PHYSICAL REVIEW D VOLUME 20, NUMBER 3 1 AUGUST 1979

Partial conservation of axial-vector current effects in the MIT bag model

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(Received 9 April 1979)

We extend the calculations of the MIT-bag-model spectrum to include the pion-field exchange energy in a manner consistent with the ideas of partial conservation of axial-vector current. We find that this interaction energy is sufficient to destabilize the quark bag whose quantum numbers are those of the pion. The η mass is also improved, while other features of the low-lying hadron mass spectrum are not significantly modified.

I. INTRODUCTION

Recently Callen, Dashen, and Gross[1] have advanced a new theory of hadronic structure in which they suggest that a more realistic variant of the MIT bag model[2] should involve a simple coupling between pion and quark fields consistent with the ideas of chiral symmetry. The proposed connection between the two fields is in the form of a boundary condition at the surface of the bag. The region exterior to this surface contains the pion field, and the boundary condition is an expression of the continuity of the axial-vector current.[3,4]

In this paper we calculate the spectroscopic effects of the implied pion-exchange energy in this model. The calculation is carried out in the same spirit as the gluon-exchange calculations of the MIT group.[5] We find that the pion-exchange energy between quarks at the boundary surface is sufficient to provide a destabilization mechanism for the pionic quark bag. The same pion-exchange energy moves the η mass, which is degenerate with the pion in the absence of the pion cloud, somewhat closer to its experimental value. Other features of the hadron mass spectrum are not changed in any significant way. Thus all of the good features of instanton effects on the bag mass spectrum that have been explicitly demonstrated by Horn and Yankielowicz[6] can also be produced by the pion-exchange energy. Any deeper connection between the two mechanisms remains unexplored.

In Sec. II we give a more explicit statement of the model. In Sec. III we obtain solutions of the semiclassical pion field equations subject to the model boundary conditions and apply these results to the calculation of the exchange energy. In Sec. IV we present and discuss results of numerical fits to the low-lying hadron data.

II. THE MODEL

The basic equations of the MIT bag model[5] include the coupled quark-gluon field equations of quantum chromodynamics, which are used in the linear approximation of the gluon field[7]:

$$(i\partial\!\!\!/ - m + gA\!\!\!/_l\lambda^l)q = 0 \,, \tag{1}$$

$$\partial_\alpha(\partial^\alpha A_l^\beta - \partial^\beta A_l^\alpha) = -g\bar{q}\lambda_l\gamma^\beta q \,. \tag{2}$$

These equations apply within the bag; outside the bag the fields vanish. The field equations are augmented by the following linear relations on the bag surface:

$$(in_\beta\gamma^\beta - 1)q = 0 \,, \tag{3}$$

$$n_\alpha(\partial^\alpha A_l^\beta - \partial^\beta A_l^\alpha) = 0 \,. \tag{4}$$

In the rigid-sphere approximation, the hadron bag is a sphere of radius R, and $n_\beta = (-\hat{r}, 0)$ is the interior normal to the surface. There is also a quadratic boundary condition whose contents are presumably satisfied by restricting the energy eigenmodes of the quarks within a specific bag to one type and by minimizing the hadron mass with respect to the radius, R.

The mass formula for a hadron bag of radius R' is given by

$$M(R) = E_V + E_0 + E_Q + E_M + E_E \,. \tag{5}$$

$E_V = 4\pi BR^3/3$ is the volume energy and $E_0 = -Z_0/R$ is the vacuum contribution.[8] The quark and antiquark energies are contained in $E_Q = (N_q + N_{\bar{q}})E_q$, where E_q is an energy eigenvalue associated with solutions of Eqs. (1) and (3) in the limit of vanishing gluon coupling ($g \to 0$).[9] Finally E_E and E_M are, respectively, the order g^2 color-electric and color-magnetic energies generated by the quarks within the bag.

To the above standard MIT bag model we add a fundamental pion field, Φ, that forms a cloud outside the surface $r = R$ but is not allowed to penetrate into the interior region to which the quarks and gluons are confined. The pion field is to satisfy the Klein-Gordon equation,

$$(\partial^2 - \mu^2)\vec{\Phi} = 0 \tag{6}$$

subject to the boundary condition

$$f_\pi n^\beta \partial_\beta \vec{\Phi} = i\bar{q}\gamma_5 \tfrac{1}{2}\vec{\tau} q \,. \tag{7}$$

