)\frac{1}{2}$, $(12)\frac{1}{2}$, $(12)\frac{3}{2}$, $(20)\frac{1}{2}$, $(20)\frac{3}{2}$, $(01)\frac{1}{2}$, $(01)\frac{3}{2}$, $(01)\frac{5}{2}$
[2111]	$(12)\frac{1}{2}$, $(20)\frac{1}{2}$, $(20)\frac{3}{2}$, $(01)\frac{1}{2}$, $(01)\frac{3}{2}$
[11111]	$(01)\frac{1}{2}$

provide an equivalent, albeit more convenient, labelling scheme. The dimension of the SU(3) representation $(\lambda\mu)$ is simply

$$g(\lambda\mu)=\tfrac{1}{2}(\lambda+1)(\mu+1)(\lambda+\mu+2)\,. \qquad (4)$$

This labelling scheme has certain advantages over the method of labelling a representation by its dimension. First, the use of $(\lambda\mu)$ provides a unique labelling scheme, whereas inequivalent representations may have the same dimensions [a trivial example of this is provided by the representations (01) and (10), both of which have dimension three, although the first is the representation for an antiquark and the second for a single quark]. Second, it provides a simple scheme of relating the representations of particles to those of antiparticles: If $(\lambda\mu)$ denotes an SU(3) representation of n quarks, then $(\mu\lambda)$ is the equivalent representation of n antiquarks. In particular, since the total $SU(3)_c$ representation must be (00) or a singlet, one has $(\lambda_c\mu_c)=(\overline{\mu}_c\overline{\lambda}_c)$. Third, powerful group-theoretical techniques—such as the Wigner-Eckart theorem—may be more easily introduced. This is of particular importance when corrections to the Jaffe approximation are calculated. Finally, this labelling scheme is in common usage among nuclear theorists, from whom one may borrow a vast amount of technology developed for the SU(3) shell model of light nuclei.

The total ket is necessarily a color singlet $(00)_c$, but it does not in general belong to a specific SU(3) flavor or SU(6) representation.

Because SU(3)⊗SU(2) is not a canonical subgroup of SU(6), the group labels of SU(3)⊗SU(2) will not, in general, provide a sufficient number of quantum numbers with which to label a state uniquely. In these instances the states are arbitrarily orthogonalized and a label ω is introduced[15] to distinguish the kets. This multiplicity first occurs for the five-quark state $[311](20)\frac{1}{2}$. In Table I are listed the allowed SU(3)⊗SU(2) representations which occur for SU(6) representations of up to five particles. A compilation has been given earlier by Hagen and MacFarlane,[18] although their tabulation contains several errors.

The label ρ which appears in the ket, Eq. (2), provides a labelling for orthogonal states which appear in cases of outer multiplicity (the familiar example is in the product $\underline{8}\times\underline{8}$, for which the representation of dimension $\underline{8}$ appears twice). In the present work ρ will not be needed to label the basis states since for the SU(3) representations of states of one or two antiquarks either λ or μ is zero, and one may easily show that in these instances there is no outer multiplicity. However, it is necessary in the $q^3\bar{q}^3$ states and more complicated states and in the recoupling calculations which occur in the calculation of decay amplitudes.

The remaining quantum numbers appearing in the basis ket, I and Y, are the isospin and hypercharge. Throughout the main body of this paper, the quantum numbers of the antiquarks will be written with a bar, as in Eq. (2).

As already indicated, the basis ket, Eq. (2), does not in general belong to an irreducible representation of SU(6). To obtain such a state requires the use of appropriate SU(6)$\supset$SU(3)$\otimes$SU(2) vector coupling (Clebsch-Gordan) coefficients.[15]

$$\begin{aligned}|q^n[f_1](\lambda_1\mu_1)_f I_1 Y_1 \times \bar{q}^m[\bar{f}](\bar{\lambda}_1\bar{\mu}_1)_f \bar{I}_1\bar{Y}_1;[f_t](\lambda_t\mu_t)_c S\omega\rho IY\rangle \\ = \sum_{\substack{(\lambda_1\mu_1)_c S_1\omega_1\\(\bar{\lambda}_1\bar{\mu}_1)_c \bar{S}_1\bar{\omega}_1}} \langle [f_1](\lambda_1\mu_1)_c S_1\omega_1 \times [\bar{f}_1](\bar{\lambda}_1\bar{\mu}_1)_c \bar{S}_1\bar{\omega}_1 \,|[f_t](\lambda_t\mu_t)_c S\omega\rho\rangle \\ \times |q^n[f_1](\lambda_1\mu_1)_f(\lambda_1\mu_1)_c S_1\omega_1 I_1 Y_1 \times \bar{q}^m[\bar{f}_1](\bar{\lambda}_1\bar{\mu}_1)_f(\bar{\lambda}_1\bar{\mu}_1)_c \bar{S}_1\bar{\omega}_1\bar{I}_1\bar{Y}_1;(\lambda_t\mu_t)_c S\omega IY\rangle\,. \quad (5)\end{aligned}$$

The ket on the right-hand side of Eq. (5) is just the basis ket from Eq. (2). A method of calculating the necessary vector coupling coefficients is given in the Appendix.

The magnetic-color contribution to the n-quark–m-antiquark state (5) may be written as[7]

$$\begin{aligned}E_g = M\left(m_s\frac{n_s}{N}R, m_s\frac{n_s}{N}R\right)\frac{\alpha_c}{R}\{8(n+m)+2\mathfrak{C}_6^{(2)}(\text{tot})-\tfrac{4}{3}S_t(S_t+1) \\ +4[\mathfrak{C}_3^{(2)}(q)_c-\mathfrak{C}_6^{(2)}(q)-\mathfrak{C}_6^{(2)}(\bar{q})]+\tfrac{8}{3}[S_q(S_q+1)+\bar{S}_q(\bar{S}_q+1)]\}\,,\end{aligned} \quad (6)$$

where $\mathfrak{C}_6^{(2)}$ and $\mathfrak{C}_3^{(2)}$ are the Casimir invariants of SU(6) and SU(3), respectively, the eigenvalues of which are[15]

$$\langle [f]\,|\,\mathfrak{C}_6^{(2)}\,|[f]\rangle = \sum_i^6 f_i^{\,2} - \tfrac{1}{6}\left(\sum_i^6 f_i\right)^2 + 3(f_2-f_5)+f_3-f_4+5(f_1-f_6) \quad (7)$$

and

$$\langle(\lambda\mu)\,|\,\mathfrak{C}_3^{(2)}\,|\,(\lambda\mu)\rangle = \tfrac{2}{3}(\lambda^2+\lambda\mu+\mu^2+3\lambda+3\mu)\,. \quad (8)$$

The values obtained using these expressions differ from those of Jaffe by factors of 4 and 2 for SU(6) and SU(3), respectively, and account for the difference in the coefficients appearing in Eq. (6) and the analogous formula in Ref. 7.

The basis states, Eqs. (2) and (5), do not belong in general to a specific representation of SU(3)$_f$. This is often advantageous because of the appreciable amount of SU(3)$_f$-symmetry breaking induced by the s quark having a nonzero mass. However, the calculations of the decay amplitudes of the $q^4\bar{q}$ states into $(q^3)(q\bar{q})$ states are performed most easily if the multiquark states have a specific SU(3)$_f$ representation. Fortunately, the unitary transformation is easily performed,

$$[\,|(\lambda_1\mu_1)I_1Y_1\rangle\,|(\lambda_2\mu_2)I_2Y_2\rangle]^I_{I_Z} = \sum_{(\lambda\mu)\rho}\langle(\lambda_1\mu_1)I_1Y_1(\lambda_2\mu_2)I_2Y_2\,|\,(\lambda\mu)IY\rho\rangle\,|\,(\lambda\mu)IY\rho\rangle\,; \quad (9)$$

the SU(3)$\supset$SU(2) Wigner coefficients may either be taken from de Swart[19] or from Akiyama-Draayer.[12] The square bracket on the left-hand side of Eq. (9) indicates the usual angular momentum coupling to I and I_Z.

The allowed values of hypercharge and isospin occurring in the SU(3)$_f$ representation $(\lambda_f\mu_f)$ are

$$\begin{aligned}Y &= p+q-\tfrac{1}{3}(2\lambda_f+\mu_f)\,,\\ I &= \tfrac{1}{2}(\mu_f+p-q)\,,\end{aligned} \quad (10a)$$

where the indices p and q are restricted by

$$\begin{aligned}0 &\le p \le \lambda_f\,,\\ 0 &\le q \le \mu_f\,.\end{aligned} \quad (10b)$$

These relations hold for systems of both quarks and antiquarks. The number of s quarks which are allowed for $(\lambda_f\mu_f)$ is

$$n_s = \tfrac{1}{3}(N+2\lambda_f+\mu_f)-p-q\,, \quad (11a)$$

where N is the total number of quarks, and the number of $\bar{s}$ quarks which occur in the state $(\bar{\lambda}_f\bar{\mu}_f)YI$ is

$$\bar{n}_s = \bar{p} + \bar{q} + \tfrac{1}{3}(\bar{N} - 2\bar{\mu}_f - \bar{\lambda}_f)\,. \qquad (11b)$$

These restrictions have important phenomenological implications on the mass of the lightest $q^4\bar{q}$ and $q^5\bar{q}^2$ baryons.[20]

The Pauli principle requires that for a group of identical particles, the $SU(3)_f$ representation be conjugate to the representation of SU(6); thus the imposition of an SU(6) symmetry on a system of n quarks or n antiquarks immediately restricts the allowed values of hypercharge and isospin which may occur for that SU(6) representation. Similarly, the number of s quarks or antiquarks is also restricted. E.g., the $SU(3)_f$ representation conjugate to [31] is $(10)_f$; from Eq. (11a) this state has at least one s quark. It is important to realize that this restriction only exists for *identical* particles and does not impose any relation whatever between the SU(6) representation and the allowed $SU(3)_f$ representation of the total state of n quark and m antiquarks.

The matrix elements of H_{bag} in a basis of n-quark–m-antiquark states labelled by irreducible representations of SU(6) are

$$\begin{aligned}
&\langle q^n[f_1]I_1Y_1;\bar{q}^m[\bar{f}_1]\bar{I}_1\bar{Y}_1;[f_t](00)_c\omega SI\rho|H_{\text{bag}}|q^n[f_1]I_1Y_1;\bar{q}^m[\bar{f}_1]\bar{I}_1\bar{Y}_1;[f_t'](00)_c\omega SIY\rho\rangle\\
&\quad=\sum_{\substack{S_1\omega_1(\lambda_1\mu_1)\\ \bar{S}_q\bar{\omega}_q(\bar{\lambda}_q\bar{\mu}_q)}}\langle[f_1](\lambda_1\mu_1)_cS_1\omega_1\times[\bar{f}_1](\bar{\lambda}_1\bar{\mu}_1)_c\bar{S}_1\bar{\omega}_1\,|[f_t](00)_cS_t\omega_t\rho\rangle\\
&\qquad\times\langle[f_1](\lambda_1\mu_1)_cS_1\omega_1\times[\bar{f}_1](\bar{\lambda}_1\bar{\mu}_1)_c\bar{S}_1\bar{\omega}_1\,|[f_t'](00)_cS_t\omega_t\rho\rangle\\
&\qquad\times\{[8(n+m)+2\mathfrak{C}_6(\text{tot})-\tfrac{4}{3}S_t(S_t+1)-4\mathfrak{C}_6(q)-4\mathfrak{C}_6(\bar{q})+E_0+E_v+E_q]\delta([f_t],[f_t'])\\
&\qquad\quad+4\mathfrak{C}_3(q)+\tfrac{8}{3}[S_1(S_1+1)+\bar{S}_1(\bar{S}_1+1)]\}.
\end{aligned}\qquad(12)$$

The $SU(3)_f$ representations of the quarks and antiquarks are not explicitly written as they are determined by the SU(6) representations. The evaluation of the relevant $SU(6)\supset SU(3)\otimes SU(2)$ coupling coefficients from known coefficients is described in the Appendix and the ones needed for the present calculations are given in Table II for the $q^4\bar{q}$ baryons and in Table VIII for $q^5\bar{q}^2$. For given SU(6) representations $[f_1]$ and $[f_2]$ the matrix is constructed from contributions of all allowed $[f_t]$ and diagonalized, the eigenvalues being the predicted masses.

