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Patterns of mass degeneracy in the baryonium spectrum

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Using group-theoretical techniques, we give SU(4) mass formulas for the flavor representations of the baryonium states. We also present the mass spectrum for the ideally mixed baryonium states. It is found that degenerate clustering of particle states separated only by isospin breaking occurs within J^P multiplets. The broad phenomenological features of the spectrum are discussed.

I. INTRODUCTION

Recently there have been extensive theoretical efforts directed towards the baryonium and multiquark spectrum.[1-29] As well there has been the discovery of a growing number of resonances which do not fit easily into the conventional two- and three-quark spectrum.[30-34] A number of these studies have concentrated on the color-magnetic spin-spin interaction with its associated semisimple SU(6) color-spin group containing the color-spin product $SU(3)_c \times SU(2)_J$.[1-11,21] The group treatments of the baryonium states have been based on the diquark structure. Two quarks couple to form a diquark in a relative s-wave state which then couples with a similar antidiquark system. The dynamic stability of a number of high-spin baryonium states against decay into $q\bar{q}$ pairs deriving from an angular momentum barrier between the two diquarks. The color representations of the diquark system can be either an SU(3) $\bar{3}$ or 6. Jaffe[1-3] has treated the $qq\bar{q}\bar{q}$ states both in the case of zero angular momentum where the color-magnetic interaction mixes the two color representations,[1] and the situation where the diquarks are sufficiently separated such that the color-magnetic interaction between them is negligible.[2] The latter approach leads to two baryonium series; one where the diquarks are both in color 3, the other with the diquarks in color 6. This approach has been combined with the bag model.[35-36] Other authors have considered the two baryonium series, in particular Chan Hong-Mo *et al.*[4 6] and Tsou[8] have noted the significance of the 6-$\bar{6}$ series and its different decay properties from the 3-$\bar{3}$ series.

In a previous work,[37] we have constructed the Clebsch-Gordan coefficients in SU(4) associated with the baryonium wave functions. Here we use the mass breaking techniques of Gell-Mann[38] and Okubo[39] to obtain the SU(4)-flavor-breaking pattern of the baryonium spectrum, thus extending the detailed studies of the flavor spectrum for the $q\bar{q}$ states.[40] With SU(4) one expects an enormous number of states to occur for each spin-parity mutiplet. Our analysis exhibits the degree and nature of the degenerate mass clustering occuring within the spectrum. The approach parallels the dynamic treatment in the bag model[1,11] and the quark model.[22,41] In the group analysis each spin-parity multiplet has a different parameter set while both the bag and quark models possess tightly constrained parameter sets, thus it does not have the same predictive power of these models. The advantage of the group-theoretic method lies in its freedom from the assumptions of these models; it is based only on a very general form assumed for the flavor breaking. Its success in conventional-hadron spectroscopic classification suggests that it will act as a valuable guide for the baryonium spectrum.

In Sec. II the mass formulas for all the representations of relevance for the $qq\bar{q}\bar{q}$ states are presented. The conjecture of charm exotics in e^+e^- reactions[42,43] as well as in the 0^{++} states[44] provides justification for working in SU(4). Furthermore, there appears only at the SU(4) level and above, a representation with tensor structure, T^{ij}_{kl}, antisymmetric with respect to i,j, and with respect to k,l.[12] This representation [20'' in SU(4)] gives rise to a unique mixing pattern in the baryonium states. In Sec. III, the mass spectrum for ideally mixed (i.e., pure diquark, antidiquark) baryonium states is given. We conclude in Sec. IV with an amplification of a number of the features of the spectrum, and with some general phenomenological aspects of the spectrum.

II. MASS FORMULAS

Following Okubo,[39,40] we will assume that the symmetry-breaking Hamiltonian, H, is given by

$$H = T_8 + \alpha T_{15}\,, \tag{2.1}$$

where T_8 and T_{15} transform as the 8th and 15th components of the adjoint representation. Throughout we will work in U(4), the results are the same as in SU(4) as long as care is taken to ensure that

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TABLE I. Diquark wave functions. The $[q_i q_j]_\pm$ represent $(q_i q_j \pm q_j q_i)/\sqrt{2}$. The antidiquarks are obtained by replacing the quarks with antiquarks, and D_i (I_z) with $\overline{D}_i$ $(-I_z)$.