In the zero-mass limit, this model is equivalent to the linear approximation of the chirally symmetric model suggested by Callan, Dashen, and Gross. The boundary condition of Eq. (7) follows from the condition that the normal component of the axial-vector current,

$$\vec{a}_\beta = \bar{q}\gamma_\beta\gamma_5\tfrac{1}{2}\vec{\tau}q + f_\pi\partial_\beta\vec{\Phi}\,, \tag{8}$$

be continuous across $r=R$. In the chiral-symmetry limit, $\mu\to 0$, $m\to 0$, the current is conserved, while for $\mu\neq 0$ we have the partial conservation of axial-vector current (PCAC) condition

$$\partial^\beta\vec{a}_\beta = \mu^2 f_\pi\vec{\Phi}\,. \tag{9}$$

III. PION FIELD AND EXCHANGE ENERGY

We calculate the pion field in the semiclassical approximation used for gluon energy calculations by the MIT group.[5] The pion field is the superposition of fields generated by each quark source. Here the quark sources appear explicitly only in the boundary condition, Eq. (7). The pion field is therefore determined by

$$(\partial^2-\mu^2)\vec{\Phi}=0\,,\quad r>R \tag{10}$$

and

$$-f_\pi\frac{\partial\vec{\Phi}}{\partial r} = i\bar{q}\gamma_5\tfrac{1}{2}\vec{\tau}q\,,\quad r=R \tag{11}$$

where in Eq. (11) we use the quark wave functions in the absence of gluon couplings. We also impose the boundary condition

$$\Phi(\infty,t)\to 0 \tag{12}$$

in order to have a bound state of matter. Since only quarks in the same mode are allowed in a bag, the right-hand side of Eq. (11) is independent of time. Equation (12) is also a time-independent boundary condition, so that Φ must be a static field.

For massless quarks (only u and d quarks are relevant here) the wave functions at the surface are given by[5]

$$q(\vec{r},0)=\frac{N}{\sqrt{4\pi}}\begin{pmatrix} ij_0(\chi)U \\ -j_1(\chi)\vec{\sigma}\cdot\hat{r}U\end{pmatrix},\quad r=R\,. \tag{13}$$

j_0 and j_1 are spherical Bessel functions, U is a spin-isospin function, and χ/R is the single-quark energy eigenvalue. The normalization factor is given by

$$N^{-2}=R^3 j_0^{\,2}(\chi)\frac{2(\chi-1)}{\chi}\,, \tag{14}$$

and the lowest energy eigenvalue is determined by

$$j_0(\chi)=j_1(\chi) \tag{15}$$

which yields $\chi=2.04$.

Assembling the preceding pieces, we find that the right-hand side of Eq. (11) is

$$i\bar{q}\gamma_5\tfrac{1}{2}\vec{\tau}q = -(4\pi)^{-1}CU^\dagger\vec{\sigma}\cdot\hat{r}\,\vec{\tau}UR^{-3}\,, \tag{16}$$

where

$$C^{-1}=2(\chi-1)/\chi\simeq 1\,. \tag{17}$$

The pion field can therefore be written in terms of spherical harmonics of order unity only,

$$\vec{\Phi}=\vec{D}_m Y_{1m}(\theta,\phi)h(\mu r)\,, \tag{18}$$

where $h(\beta)$ is the Hankel function

$$h(\beta)=(\beta^{-1}+\beta^{-2})\exp(-\beta)\,. \tag{19}$$

Combining Eqs. (11), (16), and (18) yields for $\vec{D}_m$

$$f_\pi\vec{D}_m\frac{dh(\mu R^3)}{d(\mu R^3)} = (12\pi)^{-1/2}CU^\dagger[\sigma_3\delta_{m0}+\sqrt{2}\sigma_-\delta_{m1}+\sqrt{2}\sigma_+\delta_{m-1}]\vec{\tau}U\,, \tag{20}$$

where $\sigma_\pm=(\sigma_1\pm i\sigma_2)/2$.

The pion field energy is given by

$$\tilde{E}_\pi=\tfrac{1}{2}\int_{r\geq R} d^3r[(\dot{\Phi})^2+(\nabla\Phi)^2+\mu^2\Phi^2]\,. \tag{21}$$