Clearly, the eigenvectors can be expressed as linear combinations of either the irreducible representations of SU(6) or of the $SU(3)\otimes SU(2)$ basis states of Eq. (2). Although the eigenstates of H_{bag} are nearly pure SU(6) states (the worst admixture for a $q^4\bar{q}$ state is less than 4%), it is in terms of the $SU(3)\otimes SU(2)$ basis states that the decay amplitudes are most easily calculated and it is in terms of these basis states that the eigenvectors of H_{bag} are given in Table III.

For a given $SU(3)\otimes SU(2)$ basis state, Eq. (2), it is straightforward to calculate the overlap with a particular baryon × meson state. When the proper

TABLE II. The $SU(6)\supset SU(3)\otimes SU(2)$ vector coupling coefficients necessary in the $q^4\bar{q}$ calculation.

	[31] × [11111]		
$[f](\lambda\mu)S$	$(10)0\times(01)\frac{1}{2}$	$(10)1\times(01)\frac{1}{2}$	
$[42111](00)\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$	
$[21](00)\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	
	[22] × [11111]		
$[f](\lambda\mu)S$	$(10)0\times(01)\frac{1}{2}$	$(10)1\times(01)\frac{1}{2}$	$(10)2\times(01)\frac{1}{2}$
$[32211](00)\frac{1}{2}$	$1/\sqrt{2}$	$-1/\sqrt{2}$	
$[21](00)\frac{1}{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$	
$[32211](00)\frac{3}{2}$		$(\frac{5}{7})^{1/2}$	$-(\frac{2}{7})^{1/2}$
$[111](00)\frac{3}{2}$		$(\frac{2}{7})^{1/2}$	$(\frac{5}{7})^{1/2}$
	all others +1		

TABLE III. Eigenfunctions of the color-spin interaction of $q^4\bar{q}$ baryons in the $SU(3)\otimes SU(2)$ basis. Allowed states of $q^4\bar{q}$ not listed are automatically eigenstates of the color-spin interaction.

	[31] × [11111] $S=\frac{1}{2}$	
E_g	(10)0	(10)1
−6.486	0.824 38	−0.566 03
−36.181	0.566 03	0.824 38
	[211] × [11111] $S=\frac{1}{2}$	
E_g	(10)0	(10)1
11.099	0.584 71	−0.811 24
−5.766	0.811 24	0.584 71
	[211] × [11111] $S=\frac{3}{2}$	
E_g	(10)1	(10)2
13.333	0.725 48	−0.688 25
−12.000	0.688 25	0.725 48

linear combinations of the SU(3)⊗SU(2) basis states are taken to produce eigenstates of $H_{\rm bag}$ in *both* initial and final channels, this overlap is the decay amplitude of the baryon into that particular baryon × meson channel.

To calculate the overlap of an n-quark–m-antiquark basis state with a particular $(q^{n-m_0}\bar{q}^{m_0})$ $(q^{m_0}\bar{q}^{m-m_0})$ baryon-meson state, one must first decouple the m_0 quarks from the remaining $n-m_0$ quarks, decouple the m_0 antiquarks from the m-antiquark configuration using SU(6)⊃SU(3)⊗SU(2) coupling coefficients, and then recouple the $SU(3)_f$, $SU(3)_c$, and SU(2) representations to obtain the appropriate final state. Although one could decouple m_0 quarks from the n-quark configuration and then recouple them to the $m-m_0$ antiquarks by a number of SU(3)⊃SU(2) coupling coefficients, there is a great advantage to first express the basis state as a sum of $SU(3)_f$ states by application of Eq. (9), for one is then able to exploit properties of SU(3) 9-$\lambda\mu$ coefficients. The overlap between a particular n-quark–m-antiquark baryon basis state and the product of a $(n-m_0)$-quark–m_0-antiquark baryon and a m_0-quark–m_0-antiquark meson basis state may be evaluated from

$$|q^n[f_1](\lambda_1\mu_1)_f(\lambda_1\mu_1)_c S_1\omega_1 \times \bar{q}^m[\bar{f}_1](\bar{\lambda}_1\bar{\mu}_1)_f(\lambda_1\mu_1)_c\bar{S}_1\bar{\omega}_1;\,(\lambda_t\mu_t)_f(00)_c SIY\rangle$$

$$= \sum_{f_2 f_2'}\left(\frac{n_{f_2}n_{f_2'}}{n_{f_1}}\right)^{1/2}\sum_{\substack{S_2\omega_2S_2'\omega_2'\\(\lambda_2\mu_2)_c(\lambda_2'\mu_2')_c}}\langle q^{n-m_0}[f_2](\lambda_2\mu_2)_cS_2\omega_2 q^{m_0}[f_2'](\lambda_2'\mu_2')_cS_2'\omega_2'\,|[f_1](\lambda_1\mu_1)_cS_1\omega_1\rangle$$

$$\times\sum_{\bar{f}_2\bar{f}_2'}\left(\frac{\bar{n}_{f_2}\bar{n}_{f_2'}}{n_{\bar{f}_1}}\right)^{1/2}\sum_{\substack{\bar{S}_2\bar{S}_2'\bar{\omega}_2\bar{\omega}_2'\\(\bar{\lambda}_2\bar{\mu}_2)(\bar{\lambda}_2'\bar{\mu}_2')_c}}\langle q^{m-m_0}[\bar{f}_2](\bar{\lambda}_2\bar{\mu}_2)_c\bar{S}_2\bar{\omega}_2 q^{m_0}[\bar{f}_2'](\bar{\lambda}_2'\bar{\mu}_2')_c\bar{S}_2'\bar{\omega}_2'\,|[\bar{f}_1](\mu_1\lambda_1)_c\bar{S}_1\bar{\omega}_1\rangle$$

$$\times\sum_{\substack{(\lambda_B\mu_B)\\(\lambda_M\mu_M)}}U_c\begin{pmatrix}(\lambda_2\mu_2) & (\lambda_2'\mu_2') & (\lambda_1\mu_1)\\ (\bar{\lambda}_2\bar{\mu}_2) & (\bar{\lambda}_2'\bar{\mu}_2') & (\mu_1\lambda_1)\\ (00) & (00) & (00)\end{pmatrix}U_f\begin{pmatrix}(\lambda_2\mu_2) & (\lambda_2'\mu_2') & (\lambda_1\mu_1)\\ (\bar{\lambda}_2\bar{\mu}_2) & (\bar{\lambda}_2'\bar{\mu}_2') & (\lambda_2\mu_2)\\ (\lambda_B\mu_B) & (\lambda_M\mu_M) & (\lambda_t\mu_t)\end{pmatrix}$$

$$\times\sum U\begin{pmatrix}S_2 & S_2' & S_1'\\ \bar{S}_2 & \bar{S}_2' & S_2\\ S_B & S_M & S\end{pmatrix}\sum_{\substack{I_BI_M\\Y_BY_M}}\langle(\lambda_B\mu_B)_fI_BY_B\times(\lambda_M\mu_M)_fI_MY_M\,|\,(\lambda_t\mu_t)_fIY\rangle\,. \tag{13}$$

In Eq. (13) the subscripts f and c on the SU(3) 9-$\lambda\mu$ symbols imply that all the representations within that symbol are $SU(3)_f$ or $SU(3)_c$ representations, respectively, n_f is the dimension of the representation $[f]$ of the symmetric group, and $(\lambda_B\mu_B)_f$ and $(\lambda_M\mu_M)_f$ are the $SU(3)_f$ representations of the baryon and meson states. Although the SU(3) 9-$\lambda\mu$ and SU(2) 9-j symbols are unitary as are the SU(6)⊃SU(3)⊗SU(2) coefficients, the transformation represented by Eq. (13) is not a unitary representation, since the only $SU(3)_c$ representation allowed for the hadrons are color singlets, $(00)_c$. This immediately implies that $(\bar{\lambda}_2\bar{\mu}_2)_c=(\mu_2\lambda_2)_c$ and $(\bar{\lambda}_2'\bar{\mu}_2')_c=(\mu_2'\lambda_2')_c$. Using Eq. (A13) from the Appendix, the $SU(3)_c$ 9-$\lambda\mu$ symbol may be expressed in terms of the dimensions of the $SU(3)_c$ representations. Further, there is no sum in Eq. (13) over $(\lambda_2\mu_2)_f$, $(\lambda_2'\mu_2')_f$, $(\bar{\lambda}_2\bar{\mu}_2)_f$, or $(\bar{\lambda}_2'\bar{\mu}_2')_f$ as the $SU(3)_f$ representations are fixed from the requirement of antisymmetry once the SU(6) representations $[f_2]$, $[f_2']$, $[\bar{f}_2]$, and $[\bar{f}_2']$ are determined.

For the $q^4\bar{q}$ case which is of greatest interest, $m=m_0=1$, $(\lambda_2\mu_2)_c=(\bar{\lambda}_2\bar{\mu}_2)_f=(00)$, $\bar{S}_2=0$, and the 9-$\lambda\mu$ and 9-j symbols reduce to Racah coefficients. Equation (13) then simplifies to

$$|q^4[f](10)_c(\lambda_1\mu_1)_fS_1\times\bar{q}[\bar{1}](01)_c(01)_f;\,(00)_c(\lambda_t\mu_t)_fIY\rangle$$

$$=\sum_{f_2S_2}\langle[f_2](00)_cS_2\times[1](10)_c\tfrac{1}{2}\,|[f_1](10)_cS_1\rangle\left(\frac{n_{f_2}}{n_{f_1}}\right)^{1/2}$$

$$\times U_f[(\lambda_B\mu_B)(10)(\lambda_t\mu_t)(01);(\lambda_1\mu_1)(\lambda_M\mu_M)]U(S_B\tfrac{1}{2}S_t\tfrac{1}{2};S_1S_M)$$

$$\times|q^3[f_2](\lambda_B\mu_B)_f(00)_cS_B\times q\bar{q}(\lambda_M\mu_M)_f(00)_cS_M;\,(00)_c(\lambda_t\mu_t)_fS\rho IY\rangle$$

$$=\sum\bar{\gamma}([f_2],[f_1],(\lambda_M\mu_M)_f,(\lambda_t\mu_t)_f,S_B,S_M,S_1S_t\rho)$$

$$\times|q^3[f_2](\lambda_B\mu_B)_f(00)_cS_B\times q\bar{q}(\lambda_M\mu_M)_f(00)_cS_M;\,(00)_c(\lambda_t\mu_t)_fS\rho IY\rangle\,. \tag{14}$$

TABLE IV. Decay amplitudes of selected $\frac{1}{2}^-$ $q^4\bar{q}$ baryons; the quantity γ is defined by Eq. (15).

$[f_1]=[31]$, $(\lambda_B\mu_B)=(11)$, $S_B=\frac{1}{2}$

$(\lambda_M\mu_M)$	S_M	$(\lambda_t\mu_t)$	ρ	$\gamma([42111];\frac{1}{2})$	$\gamma([32111];\frac{1}{2})$	$\gamma([42111];\frac{3}{2})$
(00)	0	(11)	1	0.184	−0.088	
(11)	0	(00)	1	0.521	−0.249	
(11)	0	(11)	1	0.391	−0.187	
(11)	0	(11)	2	0.291	−0.139	
(00)	1	(11)	1	−0.088	−0.184	0.204
(11)	1	(00)	1	−0.249	−0.521	0.577
(11)	1	(11)	1	−0.187	−0.391	0.433
(11)	1	(11)	2	−0.139	−0.291	0.323

$[f_1]=[22]$, $(\lambda_B\mu_B)=(11)$, $S_B=\frac{1}{2}$

$(\lambda_M\mu_M)$	S_M	$(\lambda_t\mu_t)$	ρ	$\gamma([21];\frac{1}{2})$	$\gamma([33111];\frac{3}{2})$
(00)	0	(11)	1	0.250	
(00)	1	(11)	1	0.144	−0.289
(11)	0	(11)	1	0.177	
(11)	0	(11)	2	−0.395	
(11)	1	(11)	1	0.102	−0.204
(11)	1	(11)	2	−0.228	0.456
(11)	0	(03)	1	−0.500	
(11)	1	(03)	1	−0.289	0.577

$[f_1]=[211]$, $(\lambda_B\mu_B)=(11)$, $S_B=\frac{1}{2}$

$(\lambda_M\mu_M)$	S_M	$(\lambda_t\mu_t)$	ρ	$\gamma([32211];\frac{1}{2})$	$\gamma([21];\frac{1}{2})$	$\gamma([111];\frac{3}{2})$
(00)	0	(11)	1	−0.319	−0.052	
(11)	0	(11)	1	0.225	0.037	
(11)	0	(11)	2	−0.101	−0.016	
(11)	0	(22)	1	−0.403	−0.065	
(11)	0	(30)	1	−0.403	−0.065	
(00)	1	(11)	1	0.124	0.398	−0.181
(11)	1	(11)	1	−0.088	−0.281	0.128
(11)	1	(11)	2	0.039	0.126	−0.057
(11)	1	(22)	1	0.157	0.503	−0.229
(11)	1	(30)	1	0.157	0.503	−0.229

