Gluonic-bound-state model and $X(2.8)$

K. Ishikawa
Department of Physics, University of California, Los Angeles, California 90024
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Gluonic states are discussed using a bound-state equation. We estimate masses and decay widths and point out the absence of a low-mass 0^+ state. Therefore, the easiest way to observe these states is through the $\gamma\gamma$ decay mode of the 0^- particle, whose mass is about 2.8 GeV in our approximation. One candidate for the 0^- gluonic state is $X(2.8)$, which was observed in a radiative decay of ψ (J).

In part due to the discovery of asymptotic freedom,[1] quantum chromodynamics (QCD) is believed to be a basic theory of the hadrons. This property makes calculation possible in the ultraviolet region and the results are consistent with the scalings of the e^+e^- reaction and lepton-hadron reactions. However, it is difficult to know the properties of the theory in detail in the infrared region. We believe that a phenomenological investigation of this theory is valuable at this time for such difficult problems.

QCD has a gluon as an additional degree of freedom beyond the quarks. To study the gluons[2] by a phenomenological method is the purpose of this paper. Experimentally we have not observed a one-gluon state which has color. The gluons might be confined in the same way as the quarks. It may be that any colored states are confined permanently and only the color-singlet states exist. Then as additional states beyond the usual $q\bar{q}$ or qqq states, the color-singlet gluon bound states (called GBS hereafter), which are composed of only gluons, could exist.

Because of the massless nature of the gluons, we need a relativistic bound-state model for an investigation of GBS. In a previous paper,[3] we discussed the bound-state equation which included the relativistic effects for ordinary mesons. Assuming a static confining force between colored charges, we obtained the equations of motion by applying the variational method. Now we apply the same method to the gluon bound states. If we suppose the universal form of the interaction between colored particles, then the force between the gluons is about twice as large as the one between the quark and the antiquark because of the different representation of the underlying gauge group.

Let us assume that the longitudinal instantaneous interaction causes the confinement in the Coulomb gauge as was shown by Bender, Eguchi, and Pagels.[4] In their paper, it was shown that the Coulomb interaction between the color charges, which is $1/r$ usually, is modified so drastically that it turns out to be the confining type interaction under some condition. Suppose that the Hamiltonian describing the motion of gluons in the Coulomb gauge is given by

$$H=\int d\vec{x}\left[\tfrac{1}{2}\vec{E}^2(x)+\tfrac{1}{2}\vec{B}^2(x)+\int dy\ \rho^a(x)\tfrac{3}{8}V(x-y)\rho^a(y)\right], \tag{1}$$

where

$$B_k^a=\tfrac{1}{2}\epsilon_{ijk}(\partial_i A_j^a-\partial_j A_i^a+gf^{abc}A_i^bA_j^c)\,, \tag{2}$$

$$\rho^a(x)=f^{acb}A_i^b(x)E_i^c(x)\,, \tag{3}$$

and $E_i^a(x)$ are conjugate to $A_i^a(x)$. The second term of the Hamiltonian represents a confining interaction between colored charges, which we assume to be the same form as the potential between quarks. An energy expectation value for the color-singlet two-gluon state with vanishing momentum is given by

$$E=2\int d\vec{p}\,|\vec{p}|\ |f_{ij}(\vec{p})|^2 +\tfrac{3}{4}\left[\int d\vec{x}\,|\tilde{f}_{ij}(\vec{x})|^2V(\vec{x})+g^2|\tilde{f}_{ij}(0)|^2\right]. \tag{4}$$

The wave function $f_{ij}(\vec{p})$ and its Fourier transform $\tilde{f}_{ij}(\vec{x})$ satisfy

$$\int d\vec{p}\ \mathrm{Tr}\left[f^\dagger(\vec{p})f(\vec{p})\right]=\int d\vec{x}\ \mathrm{Tr}\left[\tilde{f}^\dagger(x)\tilde{f}(x)\right]=1\,. \tag{5}$$

The last term of Eq. (4) comes from the contact four-gluon coupling and is essential to prove the absence of the scalar 0^+. Requiring the energy E to become a minimum, we obtain the equations for $\tilde{f}_{ij}(\vec{p})$ $[f_{ij}(\vec{x})]$. As a simple example, we consider an r^2 potential, $V=cr^2$, and show the absence of the 0^+ state.