10(2,0,0,0)			
$C=0$	$D_1(+1)=\mathcal{P}\mathcal{P}$	$C=1$	$D_4(+\frac{1}{2})=[\mathcal{P}c]_+$
	$D_1(0)=[\mathcal{P}\mathcal{N}]_+$		$D_4(-\frac{1}{2})=[\mathcal{N}c]_+$
	$D_1(-1)=\mathcal{N}\mathcal{N}$		$D_5=[sc]_+$
	$D_2(+\frac{1}{2})=[\mathcal{P}s]_+$	$C=2$	$D_6=cc$
	$D_2(-\frac{1}{2})=[\mathcal{N}s]_+$		
	$D_3=ss$		
6(1,1,0,0)			
$C=0$	$S_1=[\mathcal{P}\mathcal{N}]_-$	$C=1$	$S_3(+\frac{1}{2})=[\mathcal{P}c]_-$
	$S_2(+\frac{1}{2})=[\mathcal{P}s]_-$		$S_3(-\frac{1}{2})=[\mathcal{N}c]_-$
	$S_2(-\frac{1}{2})=[\mathcal{N}s]_-$		$S_4=[sc]_-$

the representations have zero baryon number. The above structure for the Hamiltonian has the property of conserving total isospin, hypercharge, and charm. In the tensor notation of Okubo[40,45] the mass operator has the form

$$H=T_{33}+yT_{44}\,, \tag{2.2}$$

where y is a measure of the U(4) breaking. It is convenient to express the states in the irreducible representations in terms of the Gelfand basis (see Appendix). In this basis the states can be described by a set of integers $\{m_{ij}\}$ which form a Gelfand pattern. Using techniques given in the Appendix, the mass spectrum from Eq. (2.2) for the states of the irreducible representations is given by

$$\begin{aligned} m=m_0&+\alpha_1(-Y+B-\tfrac{1}{3}C+yC)\\ &+\alpha_2(L_1+\langle e_{34}e_{43}\rangle+yL_2)\,, \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} L_1=\tfrac{1}{2}[&I_2^{(3)}+2F(Y,I)\\ &+Y(\tfrac{4}{3}C-4B-3)-(B-\tfrac{1}{3}C)^2]\,, \end{aligned} \tag{2.4}$$

$$L_2=\tfrac{1}{2}[I_2^{(4)}-I_2^{(3)}-I_1^{(4)}+4C+C^2]\,, \tag{2.5}$$

and

$$F(Y,I)=\tfrac{1}{4}Y^2-I(I+1)\,. \tag{2.6}$$

The quantities $I_1^{(n)}$, $I_2^{(n)}$ are Casimir invariants of the U(n) group.[46] They, together with the quantum numbers can be evaluated using Eqs. (A4)–(A7). The group generators e_{34} and e_{43} in Eq. (2.3) can be expressed as functions of the $\{m_{ij}\}$.[46] For the baryonium states, the baryon number, B, is zero. The effect of the term $\langle e_{34}e_{43}\rangle$ is to mix states in the same U(4) representation (intramultiplet mixing) at the same point in weight space, but belonging to different U(3) representations.

The diquarks lie in the 6 and 10 representations; the antidiquarks in their conjugates. The diquark wave functions are given in Table I. The baryonium states lie in the products:

$$6\times\overline{6}=1+15+20''\,, \tag{2.7}$$

$$10\times\overline{6}=15+45\,, \tag{2.8}$$

$$6\times\overline{10}=15'+\overline{45}\,, \tag{2.9}$$

$$10\times\overline{10}=1+15+84\,. \tag{2.10}$$

Table II contains the U(3) subreductions of these representations. The mass formulas for these representations are given in Tables III–IV. For the products, $6\times\overline{6}$, and $10\times\overline{10}$, the conjugate states lie in the same representations, and from the invariance of the mass formulas under charge conjugation, terms linear in Y or C alone must vanish. This leads to the condition, $\alpha_1+2\alpha_2=0$. for the 15 and 84 representations, while for the 20'', a complete degeneracy; $\alpha_1=\alpha_2=0$. The sit-

TABLE II. U(3) decompositions associated with the U(4) representations.

U(4) Rep.	C	U(3) Rep.	U(4) Rep.	C	U(3) Rep.
4(1,0,0,0)	0	3	20''(1,1,−1,−1)	−1	6
	1	1		0	8
				1	$\overline{6}$
6(1,1,0,0)	0	$\overline{3}$			
	1	3	45(2,0,−1,−1)	−1	15
				0	8 + 10
10(2,0,0,0)	0	6		1	$\overline{3}+6$
	1	3		2	3
	2	1			
			84(2,0,0,−2)	−2	6
15(1,0,0,−1)	−1	3		−1	3 + 15
	0	1 + 8		0	1 + 8 + 27
	1	$\overline{3}$		1	$\overline{3}+\overline{15}$
				2	$\overline{6}$

TABLE III. Mass formulas and intramultiplet mixing elements for the $6\times\bar{6}$ and $10\times\overline{10}$ representations. The constants differ in each U(4) representation. $F(Y,I)$ is defined in Eq. (2.6).