$[f_1]=[211]$, $(\lambda_B\mu_B)=(30)$, $S_B=\frac{3}{2}$

$(\lambda_M\mu_M)$	S_M	$(\lambda_t\mu_t)$	ρ	$\gamma([322211];\frac{1}{2})$	$\gamma([21];\frac{1}{2})$	$\gamma([111];\frac{3}{2})$	$\gamma([32211];\frac{5}{2})$
(00)	0	(30)	1			−0.094	
(11)	1	(11)	1	0.382	−0.275	0.513	−0.577
(11)	1	(22)	1	0.382	−0.275	0.513	−0.577
(11)	1	(30)	1	0.270	−0.195	0.363	−0.408
(11)	0	(11)	1			0.132	
(11)	0	(22)	1			0.132	
(11)	0	(30)	1			0.094	
(00)	1	(30)	1	−0.270	0.195	−0.363	0.408

Combining the $\bar{\gamma}$'s with the coefficients of the SU(3)⊗SU(2) basis states in the eigenstates of H_{bag}, one has

$$\gamma([f_2],[f_1],(\lambda_M\mu_M)_f,(\lambda_t\mu_t)_f,S_B,S_M,S_t,\rho)=\sum_i C(\alpha,S_i)\bar{\gamma}([f_2],[f_1],(\lambda_M\mu_M)_f,(\lambda_t\mu_t)_f,S_B,S_M,S_i,S_t,\rho)\,, \tag{15}$$

where α labels the eigenfunctions of H_{bag}. With a knowledge of the γ's which are tabulated in Table IV, the overlap between any $q^4\bar{q}$ basis state and a q^3 baryon $q\bar{q}$ meson product state may be calculated using only readily available SU(3)⊃SU(2) coupling coefficients.[12,19] The outer multiplicity ρ enters only in the octet-octet example; the definition used in Table IV is that of Ref. 12.

III. THE $\frac{1}{2}^-$ BARYONS FROM $q^4\bar{q}$

In this section a comparison between the masses and partial decay rates of the $\frac{1}{2}^-$ baryons calculated within the $q^4\bar{q}$ configuration and experiment will be given. A comparison when possible will also be made with predictions of Horgan *et al.*[2,21] who describe these same baryons as P-wave excitations in the SU(6)⊗O(3) quark model in which a phenomenological form for the mass operator containing a number of parameters is postulated and the parameters are then varied to obtain a reasonable fit to the experimental input. As already mentioned, the present calculations using the MIT bag have *no* free parameters, the parameters being fixed from other hadrons. The masses of all states which can be constructed from a $q^4\bar{q}$ configuration are given in Table V.

Surprisingly, it will emerge that the agreement for the masses is often as good, and in some places better, than with the predictions of the SU(6)⊗O(3) model. Admittedly, the SU(6)⊗O(3) calculations were concerned with more than just masses, but nevertheless, this apparent agreement with experiment is certainly very encouraging. It will also emerge that while $SU(3)_f$ is often badly broken [the admixtures due to the nonzero s quark mass being determined in this model by Eq. (9)], the SU(6) color-spin symmetry is very good with the worst admixture being less than 4%. All of the $\frac{1}{2}^-$ hadrons with mass less than 1900 MeV belong to the single SU(6) represenation [21]. (Because a column of six boxes in a Young tableaux transforms as a scalar under SU(6), [21] is equivalent to [543333] which would be the appropriate U(6) representation for the baryons.) The masses of states which can be constructed from a $q^4\bar{q}$ configuration are shown in Fig. 1. Because of the approximation made in passing from Eq. (1) to Eq. (6), the masses are quoted to the nearest 50 MeV.

The comparison between the experimental partial decay rates and the calculated decay probabilities must be taken *cum grano salis*; all dependence on the dynamics, e.g., phase space or coupling constants, has been ignored. Further, the possibility that some decays would either violate the Zweig rules or decay via the emission of an extra gluon has been ignored, although it is anticipated that such decays should be hindered.

TABLE V. The masses (in GeV) of all baryons which may be constructed from a $q^4\bar{q}$ configuration as calculated using the Jaffe approximation to the MIT bag. The SU(6) representation given is that of the four quarks. The masses are given to the nearest 50 MeV.

[31] (I, Y)	$S=\frac{1}{2}$	$S=\frac{3}{2}$
(0,0), (1,0)	1.40, 1.85	1.85
$(\frac{1}{2},-1)$, $(\frac{1}{2},1)$	1.65, 2.05	2.05
(0,0)	1.90, 2.25	2.25

[22] (I, Y)	$S=\frac{1}{2}$	$S=\frac{3}{2}$
$(\frac{1}{2},1)$	1.50	1.80
(0,0), (1,0), (0,2)	1.70	2.00
$(\frac{1}{2},-1)$, $(\frac{3}{2},-1)$, $(\frac{1}{2},1)$	1.90	2.15
(1,0)	2.10	2.35

[211] (I, Y)	$S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$
$(\frac{1}{2},1)(\frac{3}{2},1)$	1.70, 1.95	1.60, 2.00	2.00
(0,0), $(1,0)^2$, (2,0), (1,2)	1.90, 2.10	1.80, 2.15	2.15
$(\frac{1}{2},-1)^2$, $(\frac{3}{2},-1)$, $(\frac{1}{2},1)$, $(\frac{3}{2},1)$	2.05, 2.30	2.00, 2.30	2.30
(0,−2), (1,−2), (0,0), (1,0)	2.25, 2.45	2.20, 2.45	2.45
$(\frac{1}{2},-1)$	2.45, 2.60	2.40, 2.65	2.65

[1111] (I, Y)	$S=\frac{1}{2}$	$S=\frac{3}{2}$
$(\frac{3}{2},1)$, $(\frac{5}{2},1)$	2.25	2.00
(1,0), (2,0), (2,2)	2.35	2.15
$(\frac{1}{2},-1)$, $(\frac{3}{2},-1)$, $(\frac{3}{2},1)$	2.50	2.30
(0,−2), (1,−2), (1,0)	2.65	2.45
$(\frac{1}{2},-3)$, $(\frac{1}{2},-1)$	2.80	2.65
(0,−2)	2.95	2.80

A. I=0, Y=0 states

According to the rules enunciated by Jaffe,[5] the lightest-mass baryons constructed from a $q^4\bar{q}$ configuration will arise from states for which the four quarks have SU(6) symmetry [31]. The conjugate $SU(3)_f$ representation is $(10)_f$ and, as already noted elsewhere,[20] this requires that all states which belong to the nonet constructed from $q^4[31]\bar{q}$ contain at least one s quark. As the construction of a hypercharge-0 state requires either one or three (i.e., $s^2\bar{s}$) s quarks, it immediately follows that the lowest member of the nonet has hypercharge 0. The mass and decay probabilities for this and other Λ* states are given in Table VI.

The calculations obtain a Λ* particle near 1400 MeV, tantalizingly near a known resonance at 1405 with the appropriate quantum numbers. This assignment is not completely uncontentious; the Λ^* (1405) is near the $N\bar{K}$ threshold and it has been suggested that it is a virtual bound state of the $N\bar{K}$ system. The wave function of the lowest particle in this calculation does have an appreciable amount of a $N\bar{K}$ component, although the $\Sigma\pi$ component and components involving vector mesons are more important:

$$|q^4[31]\bar{q}; I=Y=0\rangle = (1/\sqrt{3})\{0.32\,|\Sigma\pi\rangle - 0.11\,|\Lambda\eta_0\rangle + 0.26\,|N\bar{K}\rangle + 0.68\,|\Sigma\rho\rangle - 0.23\,|\Lambda\phi_0\rangle + 0.55\,|N\bar{K}^*\rangle\}$$
$$+\text{components involving nonsinglet color mesons and baryons.} \qquad (16)$$

Here η_0 and ϕ_0 represent that part of the physical η and ϕ mesons having no strange quarks. The harmonic-oscillator quark model also is fraught with difficulty[22] if the $\Lambda^*(1405)$ is not included in the lowest $L=1$ [70] multiplet.

A second difficulty shared with the usual quark model involves the ratio of the coupling constants $g_{\Lambda^*\Sigma\pi}$ and $g_{\Lambda^*N\bar{K}}$; if one assumes the ratio of the amplitudes in the wave function, Eq. (16), is just the ratio of the coupling constants, then

$$\frac{g_{\Lambda^*N\bar{K}}}{g_{\Lambda^*\Sigma\pi}} = \left(\frac{2}{3}\right)^{1/2}. \qquad (17)$$

This is identical to the result—not surprisingly since, for both models, the $SU(3)_f$ representation is a nonet—obtained in the usual quark model[22] where it is usually assumed the $\Lambda^*(1405)$ is a $SU(3)_f$ singlet. However, this result is in conflict with analyses using the K-matrix formalism; the ratio of the coupling constants preferred from these analyses is a factor of 3 larger than Eq. (17), and apparently is independent of the parametrization used for the K matrix.[23]

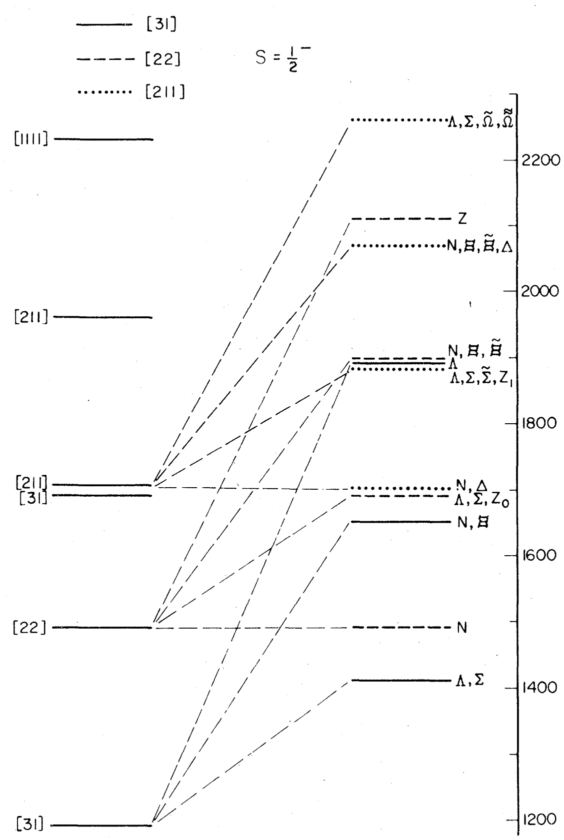

FIG. 1. The energy spectrum (in MeV) of $\frac{1}{2}^-$ $q^4\bar{q}$ baryons. The left column is the spectrum which would result were the strange quark massless, and the right column is the result assuming a strange-quark mass of 279 MeV. The $\tilde{\Xi}$ and $\tilde{\Sigma}$ are exotic particles with $(I, Y) = (\frac{3}{2}, -1)$ and $(2, 0)$, respectively. Other particles may be found from Table V.

The next heavier Λ^* particle emanates from the $q^2[22]\bar{q}$ configuration. It therefore belongs to a *different* $SU(3)_f$ multiplet than does $\Lambda^*(1405)$, unlike the usual quark model for which the two lightest Λ^* particles are members of the same nonet. Both particles belong to the same SU(6) representation [21]; however, the amplitude of the [21] for the $\Lambda^*(1405)$ is actually only 0.997 because of the mixing induced by the color-spin interaction. The $q^4[22]\bar{q}$ particle is predicted to have a mass of approximately 1700 MeV, near a $\frac{1}{2}^-$ Λ^* resonance observed at 1670 MeV. The Λ^* (1670) is observed to decay into $N\bar{K}$, $\Lambda\eta$, and $\Sigma\pi$ with approximately equal rates[24,25]; this supports the assignment of this state as deriving from $q^4[22]\bar{q}$ as the predicted rates from this state into the three channels are also similar in magnitude. The experimental decay rates suggest the ordering of states predicted by the bag model is correct since the $q^4[31]\bar{q}$ state which is here assumed to be the $\Lambda^*(1405)$ is predicted to have a very small amplitude for decay into $\Lambda\eta$. This in turn implies that the form of the contribution from the color-spin interaction, Eq. (6), is of the correct form.