The scalar and pseudoscalar are expressed by

$$f_{ij}^S(\vec{p})=\delta_{ij}f^S(\vec{p})\,,\qquad f_{ij}^P(\vec{p})=\epsilon_{ijk}p_kf^P(\vec{p})\,, \tag{6}$$

where the functions $f^S(\vec{p})$ and $f^P(\vec{p})$ are even functions of $\vec{p}$ from Bose-Einstein statistics and satisfy

$$\frac{9}{4}c\sum_i\left(\frac{\partial}{\partial p_i}\right)^2 f^S(\vec{p}) - 2|\vec{p}|f^S(\vec{p}) + \frac{9}{4}g^2\int d\vec{p}\,' f(\vec{p}\,') + E^S f(\vec{p}) = 0\,,$$
$$\frac{9}{4}c\sum_i\left[\left(\frac{\partial}{\partial p_i}\right)^2 - \frac{1}{p^2}p_i\frac{\partial}{\partial p_i}\right]f^P(\vec{p}) - 2|\vec{p}|f^P(\vec{p}) + E^P f^P(\vec{p}) = 0\,. \tag{7}$$

From Eq. (7) we can prove

$$\int d\vec{p}\,' f^S(\vec{p}\,') = 0\,. \tag{8}$$

If we assume that this integration does not vanish, the following relation should be satisfied from Eq. (7):

$$f^S(\vec{p}) \underset{|\vec{p}|\to\infty}{=} \frac{9}{8}\frac{1}{|\vec{p}|}g^2\int d\vec{p}\,' f^S(\vec{p}\,')\,, \tag{9}$$

which contradicts both itself and the normalization condition. Then we conclude that Eq. (8) should be satisfied. The states which satisfy Eq. (8) cannot be s states. There is no 0^+ state in an r^2-potential case. For the other potential cases that are regular at the origin, too, it is possible to prove Eq. (8) in the same manner, if the equations are reduced to the differential equations in momentum space. For other cases such as r^{2n-1} ($n \geq 0$), if we are allowed to use the inequality

$$\lim_{|p_1|\to\infty}\int d\vec{p}_2\,\frac{1-\cos L|\vec{p}_1-\vec{p}_2|}{|\vec{p}_1-\vec{p}_2|^2}f(\vec{p}_2) \leq \frac{N}{|p_1|^2}\,, \tag{10}$$

which might be true for a locally concentrated function $f(\vec{p})$, the integration should vanish. Accidentally, the high excited s state could satisfy Eq. (8).

We may imagine three-gluon or four-gluon 0^+ states; however, the masses of these many-gluon states are expected to be much greater than those of the two-gluon states. The discussion about these states will be given in a separate communication.

The linear plus Coulomb potential is most popular for the charmonium problems.[5] Now we consider this potential, which is parametrized as $cr-\kappa/r$, for the discussion of the GBS. We apply a Gaussian approximation[6] for the wave function as follows:

$$\tilde{f}^P(x) = \left(\frac{\pi^3}{18\alpha}\right)^{1/4} e^{-\alpha x^2}\,, \tag{11}$$

where α is a parameter which we determine later. The energy expectation value of this state has a minimum value $[8c(16-9\kappa)/\pi]^{1/2}$ at $9c/(16-9\kappa)$ of α, when the coefficient κ of the Coulomb part is smaller than $\frac{16}{9}$. In other cases the energy does not have the minimum value.[7]

Let us choose 0.233 GeV^2 as c, which has been used by the Cornell group[5] for the discussion of charmonium. The mass is a function of κ and is given in Fig. 1. For the small value of κ, the mass is far larger than those of the ordinary mesons and is equal to be 3.1 GeV at zero κ. Conversely, κ is a function of a typical mass scale of the system from the renormalization-group argument. There might be two ways to include this effect. In the first, κ is simply considered as a function of the mass of the system as follows:

$$\kappa = \frac{a_0}{\ln(M^2/\Lambda^2)}\,, \tag{12}$$

where $a_0 = 1.09$ and $\Lambda = 0.5$ GeV. Λ is determined from the analysis of electron or neutrino reactions; a_0 is determined in such a way that the κ becomes 0.3 (0.5) at 3.1 GeV.

One possible solution from Fig. 1 is at $M \simeq 2.8$ GeV and $\kappa \simeq 0.3$ (0.5), and the other is at $M \simeq 0.7$ GeV and $\kappa = 1.7$, which is too large to be regarded as a small coupling. In this paper, we will investigate the former possibility, which may have something to do with the $X(2.8)$ (Ref. 8) observed in the radiative decay of ψ (J).