We write the pion field explicitly as a sum of terms arising through the boundary condition from each of the quark sources in the hadron bag:

$$\vec{\Phi}=\sum_a\vec{\Phi}_a\,. \tag{22}$$

As in the gluon energy calculation of the MIT group, we argue that the terms in E_π of the type $\Phi_a\Phi_a$ are part of the quark self-energy which lead to quark-mass renormalization and are therefore absorbed into the phenomenological quark energy term E_Q in Eq. (5). We therefore keep only the exchange terms and let

$$\tilde{E}_\pi\to E_\pi=\sum_{a\neq b}E^{ab}\,, \tag{23}$$

where

$$E^{ab}=\tfrac{1}{2}\int_{r\geq R} d^3r(\nabla\vec{\Phi}_a\cdot\nabla\vec{\Phi}_b+\mu^2\vec{\Phi}_a\cdot\vec{\Phi}_b)\,. \tag{24}$$

Gauss's theorem allows us to reduce the exchange integral to the simple surface term

$$E^{ab}=-\tfrac{1}{2}\int d\Omega\, r^2\vec{\Phi}_b\cdot\frac{\partial\vec{\Phi}_a}{\partial r}\bigg|_{r=R}\,. \tag{25}$$

Inserting Eq. (11) for $\partial\Phi_a/\partial r$ and the spherical harmonic expansion for Φ_b yields

$$-(48\pi)^{1/2}f_\pi E_\pi^{ab}=Ch(\mu r)U_a^\dagger[\vec{D}(b)\sigma_3+\sqrt{2}\vec{D}_{-1}(b)\sigma_{-1} + \sqrt{2}\vec{D}_1(b)\sigma_+]\cdot\vec{\tau}U_a\,. \tag{26}$$

TABLE I. Masses of hadrons (in GeV) with and without the pion interaction term. The masses of p, $N^*_{3/2}$, Ω^-, and ω are used as input in both cases. The values of parameters are discussed in the text.

Particle	$\langle\hat{\Omega}\rangle$	$\langle E_\pi(R_{\rm MIT})\rangle$	Exp. mass	Bag-model fit without E_π [a]	Bag-model fit with E_π
p	30	0.180	0.938	0.938	0.938
Λ	18	0.111	1.116	1.105	1.077
Σ	2	0.012	1.189	1.144	1.081
Ξ	0	0	1.321	1.289	1.219
$N^*_{3/2}$	6	0.027	1.236	1.233	1.236
Σ^*	2	0.009	1.385	1.382	1.373
Ξ^*	0	0	1.533	1.529	1.518
Ω^-	0	0	1.672	1.672	1.672
ρ	2	0.014	0.77	0.783	0.816
K^*	0	0	0.892	0.928	0.951
ω	−6	−0.043	0.783	0.783	0.783
ϕ	0	0	1.019	1.068	1.100
K	0	0	0.495	0.497	0.386
π	−6	−0.121	0.139	0.280	···

[a] From Ref. 5.

We define the effective pion-exchange operator $\hat{E}_\pi^{ab}$ by

$$E_\pi^{ab} = U_a^\dagger U_b^\dagger \hat{E}_\pi^{ab} U_a U_b \,. \tag{27}$$

Equations (20) and (26) then give

$$\hat{E}_\pi^{ab} = (48\pi)^{-1} C^2 f_\pi^{-2} R^{-3} G \vec{\sigma}_a \cdot \vec{\sigma}_b \vec{\tau}_a \cdot \vec{\tau}_b \,, \tag{28}$$

where

$$G = [-2h(\mu R)]/[\mu R h'(\mu R)] \tag{29}$$

and note that as $\mu \to 0$, $G \to 1$.

A useful form for the pion-exchange energy is

$$E_\pi = \sum_{a\neq b} \langle \hat{E}_\pi^{ab} \rangle = (48\pi)^{-1} C^{-2} f_\pi^{-2} R^{-3} G \langle \hat{\Omega} \rangle \tag{30}$$

where

$$\hat{\Omega} = \sum_{a\neq b} (2\hat{S}_{ab}^{\,2} - 3)(2\hat{I}_{ab}^{\,2} - 3) \,. \tag{31}$$

$\hat{S}_{ab}$ and $\hat{I}_{ab}$ are, respectively, the spin and isospin operators of the two-particle subsystem. The results for pion exchange between two antiquarks or between a quark and an antiquark are the same; the reader should be reminded that strange quarks are not counted in the above summation.