$84(2,0,0,-2)$

$C=\pm 2\quad m(\bar{6})=m_0+\frac{1}{7}\beta\,[48+18F(Y,I)+56y]$

$C=\pm 1\quad m(\overline{15})=m_0+\frac{1}{12}\beta\,[92+3YC+18F(Y,I)+60y]$

$m(\bar{3})=m_0+\frac{1}{15}\beta\,[83+63F(Y,I)+135y]$

$\langle 15\,|\,\bar{3},\tfrac{1}{2},-\tfrac{1}{3}\rangle=\tfrac{3}{2}\beta,\quad \langle 15\,|\,\bar{3},0,\tfrac{2}{3}\rangle=\sqrt{3}\beta$

$C=0\quad m(27)=m_0+\frac{1}{5}\beta\,[46+6F(Y,I)+10y]$

$m(8)=m_0+\frac{1}{15}\beta\,[97+32F(Y,I)+105y]$

$m(1)=m_0+\frac{1}{3}\beta\,(10+30y)$

$\langle 27\,|\,8,\tfrac{1}{2},\pm 1\rangle=\tfrac{6}{5}\beta,\quad \langle 27\,|\,8,0,0\rangle=\tfrac{3}{5}\sqrt{6}\beta$

$\langle 27\,|\,8,1,0\rangle=\tfrac{2}{5}\sqrt{6}\beta,\quad \langle 8\,|\,1,0,0\rangle=\tfrac{5}{3}\sqrt{2}\beta$

$20''(1,1,-1,-1)$

$m(20'')=m_{20''}$

$15(1,0,0,-1)$

$C=\pm 1\quad m(\bar{3})=m(3)=m_0+\frac{1}{10}\beta(7-18F(Y,I)-15y)$

$C=0\quad m(8)=m_0-\beta F(Y,I)$

$m(1)=m_0+\frac{1}{4}\beta\,(7-9y)$

$\langle 8\,|\,1,0,0\rangle=-\frac{1}{\sqrt{2}}\beta$

$1(0,0,0,0)$

$m(1)=m_1$

uation is more complex for the cross products $10\times\bar{6}$ and $6\times\overline{10}$; the charge-conjugate states now lie in different representations with different constants in the mass formulas. In these cases terms linear in Y can occur, the condition of charge-conjugation invariance leading only to an expressing of the constants of one product in terms of the other. A further feature of the cross products is that the $C=Y=0$ states of each product are no longer eigenstates of G parity.[1] We will consider this when dealing with the mixing of the products. Along with the mass formulas, the nondiagonal terms resulting from the intramultiplet mixing are given in the form $\langle r_1\,|\,r_2, I, Y\rangle$ where I and Y specify the mixed states in the U(3) representations r_1 and r_2.

III. IDEAL MIXING

Since multiple-weight points occur in the products in Eqs. (2.7)–(2.10), we need to consider the physical states at these points as mixtures of the representation states. Consider a general mixing of n states, then following conventional treatment of mixing, the masses of the physical states will be given by the eigenvalues of a symmetric $n\times n$ mass matrix in representation space,

$$M_{ij}=\langle R_i\,|\,H\,|\,R_j\rangle,\quad i,j=1\cdots n\,. \tag{3.1}$$

H in Eq (3.1) is a phenomenological Hamiltonian including Eq. (2.2) as well as a term responsible for mixing between the various U(4) representations. We will assume that the mixing between the different radial states is negligible.[47] The diagonal terms and those given by the intramultiplet mixing term $\langle e_{34}e_{43}\rangle$ are given by Eq. (2.3). [We assume that these terms are unaffected by the extra U(4) mixing term in H]. The other elements are parameters introduced to allow for intermultiplet U(4) mixing. The physical states will be mixtures of the representation states,

$$P_i=\sum_{k=1}^{n}a_i^k\,|\,R_k\rangle,\quad i=1\cdots n\,. \tag{3.2}$$

The problem of determining the a_i^k in Eq. (3.2)

TABLE IV. Mass formulas and intramultiplet mixing elements for the representations in the $10\times\bar{6}$. The formulas for the $6\times\overline{10}$ can be obtained by replacing Y with $-Y$, and C with $-C$.

$45(2,0,-1,-1)$

$C=+2\quad m(3)=m_0-2\alpha+4\beta-3Y(\alpha+\beta)+6y(\alpha+6\beta)$

$C=+1\quad m(6)=m_0-\alpha+9\beta-3Y(\alpha+3\beta)+3y(\alpha+7\beta)$

$m(\bar{3})=m_0-\alpha+7\beta-3Y(\alpha+2\beta)+3y(\alpha+9\beta)$

$\langle 6\,|\,3,\tfrac{1}{2},-\tfrac{1}{3}\rangle=3\sqrt{2}\beta$

$C=0\quad m(10)=m_0+14\alpha-Y(3\alpha+11\beta)+6\beta y$

$m(8)=m_0+15\alpha-Y(3\alpha+10\beta)+4\beta F(Y,I)+6\beta y$

$\langle 10\,|\,8,1,0\rangle=\langle 10\,|\,8,\tfrac{1}{2},-1\rangle=2\sqrt{2}\beta$