In the second method, the coupling constant is considered to be a function of momentum transfer. The potential $V(r)$ is given by a Fourier transform of

$$-\frac{4}{3}\frac{12\pi}{33-2n_f}\frac{1}{\vec{q}^{\,2}}\ln\left(\frac{1}{1+\vec{q}^{\,2}/\Lambda^2}\right)\,, \tag{13}$$

where n_f is a number of flavors. Let us postulate Eq. (13) is applicable for a small $\vec{q}^{\,2}$, too, because the behavior of this function at small $\vec{q}^{\,2}$

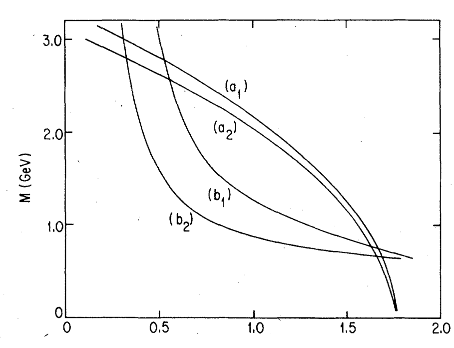

FIG. 1. Masses are given as functions of κ by using (a) our scheme and (b) the first of the renormalization-group arguments. The parameters are given by (1) κ(3 GeV)=0.3, C=0.233, (2) κ(3 GeV)=0.5, C=0.27.

is given by $(\vec{q}^2)^{-2}$ whose Fourier transform is proportional to r. Richardson[9] has discussed the heavy quarkonium by using this potential and has obtained remarkable results. By using this potential to the GBS, the mass of GBS is obtained and given in Fig. 2 as a function of Λ. From Fig. 2 we can see the mass is around 2–3 GeV, if we choose 0.5–8 as Λ. The two methods give consistent results.

The $X(2.8)$ has been regarded as an ^1S state of $c\bar{c}$, although there is a large discrepancy in a radiative $M1$ transition. This problem disappears if we regard $X(2.8)$ predominantly as the GBS. The estimation of the width of $\psi\ (J) \to X(2.8)\gamma$ will be done after a discussion of a mixing problem.

Before a study of the GBS decay, we should notice the fact that the gluon does not have any flavor. Therefore, any transitions of these states to the ordinary hadrons are a sort of Okubo-Zweig-Iizuka (OZI) suppression process.[10] For the origin of OZI violation there might be two main possible mechanisms, gluon-exchange effects[11] and unitarity-correction effects.[12] The unitarity-correction effects are not important in the GBS decay because there is no mechanism with which the GBS decay to the ordinary mesons except the OZI-violation processes. We investigate only the gluon exchange effects.

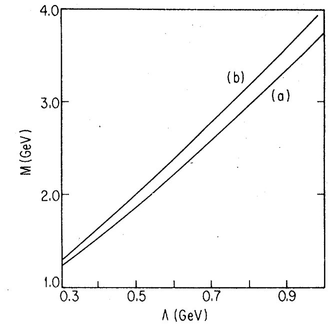

FIG. 2. Masses are given as functions of Λ in the second of the renormalization-group arguments. The n_f are 3 (a) or 5 (b).

A matrix element for the GBS transition to a $q\bar{q}$ state through the lowest-order perturbative effect is given by

$$ {}_{\rm out}\langle q_i'(p)\bar{q}_i'(q)\,|\,{\rm GBS}\rangle_{\rm in} = \frac{1}{(2\pi)^3}\left(\frac{m^2}{p^0q^0}\right)^{1/2}\frac{1}{M_{\rm GBS}}\frac{1}{\sqrt{8}}\sum_{a,\,j'}\left(\frac{\lambda^a}{2}\right)_{i'j'}\left(\frac{\lambda^a}{2}\right)_{j'i'} g^2\bar{U}(p,s)\gamma_i \times \int d\vec{l}\,\frac{l\!\!/ + m}{2l^0}\gamma_j v(\vec{q},s')\frac{1}{2|\vec{l}-\vec{p}|} f^{ij}(\vec{l}-\vec{p})\,\delta(M_{\rm GBS}-p_0-q_0)\,\delta(\vec{p}+\vec{q})\,, \quad (14)$$

where m and $M_{\rm GBS}$ stand for a quark and a GBS mass respectively. Using this matrix element we calculate the width of the GBS based on (1) a free quark model, (2) a pole dominance model. The width becomes 1.8 GeV in (1) per flavor when we use 0.19 as α_s and neglect m. To obtain this value, we made the Gaussian approximation for $f^{ij}(\vec{l}-\vec{p})$ and took a leading order term of the integration of Eq. (12) about α. If the value calculated is true, it is impossible to detect the GBS clearly. However, to treat the quarks as if free particles would be too simplified. We calculate by using the second picture, the pole-dominance model, where quarks are bound permanently. The ordinary meson is regarded with the $q\bar{q}$ state coming from the decay of the GBS. The transition matrix element from the GBS to a meson α is calculated and reduced to

$$\frac{1}{\pi}\frac{1}{M_\alpha}\delta(M_\alpha - M_{\rm GBS})\,\delta(\vec{0})\,\alpha_s\frac{\overline{1}}{3}\psi_\alpha(0)\left(\frac{8\pi\alpha}{9}\right)^{1/4} \quad (15)$$

in the same approximation as in the previous case for $f^{ij}(k)$ and the nonrelativistic approximation for the meson α. Actually, $\psi_\alpha(0)$ is a wave function at the origin in a Schrödinger equation model.