IV. NUMERICAL RESULTS AND DISCUSSION

The numerical values of $\langle\hat{\Omega}\rangle$ are given in Table I. We use the usual SU_6 contents with the exception of ϕ for which we use $\bar{s}s$, and of ω, which we take to have no $\bar{s}s$ content. We also show in Table I the value for the expectation value of $\hat{E}_\pi$ in the states of Ref. 5, with radii as given in their Table III. We use $f_\pi = 0.94\mu/\sqrt{2}$. A number of these energies are substantial. In particular, the contribution of this term to the mass of the bag state with pion quantum numbers is considerable and negative, which tends to improve the agreement with experiment. However, in practice such a large negative term removes the minimum in the expression for the pion mass as a function of the radius of the bag. Therefore the quadratic boundary condition at the bag surface cannot be satisfied, and there is no bag state with the quantum numbers of the pion. Thus the pion-exchange interaction alone is capable of destroying the pionlike bag state, just as the instanton term of Refs. 1 and 6 can remove the pion state. We note that for $\mu \to 0$ our pion-exchange energy E_π has the same R dependence ($\sim R^{-3}$) as the instanton-induced interaction energy E_I obtained by Horn and Yankielowicz. However, there is the important difference that E_I affects only the proton, Λ, pion, and η states, while E_π affects all states with two or more nonstrange constituents.

This perturbative calculation of the expectation of $\hat{E}_\pi$ is unrealistic, since the large $1/R^3$ term will change the value of the equilibrium radius, thus changing all contributions to $M(R)$. Consequently, a new fit to the bag-model phenomenological parameters is required. We have therefore done a complete refitting of parameters and radii, including the pion-interaction term using $f_\pi = 0.94\mu/\sqrt{2} = 0.092$ GeV. The procedure is exactly parallel to that of Ref. 5, and our program does reproduce their fit when the pion term is not included. With the pion term included, the bag parameters Z_0, B, α_c, and m_s were determined using the proton,

$N^*_{3/2}$, Ω^-, and ω experimental masses. We used zero mass for the nonstrange quarks and neglected the very small contribution of color-electric gluon interactions. The results are given in Table I, along with experimental masses and the fit of Ref. 5 for comparison. The parameters obtained in our fit are $Z_0=1.53$, $B=3.00\times10^{-4}$ GeV^4, $\alpha_c=0.790$, and $m_s=0.291$ GeV.

For comparison, the parameters obtained in Ref. 5 in the fit using zero for the mass of the nonstrange quark are $Z_0=1.84$, $B=4.42\times10^{-4}$ GeV^4, $\alpha_c=0.55$, and $m_s=0.279$ GeV. The radii were not changed greatly (except of course for the pion) and increased for most states (those with positive $\langle\hat{\Omega}\rangle$ in the table). There is indeed no stable pion in this fit and there would not be even for substantially larger values of f_π. Most other masses change only slightly and no mass changes by an amount comparable to the perturbative calculation. We have also made a fit including the instanton energy but not the pion-interaction energy (not shown in the table). The masses resulting from this fit are qualitatively much like those found here, although of course only the proton, Λ, pion, and η are shifted. The strength of the instanton interaction has only been estimated by theoretical considerations; our numerical fits prefer the minimum strength that will destroy the pion-like bag state.

We have also looked at the effect of the pion-exchange energy on the η and η'. In the absence of the higher-order gluon exchanges discussed by DeGrand *et al.*,[5] one of these would have a quark content of $\bar{s}s$ and the other would be $\bar{u}u+\bar{d}d$. The latter would be degenerate with the pion in the absence of the pion-exchange energy, thus having a mass of 0.280 GeV using the MIT parameters, in bad agreement with either experimental mass. The perturbation arguments indicate a contribution of 0.362 GeV from the pion-exchange energy, leading to the hope that at least one of the η and η' masses might agree with experiment. Unfortunately, when $\langle E_\pi(R)\rangle$ is properly included in the minimization of $M(R)$, the $\bar{u}u+\bar{d}d$ combination has a mass of only 0.302 GeV, in bad agreement with either the 0.549 GeV mass of the η or the 0.958 GeV mass of the η'. The corresponding mass of the $\bar{s}s$ combination changes only slightly to 0.645 GeV and remains in poor agreement with either experimental mass. Thus the mixing effects of Ref. 5 must be invoked to fit the η and η' masses.

ACKNOWLEDGMENT

This work was supported in part under USDOE Contract No. C00-5007.

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[7]Our metric is $g_{ii}=-g_{00}=+1$.

[8]In Ref. 5 it is argued that Z_0 is positive. However, it is worth pointing out that in QED it has been shown that $Z_0>0$ for a slab cavity but of opposite sign for a spherical cavity. In our numerical work we have allowed either sign for Z_0 and find that a positive Z_0 does indeed lead to a better fit with and without the inclusion of the pion-exchange energy.

[9]However, the effect of the gluon field on the numerical value of R is accounted for through the inclusion of E_E and E_M in the minimization of $M(R)$.