$C=-1\quad m(15)=m_0+\alpha+\frac{61}{3}\beta-Y(3\alpha+\frac{19}{2}\beta)+3\beta F(Y,I)+3y(\beta-\alpha)$

$15(1,0,0,-1)$

$C=+1\quad m(3)=m_0-\alpha+3\beta-3\beta Y+3y(\alpha+5\beta)$

$C=0\quad m(8)=m_0+11\beta-3Y(\alpha+2\beta)+4\beta F(Y,I)+3\beta y$

$m(1)=m_0+4\beta+12\beta y$

$\langle 8\,|\,1,0,0\rangle=2\sqrt{2}\beta$

$C=-1\quad m(3)=m_0+\alpha+7\beta-3Y(\alpha+4\beta)+3y(\beta-\alpha)$

TABLE V. Mass spectrum for the ideally mixed $6\times\bar{6}$. The masses are with respect to m_0. Distinct conjugate states are obtained by $S_i\times S_j\to\bar{S}_i\times S_j$ for $i\neq j$. The multiplicity includes the conjugate states.

Mass	States	Multiplicity
$+\epsilon_5$	$S_4\bar{S}_4$	1
$+\epsilon_4$	$S_4\times\bar{S}_3$	4
$+\epsilon_3$	$(S_3\times\bar{S}_3)_{I=0}$	1
$+\epsilon_2$	$(S_3\times\bar{S}_3)_{I=1}$	3
$+\epsilon_1$	$S_4\times\bar{S}_2$	4
m_0	$S_4\bar{S}_1$, $S_3\times\bar{S}_2$	10
$-\epsilon_1$	$S_3\times\bar{S}_1$	4
$-\epsilon_2$	$(S_2\times\bar{S}_2)_{I=1}$	3
$-\epsilon_3$	$(S_2\times\bar{S}_2)_{I=0}$	1
$-\epsilon_4$	$S_2\times\bar{S}_1$	4
$-\epsilon_5$	$S_1\bar{S}_1$	1

lies at the heart of the group-theoretical approach to flavor breaking. The maximum number of mixing parameters in Eq (3.1) will be $n(n-1)/2$, hence for $n\leq 3$ the input masses will be sufficient for determining the mixing parameters and the coefficients in Eq. (3.2) provided the unmixed diagonal terms are known. For $n\geq 4$ the input masses for the type of products involved in $qq\bar{q}\bar{q}$ mixing will in general be insufficient for determining the parameters in the mixing. This is unlike the $q\bar{q}$ mixing where the products involve the adjoint and singlet representations, and where sufficient of the mixing parameters are determined by the intramultiplet mixing to enable the masses alone to determine the full mass matrix for any SU(n).[48] To proceed for the baryonium mixing either some assumptions are required regarding the mixing terms in Eq. (3.1) or about the a_i^k. In principle the a_i^k could be determined from the decay properties of baryonium. In practice, the extraction of such information is difficult for $q\bar{q}$ mesons, and is not feasible at present for bar-

TABLE VII. Mass spectrum for the $10\times\overline{10}$. Conjugation follows the pattern in Table V.

Mass	States	Multiplicity
$+12\beta y$	$D_6\bar{D}_6$	1
$+3\beta(1+y)$	$D_6\bar{D}_5$	2
$+9\beta y$	$D_6\times\bar{D}_4$	4
$+6\beta(1+y)$	$D_6\times\bar{D}_3$, $D_5\bar{D}_5$	3
$+3\beta(1+2y)$	$D_6\times\bar{D}_2$, $D_4\times\bar{D}_5$	8
$+6\beta y$	$D_6\times\bar{D}_1$, $(D_4\times\bar{D}_4)_{I=0,1}$	10
$+3\beta(3+y)$	$D_5\bar{D}_3$	2
$+3\beta(2+y)$	$D_4\times\bar{D}_3$, $D_5\times\bar{D}_2$	8
$+3\beta(1+y)$	$D_5\times\bar{D}_1$, $(D_4\times\bar{D}_2)_{I=0,1}$	14
$+3\beta y$	$(D_4\times\bar{D}_1)_{I=1/2,3/2}$	12
$+12\beta$	$D_3\bar{D}_3$	1
$+9\beta$	$D_2\times\bar{D}_3$	4
$+6\beta$	$D_1\times\bar{D}_3$, $(D_2\times\bar{D}_2)_{I=0,1}$	10
$+3\beta$	$(D_1\times\bar{D}_2)_{I=1/2,3/2}$	12
m_0	$(D_1\times\bar{D}_1)_{I=0,1,2}$	9

TABLE VI. Mass spectrum for the $10\times\bar{6}$. The quantity in [ ] indicates the fine splitting from the mass of that level. Underlined states mix with their conjugates to form G-parity eigenstates. The (*), (**), pairs mix according to Eq. (3.4). See text regarding their assignment.

