Charmonium and Υ systems with small hyperfine splittings

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(Received 27 December 1978)

The parameters of the simple Coulomb plus linear effective potential with lowest-order relativistic corrections are determined by the charmonium states *other* than the ${}^1S_0$ states and by the leptonic decay widths of J/ψ and ψ'. The effective coupling constant α_s is found to be 0.21. Within this framework the hyperfine splittings are small and so are the consequent $M1$ transitions (well below the experimental upper limit of 1.2 keV). The ${}^1S_0$ partners of ψ and ψ' are predicted to be $\eta_c(3020)$ and $\eta_c'(3638)$ and are not to be identified with the experimentally uncertain $X(2830)$ and $\chi(3450)$. The same effective potential, changing only the constituent quark mass, gives a good description of the Υ system. The spin-independent relativistic corrections play an important role in obtaining the equality $m_{\Upsilon'} - m_{\Upsilon} \approx m_{\psi'} - m_{\psi}$.

I. INTRODUCTION

The description of mesons as bound quark-antiquark systems has been enormously successful. Particularly remarkable is the "new spectroscopy" of the charmonium states where the heavy-quark mass makes the system amenable to analysis in terms of potential theory.[1] There have been several successful analyses of the charmonium spectrum with a central (spin-independent) part of the effective potential which has a typical Coulomb-type term from single-gluon exchange in quantum chromodynamics, and, at large distances, a linear behavior appropriate for a confined system.[1,2] Spin-independent and spin-dependent corrections of order v^2/c^2 are usually included by using one of several techniques of reducing these from the central potential.[3,4]

This has raised the exciting possibility of having a single effective potential for all the heavy-quark systems (e.g. charmonium and the Υ system), the only difference between them being the masses of the constituent quarks.

Perhaps the major difficulty in executing this unification program is the problem of the ${}^1S_0$ partners of the ${}^3S_1$ states $J/\psi(3095)$ and $\psi'(3686)$. This is beyond fine tuning of the potential. More precisely, the problem arises when the ${}^1S_0$ states are assumed to be the $X(2830)$ and $\chi(3450)$.[5,6] Basically the difficulty is that the $\vec{S}_1 \cdot \vec{S}_2$ splitting of 250 MeV is abnormally large. Reasonable estimates based on general scaling and the leptonic width of ψ would put ${}^1S_0$ states about 65 MeV below the ${}^3S_1$ partners.[1,7]

Although one can fit the large splitting by making the $\vec{S}_1 \cdot \vec{S}_2$ forces independent of the central potential,[8] or by invoking anomalous splittings,[2] we run headlong into the second problem. All theoretical estimates for the width of the magnetic dipole ($M1$) transition are about 20 times too large.[1]

Recently Meshkov and Samios[9] have reexamined the Dalitz plot for three-photon events at the ψ mass. They found that the significance of $X(2830)$ is reduced to about 2 standard deviations. Thus the ${}^1S_0$ partner of $\psi(3095)$ may not be at 2830 MeV. At the same time a recent experiment studying the radiative decays of ψ' found no evidence for a peak at 3450.[10] In the light of these analyses, we adopt the prescription of using the simple Coulomb plus linear potential with relativistic corrections to fit all the states of charmonium *other* than the ${}^1S_0$ states. Having established the parameters of the potential, we can then predict the masses of the ${}^1S_0$ states.

Next we consider the Υ system characterized by heavy constituent quarks and narrow widths. We use the *same* potential which fits the charmonium, but with constituent quarks of different mass.

In Sec. II we describe in detail the effective potential. In Sec. III we present the analysis of the data on ψ and Υ systems and discuss the results, while Sec. IV contains a summary and the concluding remarks.

II. THE EFFECTIVE POTENTIAL

In this section we shall outline the effective potential to be used in the analysis of the spectrum of the bound quark-antiquark system. The basic central potential has the simple form

$$V_c(r) = -\frac{4}{3}\frac{\alpha_s}{r} + V_0 + (g_V + g_S)r\,, \qquad (2.1)$$

where the first term is the standard Coulomb-type term from one-gluon (vector) exchange, and V_0 is a constant. The third term is the confinement potential, linear in r, where we allowed the possibility of both scalar[11] and vector contributions.[12]