There is also an observation[26] of a small amount of decay of $\Lambda^*(1670)$ to $\Sigma^*(1385)\pi$, an impossible feat for states constructed from either $q^4[31]\bar{q}$ or $q^4[22]\bar{q}$, both of which can decay only to the baryon octet. There must, therefore, be a small admixture from the $q^4[211]\bar{q}$ configuration. Such admix-

TABLE VI. Decay probabilities for selected $q^4\bar{q}$ particles as predicted using the MIT bag model. The SU(6) representations given are first that of the four quarks, and second, that of the total state or the dominant SU(6) representation in those cases where the color-spin interaction breaks SU(6). The SU(6)⊗O(3) mass estimates are from Refs. 2 and 21.

Decay probabilities for N^*					
	[22]	[31]	[211]	[22]	[211]
SU(6)	[21]	[21]	[21]	[21]	[32211]
Masses (MeV)					
Bag	1500	1650	1700	1900	1950
SU(6)⊗O(3)	1516	1670			
Observed	1535	1700			
Decay channel					
$N\eta_S$		2.3		6.3	0.0
$N\eta_0$	6.3		0.3		
ΛK		0.4		9.4	0.0
ΣK		3.5		9.4	0.0
$N\pi$	18.7		0.1		
$N\phi_S$		9.5		2.1	9.5
$N\phi_0$	2.1		19.0		
ΛK^*		1.7		3.1	14.1
ΣK^*		9.5		3.1	1.6
$N\rho$	6.3		6.4		
$\Delta\rho$			7.6		
$\Sigma^* K$					7.6

Decay probabilities of Σ^*				
	[31]	[22]	[31]	[211]
SU(6)	[21]	[21]	[42111]	[21]
Masses (MeV)				
Bag	1400	1700	1850	1900
SU(6)⊗O(3)	1587	1639	1745	
Observed	(1480)	1620	1750	
Decay channel				
$\Sigma\eta_0$	1.2	3.1	5.1	0.0
$\Sigma\pi$	2.3	6.3	10.2	0.0
$\Lambda\pi$	0.4	9.4	1.7	0.2
$N\bar{K}$	2.3	6.3	10.2	0.2
$\Sigma\phi_0$	5.1	1.0	1.2	0.5
$\Sigma\rho$	9.5	2.1	2.3	1.1
$\Lambda\rho$	1.7	3.1	0.4	14.2
$N\bar{K}^*$	9.5	2.1	2.3	9.5
$\Sigma^*\phi_0$				2.3
$\Sigma^*\rho$				5.1

Decay probabilities of Λ^*				
	[31]	[22]	[211]	[31]
SU(6)	[21]	[21]	[21]	[42111]
Masses (MeV)				
Bag	1400	1700	1900	1850
SU(6)⊗O(3)	1405	1675		
Observed	1405	1670	1870	
Decay channel				
$\Sigma\pi$	3.5	9.4	0.0	15.3
$\Lambda\eta_0$	0.4	9.4	0.2	1.7
$N\bar{K}$	2.3	6.3	0.2	9.5
$\Sigma\rho$	15.3	3.1	1.6	3.5
$\Lambda\phi_0$	1.7	3.1	14.9	0.4
$N\bar{K}^*$	9.5	2.1	9.5	2.3
$\Sigma^*\rho$			7.6	

Decay probabilities of Ξ^*			
	[31]	[22]	[22]($I=\frac{3}{2}$)
SU(6)	[21]	[21]	[21]
Masses (MeV)			
Bag	1650	1900	1900
SU(6)⊗O(3)	1724	1760	1924
Observed	(1630)	(1940)	
Decay channel			
$\Xi\eta_0$	1.2	9.4	
$\Xi\pi$	3.5	3.1	12.5
$\Lambda\bar{K}$	1.5	12.5	12.5
$\Xi\phi_0$	5.2	3.1	
$\Xi\rho$	15.3	1.0	4.2
$\Lambda\bar{K}^*$	6.8	4.2	4.2

Decay probabilities of sundry exotic particles			
	$Y=-2,\ I=0$	$Y=-2,\ I=1$	$Y=-3,\ I=\frac{1}{2}$
Masses (MeV)			
Bag	2250	2250	2800
Decay channel			
$\Xi\bar{K}$	0.4	0.4	
$\Xi\bar{K}^*$	25.3	25.3	
$\Omega^-\phi$	5.2	5.2	
$\Xi^*\bar{K}^*$	1.9	1.9	
$\Omega^-\bar{K}^*$			33.3

Decay probabilities of $Y=+2$ particles			
	Z_0^*[22]	Z_1^*[2111]	Z_2^*[1111]
SU(6)	[21]	[21]	[21]
Masses (MeV)			
Bag	1700	1900	2400
Decay channel			
NK	25.0	0.4	
NK^*	8.3	25.3	
ΔK^*		7.6	100

tures would arise by including in the Hamiltonian the small terms dropped from the color-spin interaction in passing from Eq. (1) to Eq. (8).

The bag model predicts three additional Λ^* particles in the region below 1900 MeV. There is one additional $\frac{1}{2}^-$ state identified at 1870 MeV. Its decays[24] to $N\bar{K}$ and to $\Sigma^*(1385)\pi$ suggest the association of this state with the configuration $q^4[211]\bar{q}$; the other two predicted states are from the $q^4[31]$ representations. One belongs to the [42111] representation of SU(6) and would decay primarily to $\Sigma\pi$; the other is a member of the same nonet as $\Lambda^*(1405)$, but has three s quarks. Because of the proximity of these three states to each other, it is likely that a certain amount of admixing will occur and that SU(6) will not be a valid symmetry in this energy region.

The $\Lambda^*(1870)$ decays to $\Lambda^*(1385)\pi$, whereas the $q^4[211]\bar{q}$ state is predicted to decay to $\Sigma^*\rho$. This illustrates a difficulty with the present formulation of the bag model, a feature which will be of even greater import in the discussion of the $\frac{3}{2}^-$ baryons. The $q\bar{q}$ pair is assumed to have zero orbital angular momentum relative to the Σ^* and are coupled to spin one. In the nonrelativistic quark model there is a clear distinction between orbital and spin angular momentum, but not so here. Although one might effect such a transformation, we shall content ourselves here with the knowledge that the bag model predicts an appreciable decay rate to the $\Sigma^*(1385)$.

There are several other Λ^* particles predicted around 2 GeV, although there is at present no firm experimental evidence for any resonance having $\frac{1}{2}^-$ angular momentum. It is apparent, however, that there is no need to invoke P-wave excitations of the bag to account for Λ^* particles below 1800 MeV.

B. $I=\frac{1}{2}$, $Y=1$; N^*

The lightest five-quark particle predicted to have the quantum numbers of the nucleon derives from the $q^4[22]\bar{q}$ configuration rather than $q^4[31]\bar{q}$ as the $\Lambda^*(1405)$ did; however, both of these particles belong to the representation [21] of SU(6). Consequently, although it might appear that there is an interweaving of SU(6) representations, in fact, all the lowest excited baryons of $J=\frac{1}{2}^-$ belong to the same SU(6) representation.

The lightest-mass N^* is predicted at 1500 MeV and may be identified with a known N^* at 1535 MeV.[27] From Table VI it is seen that the largest decay probability for this N^* is for $N\pi$ decay with the rate for $N\rho$ a factor of 3 smaller. Experimentally, the $N\rho$ rate appears to somewhat larger than that for $N\pi$. Again, the usual quark model[22] shares this apparent failure.

The next heavier N^* is that due to $q^4[31]$ and belongs to the same nonet as the $\Lambda^*(1405)$; it is predicted to have a mass of 1650 MeV. It is natural to associate it with the $N^*(1700)$, although difficulties present themselves when the predicted decay probabilities are compared with experiment. As already noted, since this N^* is derived from $q^4[31]\bar{q}$, it has an $s\bar{s}$ pair and its decay to $N\pi$ is, therefore, only allowed if the Okubo-Zweig-Iizuka rules are violated; one anticipates that such a decay process to be at least diminished. Unfortunately, the $N^*(1700)$'s primary decay mode appears to be $N\pi$. Further, a large decay rate to ΣK is predicted, although only a small amount is observed. The agreement is in this case, clearly less than satisfactory.

A further heavier N^* is predicted near 1700 MeV which should have a large coupling to $\Delta\rho$; no such state is reported in this region. Three additional N^* particles are predicted to have a mass of less than 2 GeV.

C. $I=1$, $Y=0$; Σ^*

The experimental information on Σ^* particles remains sparse. A Σ^* has been reported[28] at an energy of 1480 MeV, but has not been confirmed.[29] The only Σ^* particles assigned spin and parity $\frac{1}{2}^-$ have energies of 1620 and 1750 MeV, although the SU(6)$\otimes$O(3) quark model[2] predicts three particles in this mass region.

In the Jaffe approximation to the color-spin interaction, the Λ^* and Σ^* particles belonging to the same multiplet and having the same number of s quarks have the same energy. In reality, correction terms which have been dropped will split the two particles by 50 to 100 MeV; hence, one anticipates a $\frac{1}{2}^-$ Σ^* particle corresponding to the $\Lambda^*(1405)$ below 1500 MeV. It is disquieting that none have been observed, and their continued absence will create considerable difficulty for the bag model. The bag wave function for the lowest Σ^* suggests that the Σ^* couples only weakly to possible decay products, perhaps offering an explanation for its elusiveness. The decay probabilities are given in Table VI. The lowest confirmed $\frac{1}{2}^-$ Σ^* lies 270 MeV above the $\Lambda^*(1405)$, a splitting that appears difficult to obtain within the bag model.

The next two Σ^* are predicted at 1700 and 1850 MeV, somewhat above the known Σ^* particles as well as above the predictions of the SU(6)$\otimes$O(3) model if one assumes that a $\frac{1}{2}^-$ Σ^* has been experimentally missed. If one were to associate the particle predicted by the bag model at 1700 MeV with $\Sigma^*(1750)$, then the $\Sigma^*(1620)$ is 200 MeV above its predicted value of 1400 MeV. Neither do the decay rates shed much light on this puzzle. A resolution must await further experimental information and a more accurate estimate of the Σ^* masses.

D. $I=\frac{1}{2}$, $Y=-1$; Ξ^*

The bag model predicts Ξ^* states of masses of 1650 and 1900 MeV plus additional states above 2 GeV. The SU(6)⊗O(3) model[2] predicts the existence of two Ξ^* resonances at 1724 and 1760 MeV, intermediate between the energies predicted for the bag states. A Ξ^* state has recently been reported by Briefel *et al.*[30] at 1630 MeV and a possible state in the 1930-to-1960-MeV region. However, in the absence of further information on their spin and parity, it would be presumptuous to identify these resonances with the bag states.

E. $Y=2$; Z_I^*

The usual quark models construct the baryons from but three quarks and the maximum hypercharge is then one. However, introducing configurations such as $q^4\bar{q}$ allows more freedom, and it is possible to construct $Y=2$ states from such configurations. Such states are of some import: Their observation would unambiguously signal the onset of a region where the usual quark models must need be modified.

Within the context of the bag model, one can construct hypercharge two states with isospin zero, one or two from configurations where the four quarks have SU(6) symmetry [22], [211], or [1111], respectively. The masses of these Z_I^* particles are predicted to be 1700, 1900, and 2400 MeV, respectively, and are members of the $SU(3)_f$ representations (03), (22), and (41), having dimensions 10, 27, and 35, respectively. The decay modes of these particles are given in Table VI. The estimation of the energies of the Z_I^* particles is one of the few clear differences between the usual quark models and the bag; for many of the other, lower-lying resonances, the bag model and SU(6)⊗O(3) have similar predictions, but as already discussed, the usual quark models with three quarks cannot produce particles of hypercharge 2.

A dynamical calculation by Aaron, Amado, and Silbar[30] corroborates the bag-model estimates; using the relativistic three-body Blankenbecler-Sugar equation, S_1 and D_3 Z_0^* states were predicted near 1830 MeV. Although this S_1 state is 100 MeV above the predicted position of the bag state, this may not be too disturbing in view of the uncertainties in both calculations. It will be seen in the next section that the bag predicts a $\frac{3}{2}^-$ state near 2000 MeV. The failure of the Gell-Mann–Okubo mass formula in Ref. 31—in which it is assumed the D_3 Z_0^* is a member of the same 10* multiplet as the D_{13} πN resonance at 1520 MeV—to establish other members of the multiplet in which the Z_0^* resides is understandable: Only the Z_0^* is a pure 10* member. All other particles formed from $q^4[22]\bar{q}$ are mixtures of a 10* and an octet, with the admixtures being induced by the mass of the strange quarks.