Mass	States	Multiplicity
$+6\beta(1+3y)$	$D_6\bar{S}_4^*[(-1+2y)\Delta]$	1
$+18\beta y$	$D_6\times\bar{S}_3[y\Delta]$	2
$+12\beta(1+y)$	$D_5\bar{S}_4$	1
$+6\beta(1+2y)$	$D_4\times\bar{S}_4[-\Delta]$, $D_5\times\bar{S}_3[\Delta]$, $D_6\times\bar{S}_2[(-1+2y)\Delta]$	6
$+12\beta y$	$(D_4\times\bar{S}_3)_{I=0,1}$, $D_6\bar{S}_1[2y\Delta]$	5
$+6\beta(3+y)$	$D_3\bar{S}_4^{**}[(1-y)\Delta]$	1
$+6\beta(2+y)$	$D_2\times\bar{S}_4[-y\Delta]$, $D_3\times\bar{S}_3[(2-y)\Delta]$, $D_5\times\bar{S}_2[y\Delta]$	6
$+6\beta(1+y)$	$D_1\times\bar{S}_4[-(1+y)\Delta]$, $(D_2\times\bar{S}_3)_{I=0**,1}[(1-y)\Delta]$ $D_5\bar{S}_1[(1+y)\Delta]$, $(D_4\times\bar{S}_2)_{I=0*,1}[-(1-y)\Delta]$	12
$+6\beta y$	$(D_1\times\bar{S}_3)_{I=1/2,3/2}[-y\Delta]$, $D_4\times\bar{S}_1[y\Delta]$	8
$+18\beta$	$D_3\times\bar{S}_2[\Delta]$	2
$+12\beta$	$(D_2\times\bar{S}_2)_{I=0,1}$, $D_3\bar{S}_1[2\Delta]$	5
$+6\beta$	$(D_1\times\bar{S}_2)_{I=1/2,3/2}[-\Delta]$, $D_2\times\bar{S}_1[\Delta]$	8
m_0	$D_1\times\bar{S}_1$	3

TABLE VIII. Spin-parity series for the flavor decompositions. The diquarks are in color 3.

$6\times\bar{6}$:	$L=J,\ P=(-1)^J,\ G=(-1)^{J+I}$
$10\times\bar{6}$:	$L=J-1,\ P=(-1)^{J+1},\ G=\pm$
$10\times\overline{10}$:	$L=J,\ P=(-1)^J,\ G=(-1)^{J+I}$
	$L=J-1,\ P=(-1)^{J+1},\ G=(-1)^{J+I}$
	$L=J-2,\ P=(-1)^J,\ G=(-1)^{J+I}$

yonium states.

Given this fact, we have assumed ideal mixing for the baryonium states (this is equivalent to the ideal mixing of Jaffe[1]). One expects deviations from ideal mixing to be small for baryonium. For $q\bar{q}$ mesons, deviations from ideal mixing can be understood qualitatively by timelike multigluon exchanges with the process $q_i\bar{q}_i \rightleftarrows q_j\bar{q}_j$ occuring predominantly by two-gluon exchange for the 0^- states and three-gluon exchange for the 1^- states.[49,50] The analogous process for $q\bar{q}$ pairs in the $qq\bar{q}\bar{q}$ states may occur with a one-gluon exchange.[1] However, since the quarks in the $q\bar{q}$ pairs annihilating come from different diquarks, one expects such a process to be small compared to the process in $q\bar{q}$ mesons especially for baryonium states relatively stable to break up into meson pairs. Also the quarks annihilating must come from a flavor singlet and a color octet. Both these features introduce factors which inhibit the process relative to that in $q\bar{q}$ mesons.

If we consider Eq. (3.1) in the basis of the physical states,

$$\langle P_i|H|P_j\rangle=\sum_{k=1}^{n}\sum_{l=1}^{n}a_i^k a_j^l\langle R_k|H|R_l\rangle\,, \tag{3.3}$$

then from the requirement of Eq. (3.3) vanishing for $i\neq j$, $i>j=1\cdots n$ we have $n(n-1)/2$ equations. These provide relationships between the mixing parameters in Eq. (3.1) and those in Eq. (3.2). The type of relationships vary from case to case. From the Clebsch-Gordan coefficients for the products in Eqs. (2.7)–(2.10);[37] the a_i^k can be determined. The mass spectrum for the physical states is then obtained from Eq. (3.3) with $i=j$. An example of mixing of particular significance occurs when Eqs. (3.2) and (3.1) have the following form:

$$a_i^k=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix},\quad M_{ij}=\begin{pmatrix}m_1 & A\\ A & m_2\end{pmatrix}. \tag{3.4}$$

In this case we obtain from Eq. (3.3), $m_1=m_2\equiv m_0$, and a physical mass spectrum,

$$M_{P_1}=m_0-A,\quad M_{P_2}=m_0+A\,. \tag{3.5}$$

Such a mixing has the disadvantage of A decoupling in Eq. (3.3), and being undetermined in the group approach. It arises in both the $6\times\bar{6}$ and $10\times\bar{6}$ mass spectra.