The relativistic corrections to the central potential (2.1) can be obtained by using the reduction formula of the corresponding Bethe-Salpeter equation.[3,4] There are spin-independent (V_{SI}) and spin-dependent (V_{SD}) corrections. To lowest order, the relativistic corrections are

$$V_{SI} = \frac{4\pi}{3m_Q^2}\alpha_s \delta(\vec{r}) - \frac{2}{3m_Q^2}\alpha_s\left[\frac{\vec{p}^2}{r} + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p})\vec{p}}{r^3}\right] + \frac{1}{m_Q^2}(g_V - g_S)[l(l+1)+2]\frac{1}{r} - \frac{\vec{p}^4}{4m_Q^3}, \quad (2.2)$$

$$V_{SD} = \frac{2}{m_Q^2}\alpha_s \frac{\vec{L}\cdot\vec{S}}{r^3} + \frac{1}{3m_Q^2}\alpha_s\frac{S_{12}}{r^3} + \frac{32\pi}{9m_Q^2}\alpha_s\delta(\vec{r})\vec{S}_1\cdot\vec{S}_2 + \frac{1}{2m_Q^2}(3g_V - g_S)\frac{\vec{L}\cdot\vec{S}}{r} + \frac{1}{12m_Q^2}g_V\frac{S_{12}}{r} + \frac{4}{3m_Q^2}g_V\frac{\vec{S}_1\cdot\vec{S}_2}{r} \quad (2.3)$$

where $\vec{S}_1$ and $\vec{S}_2$ are the spins of the individual quarks, $\vec{S}$ is the total spin, $\vec{L}$ is the relative orbital angular momentum, and S_{12} is the usual tensor operator

$$S_{12} = 3\,\frac{(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2 .$$

Thus the total effective potential is

$$V(\vec{r}) = V_c(r) + [V_{SI}(\vec{r}) + V_{SD}(\vec{r})] , \quad (2.4)$$

which we use in the Schrödinger equation to obtain the spectrum of the quark-antiquark system. We treat $V_c(r)$ exactly but use first-order perturbation for the relativistic-correction term $[V_{SI}(\vec{r}) + V_{SD}(\vec{r})]$.

III. ANALYSIS OF THE DATA

In the effective potential $V(\vec{r})$ there are five parameters: α_s, V_0, g_V, g_S, and m_Q. We determine the values of these parameters by fitting the well established states $J/\psi(3095)$, $\psi'(3686)$, $\psi(3772)$, $^3P_0(3413)$, and $^3P_1(3554)$. We obtain $\alpha_s = 0.21$, $g_V = 0.75$ GeV/fm, $g_S = 0.65$ GeV/fm, $V_0 = -1.1$ GeV, and the charm-quark mass $m_c = 1.8$ GeV/c^2.

Our results for the various states of charmonium are given in Table I along with the experimental values. We have also indicated the effects of the spin-independent and spin-dependent corrections.

Keeping all the parameters of the effective potential the same except for the constituent quark mass, we can extend our analysis to the Υ system. We find good agreement with the known states of Υ. The b-quark mass we obtained is $m_b = 5.15$ GeV/c^2.

In the above analysis, for the matrix elements of $\delta(\vec{r})$ in V_{SI}(2.2) and V_{SD}(2.3), it has been pointed out[2,13] that because the inelastic channels and higher-order effects become important at small distances, the magnitude of the wave function at $r = 0$ will not be accurately given by the Schrödinger equation with potentials that do not take these effects into account. Thus following these authors we determine the matrix element of $\delta(\vec{r})$ by the Weisskopf–Van Royen formula and the experimental value of the leptonic width,[1]

$$\langle\psi(\vec{r})|\delta(\vec{r})|\psi(\vec{r})\rangle = |\psi(0)|^2 = \frac{M^2}{16\pi\alpha^2 e_q^2}\Gamma_{ee} . \quad (3.1)$$

For the S states of charmonium (3.1) gives

$$|\psi(0)|_{1S}^2 = 64.1 \text{ fm}^{-3} , \quad (3.2)$$

$$|\psi(0)|_{2S}^2 = 39.5 \text{ fm}^{-3} . \quad (3.3)$$

We now discuss the various features of our charmonium analysis: The $^1S_0$ partners (η_c and η_c') of $J/\psi(3095)$ and $\psi'(3686)$ are obtained at masses 3020 and 3638 MeV/c^2, respectively. The proximity of these states to ψ and ψ' makes the $M1$ transition rate quite small, in agreement with experiment. The $M1$ transition rate we obtained is 0.56 keV which is below the experimental upper limit of 1.2 keV.[1] Here and in the rest of this paper, to avoid confusion, the $^1S_0$ partners of ψ and

TABLE I. Spectrum of the charmonium system (in GeV/c^2).