This kind of transition causes a mixing between ordinary isosinglet particles and the GBS. We consider the pseudoscalar, which we write as η. The mixing of $\eta^0_{\rm GBS}$ with η^0_c is most important because of the close values of both masses. The mixing angles of a heavy GBS such as ours to the light pseudoscalar particles are so small that they could be neglected when we use the nonrelativistic calculations for these light mesons. This is because the mass differences between the light mesons and GBS is so large. If there were light GBS whose mass was close to that of the η or η', this mixing could have been important. Actually, the mixing of a particle with the other heavy particle causes the mass of the particle to decrease. Experimentally, η and η' are much heavier than indicated by a naive quark-model calculation. Therefore, the mixing of the heavy GBS with η or η' may not be related to η or η' problems. Neglecting the other mixing angles, the real states, η_c and $\eta_{\rm GBS}$, are given by rotating η^0_c and $\eta^0_{\rm GBS}$ by angle θ, which satisfies

$$\tan 2\theta = \frac{2\sqrt{2}}{\pi}\frac{1}{m^2(\eta_c^0) - m^2(\eta_{\mathrm{GBS}}^0)}\alpha_s |\psi_\alpha(0)| \left(\frac{8}{9}\pi\alpha\right)^{1/4} . \quad (16)$$

The total width of η_{GBS} is given by $\sin^2\theta\,\Gamma(\eta_c^0)$, which is smaller than 5.9 MeV assuming a naive quark-model calculation for $\Gamma(\eta_c^0)$. We regard the second method as more reliable from many other experiences. Of all the decay models, the $\gamma\gamma$ decay of the 0^- GBS may be the easiest way to observe the GBS because the 0^- is the lightest in our scheme. The $\gamma\gamma$ decay width is about the width of another 0^- isosinglet meson that has the same mass times the mixing angle squared.

There are two mechanisms which contribute to the radiative decay of ψ (*J*) to $\eta_{\mathrm{GBS}}\gamma$. One is through the mixing between the GBS and η_c and the width from this is smaller than 1 keV when we use the previous parameter for C and κ and a naive quark model 28 keV as $\Gamma(\psi \to \eta_c^0\gamma)$. The other is through a direct transition of ψ (*J*) to γ plus two gluons and is estimated by Koller and Walsh[13] (Fig. 3). The value is about 0.7 keV and is resonable from an experimental viewpoint. Therefore, there is no problem[14] about the radiative decay of ψ (*J*) $\to X(2.8)\gamma$, if we assume that $X(2.8)$ is η_{GBS}. The η_c is heavier than both of η_{GBS} and η_c^0 if η_c^0 is heavier than η_{GBS}^0 (2.8 GeV).

As a final problem, we consider the GBS production. The heavy quarkonium ($c\bar{c}, b\bar{b}, \cdots$) decay is one method to produce the GBS. In the decay of ψ (*J*), ψ', and other heavy mesons like Υ, etc., the GBS may be produced and the branching ratio for it may not be small. Then in the radiative decay of Υ the GBS can be produced and the rate is expected to be about the same as that of ψ (*J*) decay (Fig. 3).

In the hadron-hadron reactions, too, the GBS can be produced via a mechanism similar to the ordinary meson production through the quarks.

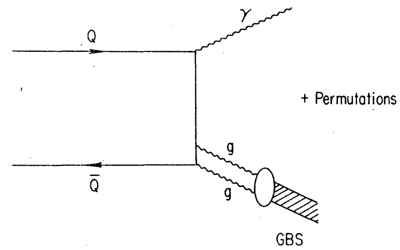

FIG. 3. Diagram which contributes to a direct transition of $Q\bar{Q}$ to the γ plus GBS.

The GBS production cross section at large p_T is estimated roughly to be the ordinary meson production cross section multiplied by the square of a ratio of the gluon distribution functions. We do not known anything about the gluon distribution functions experimentally except the following relation:

$$\sum_a \int_0^1 dx\, x\, G_{ga}(x) \simeq 0.5 . \quad (16)$$

The magnitude is of the same order as that of the quarks. Therefore, the GBS production cross section may be the same order of magnitude as the ordinary hadron production cross section at large p_T, and probably at small p_T, too.

To regard $X(2.8)$ as predominantly a GBS will be justified if it is produced in the hadron-hardon reaction much more than expected from the charmonium assumption[15] or it is produced in the radiative decay of Υ in the same ratio as that of ψ (*J*).

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