Tables V–VIII present the mass spectra of the ideally mixed states in Eqs. (2.7)–(2.10). All the mixing in $6\times\bar{6}$ except that involving the singlet is of the form in Eq. (3.4). Thus at each point where mixing occurs there arises a parameter equivalent to A in Eq (3.4) where we have labeled ϵ_i. The ordering is based on quark content. Hence around a mean mass, equal to the mass of the unmixed 20'', we have a symmetrically placed mass clustering. In the $10\times\overline{10}$ all the states can be expressed in terms of three constants, these are the same as those in the 84 in Table III. The $10\times\bar{6}$ involves features of both these spectra; the majority of states expressible in the form in $10\times\overline{10}$, while two pairs mix according to the form in Eq. (3.4). We have indicated these pairs by asterisks. Their assignment in the spectrum is based on Eq. (4.1) below. The underlined states in the $10\times\bar{6}$ mix further with their conjugates to form eigenstates of G parity, for example,

$$X_{(5,4)G=\pm}=\frac{1}{\sqrt{2}}(D_5\bar{S}_4\pm\bar{D}_5S_4)\,. \tag{3.6}$$

Associated with this mixing there will be a further splitting of their masses according to Eq. (3.5). The constants in Table VI are the same as those in the 45 in Table IV, with $\Delta=3\alpha+9\beta$. In all cases the breaking parameter, y, has been assumed to be constant for a given decomposition.

IV. DISCUSSION

The masses of the states with a determined mixing in the $10\times\bar{6}$, $6\times\overline{10}$, and $10\times\overline{10}$, can be expressed in terms of their quark content

$$\begin{aligned}m(q)=m_0&+A[n_s(T)+yn_c(T)]\\&+\Delta\{n_s(S)-n_s(A)+y[n_c(S)-n_c(A)]\}.\end{aligned} \tag{4.1}$$

The constants m_0, A, and y differ for the two representations, $n_i(T)$ is the total number of quarks of type i and $n_i(S)$ $[n_i(A)]$ is the number of quarks of type i in the symmetric (antisymmetric) diquark pair. For the $10\times\overline{10}$, Δ is zero, as all the quarks belong to symmetric diquark systems. The Δ term arises from the form of flavor breaking in Eq. (2.1), and is a splitting analogous to the ortho, para splitting in a two-fermion system. It is a feature present for color-singlet systems only involving four quarks or more. The Δ term is expected to be small. For example, when the mass of the U(3) 6 representation in the 45 is calculated in the naive quark model, and a universal quark-quark interaction assumed, then a degeneracy for the entire 6 is obtained. In terms of the mass formulas this implies $\Delta=0$. The presence of this splitting in the flavor group is similar to that due to the color-spin interaction where the splitting induced is such that the antisymmetric state [the SU(3) $\overline{3}$] is lighter.[4,5]

The phenomenon of clustering of states is evident experimentally, both near the threshold in $N\overline{N}$[31,51] and in the three types of hadronic interactions; at rest, formation, and production.[52] A similar situation is observed in e^+e^- annihilation.[53] In Table VIII, spin-parity and associated G-parity series for each decomposition are presented. The e^+e^- situation is particularly interesting since the J^{PC} values are well defined unlike the ambiguity present for the other exotic candidates (for example, the S, T, U resonances).[30] The $J^P=1^-$ states can occur only with $L=1$ and above. For $L=1$ they can occur once in the $6\times\overline{6}$ and $10\times\overline{6}$ states and twice in the $10\times\overline{10}$ states. From the color-magnetic interaction one expects the $6\times\overline{6}$ states to have lower mass. The four possible $I=0$ states in the $6\times\overline{6}$ are the S_4S_4, $(S_3\times\overline{S}_3)_{I=0}$, $(S_2\times\overline{S}_2)_{I=0}$, and $S_1\overline{S}_1$. These states have a mass separation of $\epsilon_5-\epsilon_2$, $2\epsilon_2$, $\epsilon_5-\epsilon_2$, respectively. As well, the $I_z=0$ states in $(S_3\times\overline{S}_3)_{I=1}$ and $(S_2\times\overline{S}_2)_{I=1}$ can also couple in e^+e^-. We note as a final point that a weak mixing phenomenon like that in the K-$\overline{K}^0$ system can also be expected to occur in the baryonium states, for example, between the $D_1\times\overline{D}_2$ and $D_2\times\overline{D}_1$ states in the $10\times\overline{10}$. We leave to a later paper a detailed phenomenological analysis based on the spectrum presented above.