The continued absence[32] of a $\frac{1}{2}^-Z_0^*$ particle is, therefore, particularly disquieting, and the importance of establishing the properties of hypercharge two particles clearly warrants continued searches, perhaps with facilities having more intense K^+ beams.

F. Sundry particles

Because of the considerable freedom allowed in the construction of states from a $q^4\bar{q}$ configuration arising from the greater number of $SU(3)_f$ representations, the possible quantum numbers of the allowed baryons is less circumscribed than in the case of baryons constructed from a q^3 configuration. In this subsection a number of particles which do not fit into any of the usual categories will be discussed.

A $\frac{1}{2}^-$ Δ^* resonance is observed at 1650 MeV; such a state may be formed from $q^4[211]\bar{q}$ and is calculated to have a mass of approximately 1700 MeV. The SU(6)⊗O(3) result[2] for this state is 1663 MeV. Other Δ^* resonances are predicted by the bag model at 1950 and 2050 MeV; the former state lies near a resonance having the appropriate quantum numbers at 1900 MeV.

The $SU(3)_f$ representation conjugate to the SU(6) representation [211] of four quarks is $(21)_f$, and the $SU(3)_f$ representations for the total state obtained from $(21)\times(01)$ are (30), (11), and (22). The former two are the familiar nonexotic decuplet and octet representations, respectively, while the latter has dimension 27 and will give rise to exotic particles. From Eq. (10a) the hypercharge may assume values from −2 to 2, the latter value being that of the Z_1^* discussed in the previous section. Particles with hypercharge −2 and with isospin zero and one are predicted near 2250 MeV.

Hypercharge −3 particles may arise from $q^4[1111]\bar{q}$ states since the $SU(3)_f$ representation of the four quarks is (40), and the product kets may have representation (41) or (30). A $Y=-3$ state is predicted to be around 2800 MeV.

IV. HIGHER-SPIN BARYONS FROM $q^4\bar{q}$

A. $\frac{3}{2}^-$ states

Baryons with angular momentum and parity $\frac{3}{2}^-$ will be discussed in this section. The discussion will be very abbreviated, in part because of the fewer number of known states, and because the agreement with experiment is less good than for the $\frac{1}{2}^-$ baryons. The calculated decay amplitude will be particularly affected by the intrinsic inseparability between orbital and spin angular momentum in a relativistic model. The predicted values of the masses using the MIT bag model assuming only S-wave quarks are given in Table V and are shown in Fig. 2.

The calculated masses of hypercharge-zero states appear to all lie approximately 200 MeV too high as compared with experiment. The two lowest observed $\frac{3}{2}^-$ Λ^* resonances have energies of 1520 and 1690 MeV, whereas the predicted values are 1800 and 1850 MeV. Similarly, the lowest $\frac{3}{2}^-$ Σ^* states are at 1580 and 1670 MeV, whereas there are several $\frac{3}{2}^-$ Σ^* states predicted in the neighborhood of 1800 MeV, but none at a lower energy.

The model appears to work somewhat better for states with hypercharge 1. The lowest N^* states are at 1520 and 1700 MeV, which is to be compared with predicted energies of 1600 and 1800 MeV. A Δ^* is known at 1670 MeV and the predicted energy is 1600 MeV.

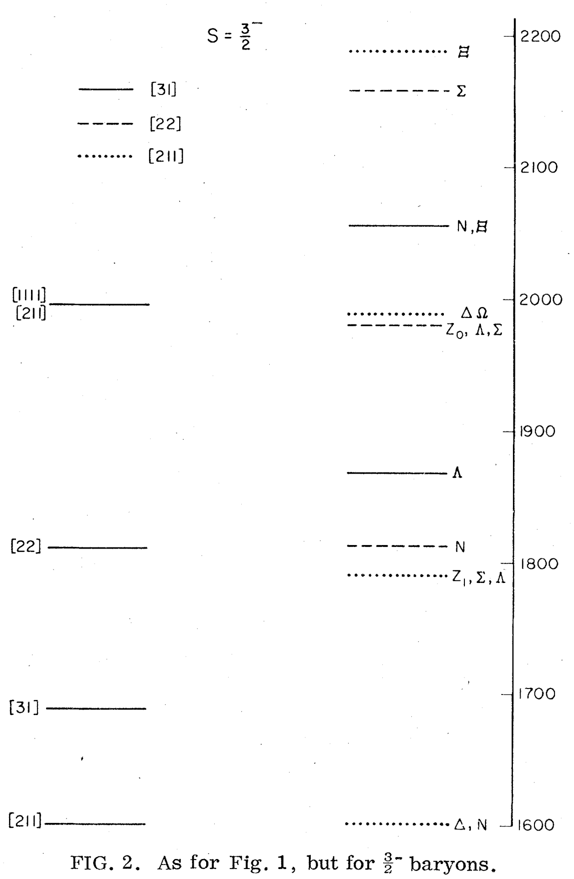

FIG. 2. As for Fig. 1, but for $\frac{3}{2}^-$ baryons.

Z_I^* states of hypercharge 2 are predicted at 2.0 and 1.8 GeV for isospin zero and one, respectively. A D_3 Z_0^* state has been reported at 1865 MeV, but needs confirmation. The calculations of Aaron *et al.*[31] predict a D_3 Z_0^* level at 1830 MeV. The near equality in answers of two such diverse approaches is encouraging.

Finally, Ω^- states are predicted at 2.2 GeV and states of hypercharge -3 at 2.8 GeV.

B. $\frac{5}{2}^-$ states

From the form of the color-spin interaction, one observes that for states originating from the same representation, the state with the higher spin will lie lower. One may anticipate this may create difficulties, as indeed it does. The lowest D_{15} N^* resonance is at 1670 MeV, whereas its predicted value is 1520 MeV. Similarly, other low-lying $\frac{5}{2}^-$ states are $\Delta^*(1960)$, $\Lambda^*(1830)$, and $\Sigma^*(1765)$, all lying lower than their predicted values of 1650, 1500, and 1600 MeV, respectively.

V. $q^5\bar{q}^2$ BARYONS

An estimate of the masses of baryons constructed from the $q^5\bar{q}^2$ configuration is of interest for several reasons. First, it provides a clue as to whether H_{bag} provides a realistic estimate of masses for large numbers of quarks; it would be disconcerting were $q^5\bar{q}^2$ baryons to have a smaller mass than $q^4\bar{q}$ baryons or that they were predicted to lie in an energy region wherein they should already have been observed. The first term in the color-spin interaction, Eq. (6), provides for a repulsion which is linear in the total number of quarks. However, the expectation value of the SU(6) Casimir operators is a *quadratic* function of the indices of the Young tableaux, is attractive and could conceivably overcome the repulsion linear in the number of quarks. That this does not occur is a manifestation of antisymmetry which requires $f_i \leq 3$, so there is an essential limit on the amount of attraction. This attraction is further offset when f_i equals three by the condition imposed by Eq. (11), namely, there must be at least one s quark. If the number of possible flavors ascribed to the quarks were increased, thereby relaxing the restriction on the f_i, more attraction might be obtained from the Casimir operators of SU(6), but only at the expense of including more massive quarks.

Secondly, the calculation of $q^5\bar{q}^2$ baryons provides an estimate of the density of baryon states at higher energies which is of interest not only in particle physics, but also in the high-energy scattering of two heavy ions.[33] Finally, one may estimate the energies of particularly exotic par-

TABLE VII. Masses (in GeV) of selected baryons constructed from $q^5\bar{q}^2$ configurations. Only the lowest two energies are given for each spin. The SU(6) representations are those of the five quarks and two antiquarks. All masses are quoted to the nearest 50 MeV.

(I, Y)	$[32]\times[\bar{2}]$ $S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$
(0, 0)	1.90, 2.30	2.15, 2.30	2.55
$(\frac{1}{2},-1)$, $(\frac{1}{2},1)$	2.10, 2.50	2.35, 2.50	2.75
(0, 0), (1, 0)	2.35, 2.70	2.60, 2.70	2.95

(I, Y)	$[32]\times[\bar{1}\bar{1}]$ $S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$
(1, 0)	2.00, 2.35	2.30, 2.45	2.55
$(\frac{1}{2},-1)$, $(\frac{3}{2},-1)$, $(\frac{1}{2},1)$	2.20, 2.50	2.45, 2.60	2.75
(0, 0), (1, 0), (0, 2)	2.45, 2.75	2.70, 2.80	2.95
$(\frac{1}{2},1)$	2.70, 2.95	2.90, 3.00	3.10

(I, Y)	$[311]\times[\bar{2}]$ $S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$	$S=\frac{7}{2}$
(1, 0)	2.10, 2.35	2.00, 2.45	2.45, 2.85	2.85
$(\frac{1}{2},-1)$, $(\frac{1}{2},1)$, $(\frac{3}{2},1)$	2.30, 2.55	2.20, 2.60	2.60, 3.00	3.00
(0, −2), (0, 0), (1, 0)	2.55, 2.75	2.45, 2.80	2.80, 3.15	3.15
$(\frac{1}{2},-1)$	2.75, 2.95	2.70, 3.00	3.00, 3.30	3.30

(I, Y)	$[311]\times[\bar{1}\bar{1}]$ $S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$
(0, 0), (1, 0), (2, 0)	2.20, 2.45	2.45, 2.55	2.45, 2.85
$(\frac{1}{2},-1)$, $(\frac{3}{2},-1)$, $(\frac{1}{2},1)$, $(\frac{3}{2},1)$	2.40, 2.60	2.65, 2.75	2.65, 3.00
(1, −2), (0, 0), (1, 0), (1, 2)	2.60, 2.80	2.85, 3.00	2.85, 3.15
$(\frac{1}{2},-1)$, $(\frac{1}{2},1)$	2.85, 3.00	3.05, 3.10	3.05, 3.30
(0, 0)	3.05, 3.20	3.25, 3.30	3.25, 3.50

(I, Y)	$[221]\times[\bar{2}]$ $S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$
$(\frac{1}{2},1)$	2.05, 2.40	2.25, 2.45	2.30
(1, 0), (0, 0), (0, 2), (1, 2)	2.25, 2.55	2.45, 2.60	2.45
$(\frac{3}{2},-1)$, $(\frac{1}{2},-1)$, $(\frac{1}{2},1)^2$, $(\frac{3}{2},1)$	2.45, 2.75	2.65, 2.80	2.65
(1, −2), $(1,0)^2$, (2, 0), (0, 0)	2.65, 2.95	2.85, 3.00	2.85
$(\frac{1}{2},-1)$, $(\frac{3}{2},-1)$	2.90, 3.10	3.05, 3.20	3.05

ticles, e.g., particles having $|Y|=3$, $Y=-4$, or isospin $\frac{7}{2}$.

The lowest $\frac{1}{2}^+$ $q^5\bar{q}^2$ baryon (see Table VII) is predicted to be a Λ^* at an energy around 1900 MeV and is a $SU(3)_f$ singlet; it is 500 MeV above the lowest $q^4\bar{q}$ state. The lowest $\frac{1}{2}^+$ Σ^* and N^* are estimated to appear at 2.0 and 2.1 GeV, respectively. The lowest $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states with hypercharge 3 are predicted at 2.65 and 2.55 GeV, respectively, with the lowest $\frac{3}{2}^+$ $Y=-3$ state at 3.15 GeV. The lowest $\frac{1}{2}^+$ $Y=-4$ state does not appear until 3.5 and 3.7 GeV for isospin zero and one, respectively.

VI. CONCLUSION

In this paper a systematic analysis of all baryons which may be constructed from four quarks and one antiquark has been made. The calculation of the masses and the decay amplitudes of these baryons—or indeed, any multiquark hadron—is made viable by the calculation of the necessary $SU(6)\supset SU(3)\otimes SU(2)$ vector coupling coefficients. The method used to calculate these coefficients employs SU(3) Racah and 9-$\lambda\mu$ recoupling coefficients and was briefly outlined. Tables of the necessary coefficients are given for configurations of $q^4\bar{q}$ and $q^5\bar{q}^2$.

A detailed comparison with experiment was made for the $\frac{1}{2}^-$ baryons and the agreement was very good. Particularly striking was the agreement with experiment for the Λ^*, N^*, and Δ^* particles; all observed states could easily be associated with states predicted by the bag model. In particular, if the basic assumptions of the MIT bag model are correct and if the parameters of the model as chosen in Ref. 4 are essentially correct, then resonances which had previously been thought to be P-wave states are rather states constructed from $q^4\bar{q}$ configurations. There is then no room for additional states and the lowest P-wave $\frac{1}{2}^-$ state must be at a much higher energy than was previously thought, say 1.8 to 2.0 GeV. Neither the bag model nor the usual quark models can say anything unambiguous about the positions of such states.