State	Spectroscopic notation	Central potential	+ spin-independent corrections	+ spin-dependent corrections	Experimental (Ref. 20)
η_c	$^1S_0$	3.119	3.076	3.020	
J/ψ	$^3S_1$	3.119	3.076	3.095	3.097 ± 2
	$^3P_0$	3.558	3.508	3.413	3.413 ± 5
	$^3P_1$	3.558	3.508	3.484	3.508 ± 4
	$^3P_2$	3.558	3.508	3.553	3.554 ± 5
	$^1P_1$	3.558	3.508	3.488	
$\eta_{c'}$	2^1S_0	3.795	3.673	3.638	
ψ'	2^3S_1	3.795	3.673	3.685	3.686 ± 3
	$^3D_1$	3.897	3.838	3.774	3.772 ± 6

TABLE II. Spectrum of the Υ system (in GeV/c^2).

State	Spectroscopic notation	Central potential	+ spin-independent corrections	+ spin-dependent corrections	Experimental (Ref. 21)
	1^1S_0	9.476	9.452	9.426	
Υ	1^3S_1	9.476	9.452	9.460	9.46 ± 0.01
	1^3P_0	9.864	9.848	9.818	
	1^3P_1	9.864	9.848	8.841	
	1^3P_2	9.864	9.848	9.861	
	1^1P_1	9.864	9.848	9.845	
	2^1S_0	10.017	9.975	9.961	
Υ′	2^3S_1	10.017	9.975	9.980	10.02 ± 0.02
	3^1S_0	10.410	10.355	10.344	
Υ′′	3^3S_1	10.410	10.355	10.358	10.38 ± 0.04
Υ′′′	4^3S_1	10.744	10.676	10.680	

ψ' will be referred to as η_c and η_c', respectively, whereas the experimentally observed effects at 2830 and 3454 whether they exist or not, will be called X and χ.

The mass difference for ψ-η_c of 75 MeV/c^2 obtained here is close to the early estimates.[7] The size of these splittings is largely determined by relating $|\psi(0)|^2$ to the leptonic width by means of the Weisskopf–Van Royen formula. Corrections to the formula will correspondingly change the ψ-η_c splitting.[1,11] However, we find that

$$\psi' - \eta_c' \simeq \tfrac{2}{3}(\psi - \eta_c)$$

over a wide range of the parameters and independent of the manner in which $|\psi(0)|^2$ is obtained. This relation is a natural consequence of the level spacing of the various ψ states.

The ${}^3P_2$-${}^3P_1$ splitting is 20 MeV too large. This splitting has often been a problem in potential models. Schnitzer[2] has proposed the introduction of a quark-gluon anomalous moment to reduce this splitting. This procedure of reducing the ${}^3P_2$-${}^3P_1$ splitting, however, causes a substantial increase in the singlet-triplet splitting which is not a desirable feature unless one believes $X(2830)$ to be the η_c. Another possibility is the introduction of a pseudoscalar[14] component to the linear potential. This will cause a decrease in the ${}^3P_2$-${}^3P_1$ splitting and simultaneously decrease the singlet-triplet splitting. We do not consider it worthwhile to further modify the potential here, since the next order of the relativistic corrections will itself be of the order of 20 MeV.

The spin-independent corrections are of the same order of magnitude as the spin-dependent corrections. They are responsible for a reduction of 80 MeV/c^2 in the $1S$-$2S$ mass difference generated by the static central potential. This makes it much easier to understand the following mass difference relation between the ψ and Υ systems:

$$m_{\psi'} - m_\psi \approx m_{\Upsilon'} - m_\Upsilon \ .$$

We now consider the Υ system. The level spacings are given in Table II. Because the quark mass here is large, the relativistic corrections are about three times smaller than the corresponding corrections in charmonium. The ${}^3P_2$-${}^3P_0$ splitting is about 40 MeV/c^2 in Υ where this same splitting is 140 MeV/c^2 in charmonium. In general, even if we fit the Υ system independent of the charmonium system the spacing ${}^3P_2$-${}^3P_0$ will still be less than 60 MeV/c^2. The center of gravity of the P multiplet should fall between 9860 and 9900 MeV/c^2.