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APPENDIX

The irreducible representations of U(4) can be described by a set of integers $\{m_{i4}\}$ with $i=1\cdots4$, and satisfying $m_{i4}\geq m_{i+1\,4}$.[46] The full structure of a representation can be contained in a basis scheme of Gelfand and Zetlin.[54] In this scheme the states are described by a triangular array of integers;

$$\begin{bmatrix} m_{14} & & m_{24} & & m_{34} & & m_{44} \\ & m_{13} & & m_{23} & & m_{33} & \\ & & m_{12} & & m_{22} & & \\ & & & m_{11} & & & \end{bmatrix} \tag{A1}$$

satisfying a "betweenness condition,"

$$m_{ij}\geq m_{ij-1}\geq m_{i+1j}\,. \tag{A2}$$

The array contains the subgroup reduction $U(4)\supset U(3)\supset U(2)$ in a transparent way; the rows $\{m_{in}\}i=1\cdots n$ describe irreducible representations of the U(n) subgroups for $n=2,3$. This subgroup reduction also corresponds to the labeling of states implicit in the flavor breaking given in Eq. (2.1).[55] The dimensions of the U(4) representations and those of its subgroups are given by the Weyl dimensionality formula,

$$D(n)=\prod_{i<j}^{n}\frac{(m_{in}-m_{jn}+j-i)}{1!\,2!\cdots(n-1)!}\,. \tag{A3}$$

The quantum numbers of the states in terms of the m_{ij} are given by

$$\begin{aligned} 3B&=\sum_{i=1}^{4}m_{i4}\,,\\ C&=\sum_{i=1}^{4}m_{i4}-\sum_{j=1}^{3}m_{j3}\,,\\ Y&=m_{12}+m_{22}-\frac{2}{3}\left(\sum_{i=1}^{3}m_{j3}\right),\\ I&=\tfrac{1}{2}(m_{12}-m_{22})\,,\\ I_z&=m_{11}-\tfrac{1}{2}(m_{12}+m_{22})\,. \end{aligned} \tag{A4}$$

B is the baryon number. The eigenvalues of the two lowest-order Casimir invariants required in the text are given by

$$I_1^{(n)}=\sum_{i=1}^{n}p_{in}-\binom{n}{2}, \tag{A5}$$

$$I_2^{(n)}=\sum_{i=1}^{n}(p_{in})^2-(n-1)\sum_{i=1}^{n}p_{in}+\binom{n}{3}, \tag{A6}$$

where

$$P_{ij}=m_{ij}+j-i\,. \tag{A7}$$

It has been shown in U(n), that a tensor operator T_{ij} can be expressed in terms of a sum of products of the group generators.[56] For the representations of relevance for the baryonium spectrum the highest product is second order in the generators, hence,

$$T_{ij}=a+b(e_{ij})+c\left(\sum_{k=1}^{4}e_{ik}e_{kj}\right). \tag{A8}$$

a, b, and c are constants, and the e_{ij}, $i,j=1\cdots4$, are the sixteen U(4) generators. The matrix elements for the diagonal generators are given by

$$\langle m|e_{ii}|\overline{m}\rangle=\left(\sum_{j=1}^{i}m_{ji}-\sum_{j=1}^{i-1}m_{ji-1}\right)\delta_{m\overline{m}}\,. \tag{A9}$$

The matrix elements for the e_{34} and e_{43} generators can be calculated from expressions given in Ref. 46.

[1]R. L. Jaffe, Phys. Rev. D 15, 267 (1977); 15, 281 (1977); R. L. Jaffe and F. E. Low, *ibid.* 19, 2105 (1979).

[2]R. L. Jaffe, Phys. Rev. D 17, 1444 (1978).

[3]R. L. Jaffe, MIT Report No. CTP 717, 1978 (unpublished); in Proceedings of Summer Institute on Particle Physics, 1977, SLAC Report No. 204 (unpublished), p. 352.

[4]Chan Hong-Mo and H. Høgaasen, Phys. Lett. 72B, 121, 400 (1978); Nucl. Phys. B136, 401 (1978).

[5]Chan Hong-Mo *et al.*, Phys. Lett. 76B, 634 (1978).

[6]Chan Hong-Mo, CERN Report No. TH. 2540, 1978 (unpublished).

[7]H. Høgaasen and P. Sorba, CERN Report No. TH. 2537, 1978 (unpublished).

[8]Tsou Sheung Tsun, Nucl. Phys. B141, 397 (1978).

[9]M. De Crombrugghe, H. Høgaasen, and P. Sorba, CERN Report No. TH. 2537, 1978 (unpublished).

[10]V. A. Matveev and P. Sorba, Nuovo Cimento 45A, 257 (1978).

[11]A. Th. M. Aerts, P. J. G. Rijken, and J. J. de Swart, Phys. Rev. D 17, 768 (1978).

[12]R. Gatto and F. Paccanoni, Nuovo Cimento 46A, 320 (1978).

[13]L. A. P. Balazs and B. Nicolescu, Phys. Lett. 72B, 240 (1977).