The bag model predicts several states not yet observed. The most disturbing of these is a

$\frac{1}{2}^-$ Σ^* state expected in the 1.4 to 1.5 GeV region. It is the only particle in the lowest $SU(3)_f$ multiplet for which no known resonance may be identified. Further particles for which searchers should be made are the perenially elusive Z_I^* states expected above 1.65 MeV and states of hypercharge −2 and −3 at 2.2 and 2.3 GeV, respectively.

The comparison with known resonances of the predicted $\frac{3}{2}^-$ and $\frac{5}{2}^-$ baryons is less auspicious. This poor agreement may reflect the possibility that H_{bag} does not adequately account for all the quark-quark spin-dependent forces or that the influence of P-wave quarks or deformations may be more important for higher-spin states. One method to circumvent such difficulties is to generalize the baryonium model[34] and to apply concepts introduced by Johnson and Thorn[35]—and applied by Jaffe[36] to the $N\bar{N}$ system—which relate the slope of the Regge trajectory to the $SU(3)_c$ Casimir operator; such attempts are under way.[37]

The masses of baryons constructed from all

TABLE VIII. The SU(6) ⊃SU(3)⊗SU(2) vector coupling coefficients necessary for a $q^5\bar{q}^2$ calculation.

		$[32]\times[\bar{2}]$		
$[f]S$	$(20)\frac{1}{2}\times(02)1$	$(20)\frac{3}{2}\times(02)1$	$(01)\frac{1}{2}\times(10)0$	$(01)\frac{3}{2}\times(10)0$
$[432]\frac{1}{2}$	$(\frac{5}{21})^{1/2}$	$-(\frac{1}{21})^{1/2}$	$-(\frac{5}{7})^{1/2}$	
$[42111]\frac{1}{2}$	$(\frac{121}{168})^{1/2}$	$(\frac{5}{42})^{1/2}$	$(\frac{9}{56})^{1/2}$	
$[321111]\frac{1}{2}$	$(\frac{1}{24})^{1/2}$	$-(\frac{5}{6})^{1/2}$	$(\frac{1}{8})^{1/2}$	
$[432]\frac{3}{2}$	$\frac{5}{6}$	$(\frac{1}{126})^{1/2}$		$(\frac{25}{42})^{1/2}$
$[42111]\frac{3}{2}$	$(\frac{8}{21})^{1/2}$	$(\frac{25}{84})^{1/2}$		$-(\frac{9}{28})^{1/2}$
$[33111]\frac{3}{2}$	$(\frac{2}{9})^{1/2}$	$-\frac{5}{6}$		$-(\frac{1}{12})^{1/2}$
$[432]\frac{5}{2}$		1		

		$[32]\times[\bar{1}\bar{1}]$		
$[f]S$	$(20)\frac{1}{2}\times(02)0$	$(01)\frac{1}{2}\times(10)1$	$(01)\frac{3}{2}\times(10)1$	$(20)\frac{3}{2}\times(02)0$
$[4311]\frac{1}{2}$	$-(\frac{3}{5})^{1/2}$	$(\frac{1}{5})^{1/2}$	$(\frac{1}{5})^{1/2}$	
$[42111]\frac{1}{2}$	$-(\frac{1}{40})^{1/2}$	$-(\frac{27}{40})^{1/2}$	$(\frac{3}{10})^{1/2}$	
$[21]\frac{1}{2}$	$(\frac{3}{8})^{1/2}$	$(\frac{1}{8})^{1/2}$	$(\frac{1}{2})^{1/2}$	
$[4311]\frac{3}{2}$		$(\frac{5}{7})^{1/2}$	$-(\frac{1}{14})^{1/2}$	$-(\frac{3}{14})^{1/2}$
$[42111]\frac{3}{2}$		0	$(\frac{3}{4})^{1/2}$	$-\frac{1}{2}$
$[33111]\frac{3}{2}$		$(\frac{2}{7})^{1/2}$	$(\frac{5}{28})^{1/2}$	$(\frac{15}{28})^{1/2}$
$[4311]\frac{5}{2}$			1	

		$[311]\times[\bar{2}]$		
$[f]S$	$(20)\frac{1}{2}\times(02)1$	$(20)\frac{1}{2}\times(02)1$	$(20)\frac{3}{2}\times(02)1$	$(01)\frac{1}{2}\times(10)0$
$[4311]\frac{1}{2}$	$-\frac{1}{3}$	$-(\frac{8}{27})^{1/2}$	$-(\frac{1}{27})^{1/2}$	$(\frac{5}{9})^{1/2}$
$[42111]\frac{1}{2}$	$\frac{3}{4}$	$(\frac{1}{24})^{1/2}$	$(\frac{1}{12})^{1/2}$	$(\frac{5}{16})^{1/2}$
$[32211]\frac{1}{2}$	$(\frac{1}{72})^{1/2}$	$-(\frac{49}{108})^{1/2}$	$(\frac{25}{54})^{1/2}$	$-(\frac{5}{72})^{1/2}$
$[21]\frac{1}{2}$	$(\frac{5}{16})^{1/2}$	$-(\frac{5}{24})^{1/2}$	$-(\frac{5}{12})^{1/2}$	$-\frac{1}{4}$

		$[311]\times[\bar{1}\bar{1}]$		
$[f]S$	$(20)\frac{1}{2}\times(02)0$	$(20)\frac{1}{2}\times(02)0$	$(01)\frac{1}{2}\times(10)1$	$(01)\frac{1}{2}\times(10)1$
$[4221]\frac{1}{2}$	$(\frac{1}{7})^{1/2}$	$(\frac{8}{21})^{1/2}$	$-(\frac{5}{63})^{1/2}$	$-(\frac{25}{63})^{1/2}$
$[42111]\frac{1}{2}$	$-(\frac{5}{16})^{1/2}$	$(\frac{5}{24})^{1/2}$	$-\frac{7}{12}$	$(\frac{5}{36})^{1/2}$
$[32211]\frac{1}{2}$	$(\frac{27}{56})^{1/2}$	$-(\frac{1}{28})^{1/2}$	$-(\frac{15}{56})^{1/2}$	$(\frac{3}{14})^{1/2}$
$[21]\frac{1}{2}$	$\frac{1}{4}$	$(\frac{3}{8})^{1/2}$	$(\frac{5}{16})^{1/2}$	$\frac{1}{2}$

TABLE VIII. *(Continued)*

	$[311]\times[\bar{1}\bar{1}]$			
$[f]S$	$(20)_{\frac{3}{2}}\times(02)0$	$(01)_{\frac{1}{2}}\times(10)1$	$(01)_{\frac{3}{2}}\times(10)1$	$(20)_{\frac{5}{2}}\times(02)0$
$[4221]_{\frac{3}{2}}$	$(\frac{1}{6})^{1/2}$	$(\frac{5}{9})^{1/2}$	$-(\frac{5}{18})^{1/2}$	
$[42111]_{\frac{3}{2}}$	$-(\frac{5}{24})^{1/2}$	$\frac{2}{3}$	$(\frac{25}{72})^{1/2}$	
$[32211]_{\frac{3}{2}}$	$(\frac{5}{8})^{1/2}$	0	$(\frac{3}{8})^{1/2}$	
$[4221]_{\frac{5}{2}}$			$(\frac{5}{7})^{1/2}$	$(\frac{2}{7})^{1/2}$
$[32211]_{\frac{5}{2}}$			$-(\frac{2}{7})^{1/2}$	$(\frac{5}{7})^{1/2}$

			$[221]\times[\bar{2}]$		
$[f]S$	$(20)_{\frac{1}{2}}\times(02)1$	$(20)_{\frac{3}{2}}\times(02)1$	$(01)_{\frac{1}{2}}\times(10)0$	$(01)_{\frac{3}{2}}\times(10)0$	$(01)_{\frac{5}{2}}\times(10)0$
$[4221]_{\frac{1}{2}}$	$-(\frac{1}{21})^{1/2}$	$-(\frac{5}{21})^{1/2}$	$(\frac{5}{7})^{1/2}$		
$[32211]_{\frac{1}{2}}$	$-(\frac{15}{28})^{1/2}$	$(\frac{3}{7})^{1/2}$	$(\frac{1}{28})^{1/2}$		
$[21]_{\frac{1}{2}}$	$(\frac{5}{12})^{1/2}$	$(\frac{1}{3})^{1/2}$	$\frac{1}{2}$		
$[4221]_{\frac{3}{2}}$	$(\frac{1}{3})^{1/2}$	$-(\frac{1}{6})^{1/2}$		$(\frac{1}{2})^{1/2}$	
$[33111]_{\frac{3}{2}}$	$(\frac{2}{3})^{1/2}$	$(\frac{1}{12})^{1/2}$		$-\frac{1}{2}$	
$[32211]_{\frac{3}{2}}$	0	$(\frac{3}{4})^{1/2}$		$\frac{1}{2}$	
$[4221]_{\frac{5}{2}}$		$(\frac{2}{7})^{1/2}$			$(\frac{4}{7})^{1/2}$
$[32211]_{\frac{5}{2}}$		$(\frac{4}{7})^{1/2}$			$-(\frac{3}{7})^{1/2}$

			$[221]\times[\bar{1}\bar{1}]$		
$[f]S$	$(20)_{\frac{1}{2}}\times(02)0$	$(01)_{\frac{1}{2}}\times(10)1$	$(01)_{\frac{3}{2}}\times(10)1$	$(20)_{\frac{3}{2}}\times(02)0$	$(01)_{\frac{5}{2}}\times(10)1$
$[3321]_{\frac{1}{2}}$	$-(\frac{1}{3})^{1/2}$	$(\frac{5}{9})^{1/2}$	$\frac{1}{3}$		
$[32211]_{\frac{1}{2}}$	$\frac{1}{2}$	$(\frac{5}{12})^{1/2}$	$-(\frac{1}{3})^{1/2}$		
$[21]_{\frac{1}{2}}$	$(\frac{5}{12})^{1/2}$	$\frac{1}{6}$	$(\frac{5}{9})^{1/2}$		
$[3321]_{\frac{3}{2}}$		$(\frac{1}{3})^{1/2}$	$-(\frac{1}{150})^{1/2}$	$(\frac{1}{2})^{1/2}$	$-\frac{2}{5}$
$[33111]_{\frac{3}{2}}$		$(\frac{2}{9})^{1/2}$	$\frac{5}{6}$	$-(\frac{1}{12})^{1/2}$	0
$[32211]_{\frac{3}{2}}$		$(\frac{8}{21})^{1/2}$	$-(\frac{361}{2100})^{1/2}$	$-(\frac{1}{28})^{1/2}$	$(\frac{72}{175})^{1/2}$
$[111]_{\frac{3}{2}}$		$-(\frac{4}{63})^{1/2}$	$(\frac{8}{63})^{1/2}$	$(\frac{8}{21})^{1/2}$	$(\frac{3}{7})^{1/2}$
$[3321]_{\frac{5}{2}}$			$(\frac{21}{25})^{1/2}$		$-\frac{2}{5}$
$[32211]_{\frac{5}{2}}$			$\frac{2}{5}$		$(\frac{21}{25})^{1/2}$

	$[2111]\times[\bar{2}]$			
$[f]S$	$(20)_{\frac{1}{2}}\times(02)1$	$(20)_{\frac{3}{2}}\times(02)1$	$(01)_{\frac{1}{2}}\times(10)0$	$(01)_{\frac{3}{2}}\times(10)0$
$[42111]_{\frac{1}{2}}$	$(\frac{25}{48})^{1/2}$	$-(\frac{1}{6})^{1/2}$	$(\frac{5}{16})^{1/2}$	
$[32211]_{\frac{1}{2}}$	$-(\frac{3}{8})^{1/2}$	0	$(\frac{5}{8})^{1/2}$	
$[21]_{\frac{1}{2}}$	$(\frac{5}{48})^{1/2}$	$(\frac{5}{6})^{1/2}$	$\frac{1}{4}$	
$[42111]_{\frac{3}{2}}$	$-(\frac{1}{12})^{1/2}$	$(\frac{5}{12})^{1/2}$		$(\frac{1}{2})^{1/2}$
$[32211]_{\frac{3}{2}}$	$-(\frac{15}{28})^{1/2}$	$(\frac{3}{28})^{1/2}$		$-(\frac{5}{14})^{1/2}$
$[111]_{\frac{3}{2}}$	$(\frac{8}{21})^{1/2}$	$(\frac{10}{21})^{1/2}$		$-(\frac{1}{7})^{1/2}$
$[32211]_{\frac{5}{2}}$		1		

allowed $q^5\bar{q}^2$ configurations were calculated; although the spectrum is considerably richer than for q^3 or $q^4\bar{q}$, and baryons with more exotic quantum numbers may be calculated, their masses are considerably heavier. The lowest $q^5\bar{q}^2$ baryon is a Λ^* expected near 1.9 GeV.