IV. SUMMARY AND CONCLUSION

Using our effective potential for the charmonium system, we have obtained excellent agreement with experiment. The potential is a simple combination of Coulomb and an essentially equal mixture of scalar and vector linear confining potentials. The maximum discrepancy is 24 MeV/c^2 for the ${}^3P_1(3508)$. In view of the fact that for this system $v^2/c^2 \approx 0.2$ and our relativistic corrections were of the order of 100 MeV/c^2, we do not consider it worthwhile to further fine-tune the potential here, since the next order of relativistic corrections will be of order 20 MeV/c^2.

We obtained a value of 0.21 for α_s. This is in agreement with the value obtained from the experimental ratio of hadronic to leptonic width of ψ using the asymptotic-freedom calculations of $\psi \to 3$ gluons $\to$ hadrons which gives a value $\alpha_s = 0.19$.[15,16]

One major consequence of the present calculation is that we predict the ${}^1S_0$ partners of ψ and ψ' to be $\eta_c(3020)$ and $\eta_c'(3638)$. Thus, the ${}^3S_1$-${}^1S_0$ splittings are predicted to be $\psi - \eta_c = 75$ MeV/c^2 and $\psi' - \eta_c' = 47$ MeV/c^2, respectively. If this prediction holds up, the problem of the missing $M1$ transition is completely resolved, by reducing the width from estimates based on $X(2830)$ by $(\frac{75}{260})^3$, more than the required reduction of $\sim\frac{1}{20}$. In this connection it is worth mentioning that the ratio $(\psi' - \eta_c')/(\psi - \eta_c) \approx \frac{2}{3}$ is practically independent of the details of the model.

We now turn to the Υ system. Changing only the quark mass and otherwise using the potential (2.4), we obtain $m_Q = 5.15$ GeV/c^2 for the heavy-quark mass by demanding the Υ to be at 9.46. The predictions for the other observed states $\Upsilon'(10.02)$, $\Upsilon''(10.38)$ are very good, considering that the mass has been extrapolated by a factor of 3. The relativistic corrections are, of course, much smaller than those for the charmonium system and are of the order of 20–40 MeV/c^2. As more Υ-system levels become known it will then be a better strategy to improve the central potential using these, since in that system the level spacing is basically determined by the central potential and cannot be changed much by relativistic corrections.

The ψ and Υ systems satisfy the following near equality:

$$m(\Upsilon') - m(\Upsilon) = m(\psi') - m(\psi) \approx 0.6 \text{ GeV}/c^2 . \quad (4.1)$$

Quigg and Rosner[17] have noted that if this equal level spacing were not a coincidence, but were to hold generally (i.e., for systems with still heavier quarks) then it would favor a potential of the logarithmic form. However, we find that the equality (4.1) is satisfied for the linear confining plus a Coulomb potential. The (spin-independent) relativistic corrections are found to play a particularly important role in obtaining (4.1). These corrections change $m(\psi') - m(\psi)$ from 676 MeV/c^2 to 597 but $m(\Upsilon') - m(\Upsilon)$ is changed from 541 MeV/c^2 to 520, only by 20 MeV/c^2. For systems with still heavier quarks, however, the approximate equal spacing will not be maintained in our potential in contrast to the logarithmic case.

The general deportment of levels in the Υ system is now predicted with some confidence and displayed in Table II. Finally we address the question of the number of narrow levels in the Υ system, that is, how many levels lie below the threshold for Zweig-rule-allowed decays into $Q\bar{q} + \bar{Q}q$ pairs, where q is a light quark. Our predictions for the $\Upsilon''(3^3S_1)$ and $\Upsilon'''(4^3S_1)$ are 10.34 and 10.68 GeV/c^2, respectively. Based on a flavor threshold of 10.5 GeV/c^2,[18,19] we therefore would predict that the 3S state is just below the threshold but the 4S state is definitely above. For heavier quark-antiquark systems the effective potential used here will predict fewer number of narrow states as compared to the estimate based on logarithmic central potential. The exact number of narrow states can be obtained by analyzing systems of quark and antiquark of unequal masses. This work is in progress.

ACKNOWLEDGMENTS

We are grateful for helpful conversations with S. Meshkov. P. K. performed part of this work while at the Aspen Center for Physics and would like to thank J. L. Rosner for useful discussions. This work was supported in part by the National Science Foundation.

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