[14]B. Nicolescu, Nucl. Phys. B134, 495 (1978).

[15]H. J Lipkin, Phys. Lett. 45B, 267 (1973); 56B, 97 (1975); 74B, 399 (1978).

[16]S.-O. Holmgren and M. R. Pennington, Phys. Lett. 77B, 304 (1978).

[17]L. V. Laparashvili, Phys. Lett. 72B, 251 (1977).

[18]D. D. Brayshaw, SLAC Reports Nos. SLAC-PUB-2154, 2159, 1978 (unpublished).

[19]P. Zenczykowsji, Acta Phys. Pol. B10, 313 (1979).

[20]G. R. Goldstein and P. Haridas, Tufts University report, 1978 (unpublished).

[21]B. G. Wybourne, Aust. J. Phys. 31, 117 (1978).

[22]S. Ono, University of California, Davis, report (unpublished).

[23]G. C. Rossi and G. Veneziano, Nucl. Phys. B123, 507 (1977); Phys. Lett. 70B, 255 (1977).

[24]M. Imachi, S. Otsuki, and F. Toyoda, Prog. Theor. Phys. 55, 551 (1976).

[25]M. Uehara, Prog. Theor. Phys. 59, 1587 (1978); Saga University Report No. 78/2 (unpublished).

[26]G. F. Chew, LBL Report No. 5391, 1976 (unpublished).

[27]C. Rosenzweig, Phys. Lett. 36, 697 (1976).

[28]H. G. Dosch and M G. Schmidt, Phys. Lett. 68B, 89 (1977).

[29]R. Anderson and G. C. Joshi, J. Phys. G 5, 199 (1979).

[30]L. Montanet, CERN Report No. CERN/EP/Phys. 77-22 (unpublished).

[31]F. Myhrer, CERN Report No. TH.2348, 1977 (unpublished).

[32]P. Pavlopoulos *et al.*, Phys. Lett. 72B, 415 (1978).

[33]C. Evangelista *et al.*, Phys. Lett. 72B, 139 (1977).

[34]T. A. Armstrong *et al.*, Phys. Lett. 77B, 447 (1978).

[35]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).

[36]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974).

[37]R. Anderson and G. C. Joshi, J. Math. Phys. (to be published).

[38]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[39]S. Okubo, Prog. Theor. Phys. 27, 949 (1962).

[40]See for example, V. S. Mathur, S. Okubo, and S. Borchardt, Phys. Rev. D 11, 2572 (1975); S. Okubo, *ibid.* 11, 3261 (1975); A. Kazi, G. Kramer, and D. H. Schiller, DESY Reports No. 75/10, No. 75/11 (unpublished).

[41]C. S. Kalman, Lett. Nuovo Cimento, 18, 201 (1977); 21, 201 (1978).

[42]M. Bander, G. L. Shaw, P. Thomas, and S. Meshkov, Phys. Rev. Lett. 36, 695 (1976).

[43]A. De Rújula *et al.*, Phys. Rev. Lett. 38, 317 (1977).

[44]H. J. Likpin, H. R. Rubinstein, and N. Isgur, Phys. Lett. 78B, 295 (1978).

[45]H. Hayashi *et al.*, Ann. Phys. (N. Y.) 101, 394 (1976).

[46]J. D. Louch, Am. J. Phys. 38, 3 (1970).

[47]H. J. Lipkin, Phys. Lett. 67B, 65 (1977).

[48]The solution to this problem can be quite difficult, especially for a completely general mixing of the 1 and n^2-1 representations. Often only the zero-quantum-number states are known, hence the breaking parameter is undetermined. For a solution in SU(5) see R. Anderson and G. C. Joshi, Univ. of Melbourne Report No. UM-P-78/95, 1978 (unpublished).

[49]H. Fritzsch and P. Minkowski, Nuovo Cimento 30, 393 (1975). H. Fritzsch and J. D. Jackson, Phys. Lett. 66B, 365 (1977).

[50]K. Hirata, T. Kobayashi, and Y. Takaiwa, Phys. Rev. D 18, 236 (1978).

[51]I. S. Shapiro, Phys. Rep. 35C, 129 (1978).

[52]For summary, see Y. Nambu, *Proceedings of the XIX International Conference on High Energy Physics, Tokyo, 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Tokyo, 1979).

[53]See, for example, R. Bernabei *et al.*, Frascati Report No. LNF-77/19 (P), 1977 (unpublished).

[54]I. M. Gelfand and M. L. Zetlin, Dokl. Akad. Nauk. USSR 71, 825 (1950); I. M. Gelfand and M. I. Graev, Am. Math. Soc. Transl., Ser. 2, 64, 116 (1967).

[55]P. D. Javis, University of Southampton Report No. THEP 77/9-21, 1978 (unpublished).

[56]S. Okubo, J. Math. Phys. 16, 528 (1975); 18, 2382 (1977).