In conclusion we note that the MIT bag model with the Jaffe approximation appears to be remarkably successful in accounting for the masses of $\frac{1}{2}^-$ baryons, particularly as there were no free parameters available. It is essential for the model to remain credible, however, that either a $\frac{1}{2}^-$ Σ^* particle be found in the 1.4- to 1.5-GeV region or that it be shown that corrections to the Jaffe approximation elevate the predicted Σ^* to a higher energy. The success of the bag model should encourage a more detailed examination of the effects of deformation[38] and of P-wave quarks[39,40] on the bag-model estimates.

ACKNOWLEDGMENT

This work was performed under the auspices of the United States Department of Energy.

APPENDIX

The $SU(6)\supset SU(3)\otimes SU(2)$ Wigner coefficients for which a single quark couples to a state of several quarks will be referred to henceforth as one-body Wigner coefficients. In this appendix it will be shown how to calculate the SU(6) Wigner coefficients appropriate for coupling states of several quarks to states of several antiquarks. Although one could make use of the procedures of Ref. 16, the procedure discussed here is simpler when more than one antiquark is involved.

Consider a basis state in which configurations of n_1 quarks and n_2 antiquarks are separately members of irreducible representations of SU(6), SU(3), and SU(2). The product state may immediately be coupled to a representation of SU(3) and SU(2) through the use of the appropriate SU(3) and SU(2) Wigner coefficients.

$$\begin{aligned}|q^{n_1}[f](\lambda_1\mu_1)S_1\omega_1\times\bar{q}^{n_2}[f_2](\lambda_2\mu_2)S_2\omega_2;(\lambda\mu)S\omega\rho\, II_3Y\rangle \\ = \sum_{\substack{I_1I_{1,3}\\ I_2I_{2,3}}}\langle(\lambda_1\mu_1)I_1Y_1(\lambda_2\mu_2)I_2Y_2|(\lambda\mu)IY\rangle_\rho \\ \times\sum_{\substack{I_{1,3}\\ I_{2,3}}}\langle I_1I_2I_{1,3}I_{2,3}|II_3\rangle|q^{n_1}[f_1](\lambda_1\mu_1)S_1\omega_1I_1I_{1,3}Y_1\rangle|\bar{q}^{n_2}[f_2](\lambda_2\mu_2)S_2\omega_2I_2I_{2,3}Y_2\rangle\,. \end{aligned}\tag{A1}$$

In Eq. (A1) ρ is the outer multiplicity. The $SU(3)_f$ representations which follow from the antisymmetry of the quark and antiquark states will not be written explicitly. The correct linear combination of the basis states to produce a product state which is labelled by SU(6) may be found by diagonalizing the Casimir operator of SU(6)

$$\mathfrak{C}_6^{(2)}=\sum_\mu^{35}\alpha_\mu\alpha_\mu;\quad \alpha_\mu\equiv\sum_{i=1}^{n_1}\alpha_\mu(i)-\sum_{i=1}^{n_2}\bar{\alpha}_\mu(i)\,,\tag{A2}$$

where the α_μ are the generators of SU(6). The Casimir operator may be divided into three terms

$$\mathfrak{C}_6^{(2)}=\mathfrak{C}_6^{(2)}(n_1)+\mathfrak{C}_6^{(2)}(n_2)-2\sum_{\substack{i\in n_1\\ j\in n_2}}\sum_\mu\alpha_\mu(i)\bar{\alpha}_\mu(j)\,,\tag{A3}$$

where the argument of $\mathfrak{C}_6^{(2)}$ in the first two terms on the right-hand side indicates that the Casimir operator acts only on the group of n_1 quarks or n_2 antiquarks. As the quark states in Eq. (A1) are separately antisymmetric, any particular quark among the first n_1 quarks—say quark n_1—may be singled out from the sum over i, and similarly antiquark n_2 may be singled out from the sum over j, and the matrix element multiplied by n_1n_2:

$$\mathfrak{C}_6^{(2)}=\mathfrak{C}_6^{(2)}(n_1)+\mathfrak{C}_6^{(2)}(n_2)-2n_1n_2\sum_\mu\alpha_\mu(n_1)\bar{\alpha}_\mu(n_2)\,.\tag{A4}$$

The last term of Eq. (A4) may be related to a two-quark SU(6) Casimir operator:

$$-2\sum_\mu\alpha_\mu(n_1)\bar{\alpha}_\mu(n_2)=\sum_\mu[\alpha_\mu(n_1)-\bar{\alpha}_\mu(n_2)]^2-\sum_\mu\alpha_\mu(n_1)^2-\sum_\mu\bar{\alpha}_\mu(n_2)^2\,.\tag{A5}$$

The expectation value of $\mathfrak{C}_6^{(2)}$ may now be evaluated with the use of Eq. (A4) and Eq. (A6)

$$\langle q^{n_1}[f_1](\lambda_1\mu_1)S_1\omega_1\otimes\bar{q}^{n_2}[f_2](\lambda_2\mu_2)S_2\omega_2;(\lambda\mu)S\omega\rho\,|\,\mathcal{C}_6^{(2)}\,|\,q^{n_1}[f_1](\bar{\lambda}_1\bar{\mu}_1)\bar{S}_1\bar{\omega}_1\otimes\bar{q}^{n_2}[f_2](\bar{\lambda}_2\bar{\mu}_2)\bar{S}_2\bar{\omega}_2;(\lambda\mu)S\omega\rho\rangle$$
$$=\delta_{1\bar{1}}\delta_{2\bar{2}}\{\langle\mathcal{C}_6^{(2)}([f_1])\rangle+\langle\mathcal{C}_6^{(2)}([f_2])\rangle-2n_1n_2\langle\mathcal{C}_6^{(2)}([1])\rangle\}$$
$$+n_1n_2\langle[f_1](\lambda_1\mu_1)S_1\omega_1\otimes[f_2](\lambda_2\mu_2)S_2\omega_2;(\lambda\mu)S\omega\,|$$
$$\times\sum_{\mu}[\alpha_\mu(n_1)-\alpha_\mu(n_2)]^2\,|\,[f_1](\bar{\lambda}_1\bar{\mu}_1)\bar{S}_1\bar{\omega}_1\otimes[f_2](\bar{\lambda}_2\bar{\mu}_2)\bar{S}_2\bar{\omega}_2;(\lambda\mu)S\omega\rangle\,, \tag{A6}$$

where

$$\delta_{1\bar{1}}\delta_{2\bar{2}}\equiv\delta((\lambda_1\mu_1),(\bar{\lambda}_1\bar{\mu}_1))\delta((\lambda_2\mu_2),(\bar{\lambda}_2\bar{\mu}_2))\delta(S_1\bar{S}_1)\delta(S_2\bar{S}_2)\delta(\omega_1\bar{\omega}_1)\delta(\omega_2\bar{\omega}_2)\,.$$

In this appendix a bar appearing over a quantum number indicates that the quantum number is a label for the right-hand state, unlike in the main body of the paper where a barred quantity referred to the antiquark system. The only exception is that $\bar{\alpha}_\mu$ is an SU(6) generator for the state of the antiquarks.

The last term in Eq. (A6) is evaluated by decoupling quark n_1 from the first group and quark n_2 from the second group of quarks by using the Wigner coefficients from Ref. 15, and recoupling using 9-j and SU(3) 9-$\lambda\mu$ coefficients. The last term of Eq. (A6) becomes

$$\sum_{\substack{(\lambda_1'\mu_1')S_1'\omega_1'\\ [f_1']}}\left\langle\begin{matrix}[f_1'] & [1]\\ (\lambda_1'\mu_1')S_1'\omega_1' & (10)\tfrac{1}{2}\end{matrix}\,\middle|\,\begin{matrix}[f_1]\\ (\lambda_1\mu_1)S_1\omega_1\end{matrix}\right\rangle\left\langle\begin{matrix}[f_1'] & [1]\\ (\lambda_1'\mu_1')S_1'\omega_1' & (10)\tfrac{1}{2}\end{matrix}\,\middle|\,\begin{matrix}[f_1]\\ (\bar{\lambda}_1\bar{\mu}_1)\bar{S}_1\bar{\omega}_1\end{matrix}\right\rangle\frac{n_{f_1'}}{n_{f_1}}$$
$$\times\sum_{\substack{(\lambda_2'\mu_2')S_2'\omega_2'\\ [f_2']}}\left\langle\begin{matrix}[f_2'] & [1]\\ (\lambda_2'\mu_2')S_2'\omega_2' & (10)\tfrac{1}{2}\end{matrix}\,\middle|\,\begin{matrix}[f_2]\\ (\lambda_2\mu_2)S_2\omega_2\end{matrix}\right\rangle\left\langle\begin{matrix}[f_2'] & [1]\\ (\lambda_2'\mu_2')S_2'\omega_2' & (10)\tfrac{1}{2}\end{matrix}\,\middle|\,\begin{matrix}[f_2]\\ (\bar{\lambda}_2\bar{\mu}_2)\bar{S}_2\bar{\omega}_2\end{matrix}\right\rangle\frac{n_{f_2'}}{n_{f_2}}$$
$$\times\sum_{(\lambda'\mu')(\lambda_{23}\mu_{23})}U\begin{pmatrix}(\lambda_1'\mu_1') & (10) & (\lambda_1\mu_1)\\ (\lambda_2'\mu_2') & (01) & (\lambda_2\mu_2)\\ (\lambda'\mu') & (\lambda_{23}\mu_{23}) & (\lambda\mu)\end{pmatrix}_\rho U\begin{pmatrix}(\lambda_1'\mu_1') & (10) & (\bar{\lambda}_1\bar{\mu}_1)\\ (\lambda_2'\mu_2') & (01) & (\bar{\lambda}_2\bar{\mu}_2)\\ (\lambda'\mu') & (\lambda_{23}\mu_{23}) & (\lambda\mu)\end{pmatrix}_{\bar{\rho}}$$
$$\times\sum_{S'S_{23}}U\begin{pmatrix}S_1' & \tfrac{1}{2} & S_1\\ S_2' & \tfrac{1}{2} & S_2\\ S' & S_{23} & S\end{pmatrix}U\begin{pmatrix}S_1' & \tfrac{1}{2} & \bar{S}_1\\ S_2' & \tfrac{1}{2} & \bar{S}_2\\ S' & S_{23} & S\end{pmatrix}$$
$$\times\left\langle q\bar{q}(\lambda_{23}\mu_{23})S_{23}\,\middle|\,\sum_\mu[\alpha_\mu(n_1)-\alpha_\mu(n_2)]^2\,\middle|\,q\bar{q}(\lambda_{23}\mu_{23})S_{23}\right\rangle\,, \tag{A7}$$

where n_f is the dimension of the representation $[f]$ of the symmetric group. The value of the two-quark matrix element in Eq. (A7) is $\mathcal{C}_6^{(2)}(q^2)$. The one-body Wigner coefficient relating the product of an antiquark and n_2-1 antiquarks to a state of n_2 antiquarks is the *same* as the coefficient relating the product of a quark and n_2-1 quarks with a state of n_2 quarks. To emphasize this the Wigner coefficients involving antiquarks are written with the SU(3)⊗SU(2) representation $(10)\frac{1}{2}$. The SU(3) representations of a $q\bar{q}$ state are (00) and (11), for each of which the spin may be either zero or one; the allowed SU(6) representations are [111111] and [21111]. Only the SU(3)⊗SU(2) representation (00)0 belongs to [111111]. Hence, the $q\bar{q}$ matrix assumes the values

$$\left\langle(\lambda_{23}\mu_{23})S_{23}\,\middle|\,\sum_\mu[\alpha_\mu(n_1)-\bar{\alpha}_\mu(n_2)]^2\,\middle|\,(\lambda_{23}\mu_{23})S_{23}\right\rangle=9-3[(-1)^{\mu_{23}}+(-1)^{S_{23}}+(-1)^{\mu_{23}+S_{23}}]\,. \tag{A8}$$

The sum over the SU(2) 9-j symbols may now be simplified by using either orthogonality or the sum rule

$$\sum_{S'S_{23}}U\begin{pmatrix}S_1' & \tfrac{1}{2} & S_1\\ S_2' & \tfrac{1}{2} & S_2\\ S' & S_{23} & S\end{pmatrix}U\begin{pmatrix}S_1' & \tfrac{1}{2} & \bar{S}\\ S_2' & \tfrac{1}{2} & \bar{S}_2\\ S' & S_{23} & S\end{pmatrix}(-1)^{S_{23}}=(-1)^{2S_2'+S_2+\bar{S}_2}U\begin{pmatrix}S_1' & \tfrac{1}{2} & S_1\\ \tfrac{1}{2} & S_2' & S_2\\ \bar{S}_1 & \bar{S}_2 & S\end{pmatrix}. \tag{A9}$$

With this simplification the matrix element becomes

$$\langle q^{n_1}[f_1](\lambda_1\mu_1)S_1\omega_1\times\bar{q}^{n_2}[f_2](\lambda_2\mu_2)S_2\omega_2;(\lambda\mu)S\omega\rho\,|\,\mathcal{C}_6^{(2)}\,|\,q^{n_1}[f_1](\bar{\lambda}_1\bar{\mu}_1)\bar{S}_1\bar{\omega}_1\times\bar{q}^{n_2}[f_2](\bar{\lambda}_2\bar{\mu}_2)\bar{S}_2\bar{\omega}_2;(\lambda\mu)S\omega\rho\rangle$$

$$=\delta_{1\bar{1}}\delta_{2\bar{2}}\{\langle\mathcal{C}_6^{(2)}([f_1])\rangle+\langle\mathcal{C}_6^{(2)}([f_2])\rangle-\tfrac{35}{3}\,n_1n_2\}$$

$$+\,n_1n_2\sum_{\substack{[f_1']\\(\lambda_1'\mu_1')S_1'\omega_1'}}\frac{n_{f_1'}}{n_{f_1}}\left\langle\begin{matrix}[f_1'] & [1]\\ (\lambda_1'\mu_1')S_1'\omega_1' & (10)\tfrac{1}{2}\end{matrix}\middle|\begin{matrix}[f_1]\\(\lambda_1\mu_1)S_1\omega_1\end{matrix}\right\rangle\left\langle\begin{matrix}[f_1'] & [1]\\ (\lambda_1'\mu_1')S_1'\omega_1' & (10)\tfrac{1}{2}\end{matrix}\middle|\begin{matrix}[f_1]\\(\bar{\lambda}_1\bar{\mu}_1)\bar{S}_1\bar{\omega}_1\end{matrix}\right\rangle$$

$$\times\sum_{\substack{[f_2']\\(\lambda_2'\mu_2')S_2'\omega_2'}}\frac{n_{f_2'}}{n_{f_2}}\left\langle\begin{matrix}[f_2'] & [1]\\ (\lambda_2'\mu_2')S_2'\omega_2' & (10)\tfrac{1}{2}\end{matrix}\middle|\begin{matrix}[f_2]\\(\lambda_2\mu_2)S_2\omega_2\end{matrix}\right\rangle\left\langle\begin{matrix}[f_2'] & [1]\\ (\lambda_2'\mu_2')S_2'\omega_2' & (10)\tfrac{1}{2}\end{matrix}\middle|\begin{matrix}[f_2]\\(\bar{\lambda}_2\bar{\mu}_2)\bar{S}_2\bar{\omega}_2\end{matrix}\right\rangle$$

$$\times\sum_{(\lambda'\mu')(\lambda_{23}\mu_{23})}U\begin{pmatrix}(\lambda_1'\mu_1') & (10) & (\lambda_1\mu_1)\\ (\lambda_2'\mu_2') & (01) & (\lambda_2\mu_2)\\ (\lambda'\mu') & (\lambda_M\mu_M) & (\lambda\mu)\end{pmatrix}U\begin{pmatrix}(\lambda_1'\mu_1') & (10) & (\bar{\lambda}_1\bar{\mu}_1)\\ (\lambda_2'\mu_2') & (01) & (\bar{\lambda}_2\bar{\mu}_2)\\ (\lambda'\mu') & (\lambda_M\mu_M) & (\lambda\mu)\end{pmatrix}$$

$$\times\left\{\delta(S_1,\bar{S}_1)\delta(S_2,\bar{S}_2)[9-3(-1)^{\mu_{23}}]+3(-1)^{2S_2'+S_2+\bar{S}_2}U\begin{pmatrix}S_1' & \tfrac{1}{2} & S_1\\ \tfrac{1}{2} & S_2' & S_2\\ \bar{S}_2 & \bar{S}_2 & S\end{pmatrix}[1+(-1)^{\mu_{23}}]\right\}. \quad \text{(A10)}$$

Because conjugate SU(3) representations are not equivalent, a sum rule similar to Eq. (A9) does not exist. However, since the total $\mathrm{SU}(3)_c$ representation must be a color singlet, one may use the identity

$$U\begin{pmatrix}(\lambda_1\mu_1) & (\lambda_2\mu_2) & (\lambda_{12}\mu_{12})\\ (\lambda_3\mu_3) & (\lambda_4\mu_4) & (\mu_{12}\lambda_{12})\\ (\lambda_{13}\mu_{13}) & (\mu_{13}\lambda_{13}) & (00)\end{pmatrix}_{\rho_{12},\rho_{34},\rho_{13},\rho_{24}}=(-1)^{\lambda_1+\mu_1+\lambda_{13}+\mu_{13}-\lambda_{12}-\mu_{12}-\lambda_4-\mu_4}$$

$$\times U[(\lambda_2\mu_2)(\lambda_1\mu_1)(\lambda_4\mu_4)(\lambda_3\mu_3);(\lambda_{12}\mu_{12})(\lambda_{34}\mu_{34})]_{\rho_{12},\rho_{34},\rho_{13},\rho_{24}}, \quad \text{(A11)}$$

which may be proven by applying the definition of the 9-$\lambda\mu$ coefficient in terms of SU(3) Racah coefficients.
In the event that $(\lambda_{13}\mu_{13})=(00)$, the SU(3) Racah coefficient becomes

$$U[(\lambda_2\mu_2)(\lambda_1\mu_1)(\mu_2\lambda_2)(\mu_1\lambda_1);(\lambda_{12}\mu_{12})(00)]=(-1)^{\lambda_1+\mu_1+\lambda_2+\mu_2-\lambda_{12}-\mu_{12}}\left(\frac{g(\lambda_{13}\mu_{12})}{g(\lambda_1\mu_1)g(\lambda_2\mu_2)}\right)^{1/2}, \quad \text{(A12)}$$

where $g(\lambda\mu)$ is the dimension of the representation $(\lambda\mu)$, Eq. (4). The 9-$\lambda\mu$ coefficient reduces to

$$U\begin{pmatrix}(\lambda_1\mu_1) & (\lambda_2\mu_2) & (\lambda_{12}\mu_{12})\\ (\mu_1\lambda_1) & (\mu_2\lambda_2) & (\mu_{12}\lambda_{12})\\ (00) & (00) & (00)\end{pmatrix}=\left(\frac{g(\lambda_{12}\mu_{12})}{g(\lambda_1\mu_1)g(\lambda_2\mu_2)}\right)^{1/2}. \quad \text{(A13)}$$

Upon diagonalization of the array resulting from calculating all the allowed matrix elements, Eq. (A6), the eigenvalues correspond to allowed values of the SU(6) Casimir operator, Eq. (7), and the components of the eigenvectors are the desired SU(6)⊃SU(3)⊗SU(2) Wigner coefficients. The SU(6)⊃SU(3)⊗SU(2) one-body Wigner coefficients are taken from Ref. 15 and the SU(3) Racah coefficients were calculated using the programs of Refs. 12 and 13. The coefficients necessary for calculations within the $q^5\bar{q}^2$ configuration are given in Table VIII.

[1]J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).
[2]R. R. Horgan, in *Proceedings of the Topical Conference on Baryon Resonances, Oxford, 1976,* edited by R. T. Ross and D. H. Saxon (Rutherford Laboratory, Chilton, Didcot, England, 1977).
[3]A. J. G. Hey, in *Proceedings of the Topical Conference on Baryon Resonances, Oxford, 1976,* edited by R. T. Ross and D. H. Saxon (Rutherford Laboratory, Chilton, Didcot, England, 1977).
[4]A. Chodos, R. L. Jaffe, K. Johnson, and C. Thorn, Phys. Rev. D 10, 2599 (1974).
[5]R. L. Jaffe, Phys. Rev. D 15, 267 (1977).
[6]S. -O. Holmgren and M. R. Pennington, Phys. Lett. 77B, 304 (1978).
[7]R. L. Jaffe, Phys. Rev. D 15, 281 (1977).
[8]J. P. Elliott, Proc. R. Soc. London A245, 128, 562 (1958).
[9]K. T. Hecht, Nucl. Phys. 62, 1 (1965).
[10]J. D. Vergados, Nucl. Phys. A111, 681 (1968).
[11]D. Strottman, Ph.D. thesis, SUNY at Stony Brook, 1969 (unpublished).
[12]Y. Akiyama and J. P. Draayer, Comp. Phys. Commun. 5, 405 (1973).
[13]D. J. Millener, J. Math. Phys. 19, 1513 (1978).
[14]P. H. Butler, Trans. R. Soc. 277A, 546 (1974).
[15]S. I. So and D. Strottman, J. Math. Phys. 20, 153 (1978).
[16]D. Strottman, J. Math. Phys. (to be published).
[17]H. Weyl, *The Theory of Groups and Quantum Mechanics*, translated by H. P. Roberson (Metheun, London, 1931; reissued Dover, New York, 1949); *The Classical Groups* (Princeton Univ. Press, Princeton, 1946).
[18]C. R. Hagen and A. J. Macfarlane, J. Math. Phys. 6, 1355 (1965).
[19]J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).
[20]D. Strottman, Phys. Rev. D 18, 2716 (1978).
[21]R. R. Horgan and R. H. Dalitz, Nucl. Phys. B66, 135 (1973); R. R. Horgan, *ibid.* B71, 514 (1974).
[22]N. P. Samios, M. Goldberg, and B. T. Meadows, Rev. Mod. Phys. 46, 49 (1974).
[23]G. C. Oades and G. Rasche, Nuovo Cimento 42A, 462 (1977).
[24]G. P. Gopal, R. T. Ross, A. J. van Horn, A. C. McPherson, E. F. Clayton, T. C. Bacon, and I. Butterworth, Nucl. Phys. B119, 362 (1977).
[25]D. F. Baxter *et al.*, Nucl. Phys. B67, 125 (1973).
[26]J. Prevost *et al.*, Nucl. Phys. B69, 246 (1974).
[27]S. Almehed and C. Lovelace, Nucl. Phys. B40, 157 (1972).
[28]Y. -L. Pan, F. L. Forman, W. Ko, V. Hagopian, and W. Selove, Phys. Rev. D 2, 449 (1970).
[29]T. S. Mast *et al.*, Phys. Rev. D 11, 3078 (1975).
[30]E. Briefel *et al.*, Phys. Rev. D 16, 2706 (1977).
[31]R. Aaron, R. D. Amado, and R. R. Silbar, Phys. Rev. Lett. 26, 407 (1971); see also R. Aaron, M. Rich, W. L. Hogan, and Y. N. Srivastava, Phys. Rev. D 7, 1401 (1973).
[32]B. R. Martin, in *Proceedings of the Topical Conference on Baryon Resonances, Oxford, 1976,* edited by R. T. Ross and D. H. Saxon (Rutherford Laboratory, Chilton, Didcot, England, 1977).
[33]N. K. Glendenning, in *Proceedings of the International Workshop on Gross Properties of Nuclei and Nuclear Excitations IV, Hirschegg, Kleinwalsertal, Austria, 1978,* edited by H. v. Groote (Technische Hochschule Darmstadt, Darmstadt, Germany, 1978).
[34]Chan Hong-Mo and H. Høgaasen, Phys. Lett. 72B, 121 (1977); 72B, 400 (1978); Nucl. Phys. B136, 401 (1977); Chan Hong-Mo, M. Fukugita, T. H. Hansson, H. J. Hoffman, K. Konish, H. Høgaasen, and Tsou Sheung Tsun, Phys. Lett. 76B, 634 (1978).
[35]K. Johnson and C. B. Thorn, Phys. Rev. D. 13, 1934 (1976).
[36]R. L. Jaffe, Phys. Rev. D 17, 1444 (1978).
[37]H. Högaasen and P. Sorba, Nucl. Phys. B145, 119 (1978).
[38]C. DeTar, Phys. Rev. D 17, 302 (1978); 17, 323 (1978).
[39]C. Rebbi, Phys. Rev. D 12, 2407 (1975).
[40]T. DeGrand and R. L. Jaffe, Ann. Phys. (N.Y.) 100, 425 (1976).