# Monte Carlo approach to multiparticle production in a quark-parton model. IV. Charge distributions and fluctuations

V. Černý, P. Lichard, Š. Oleiník, and J. Pišút

Department of Theoretical Physics, Comenius University, Bratislava, Czechoslovakia (Received 28 November 1978)

.A Monte Carlo quark-parton model is shown to give a reasonable qualitative description of a representative set of data on charge transfer and fluctuations in multiparticle production in hadronic collisions. In particular this is shown for the charge distribution in rapidity, charge fluctuations across a given value of rapidity, and the energy and multiplicity dependence of charge fluctuations across  $y = 0$ . Some disagreement is found with the data on the distribution of lengths of gaps with a fixed charge transfer. The results obtained in this series of papers are briefly summarized.

#### I. INTRODUCTION

The present paper concludes the series $^{1-3}$  (referred to as I, II, and III) in which we have studied a Monte Carlo quark-parton model of multiparticle production in hadronic collisions and in  $e^+e^-$  annihilation. Here we shall compare the results following from this model with a representative set of data on charge distributions and fluctuations in multiparticle production in hadronic collisions. These data are particularly relevant, since a few years ago they provided a substantial motivation for the cluster models' of multiparticle production. In making an attempt at describing these data. by a quark-parton model we can learn whether the concept of a "cluster" is really required by the data, or whether the effects considered as being of a "cluster origin" follow also from another scheme. As will. be shown below, the basic features of the data on charge transfers and fluctuations do follow from a simple quarkparton model. The only exception is perhaps the data on the distribution of lengths of gaps with a fixed charge transfer.

In comparing the model with the data we are not trying to make fits. We prefer to keep the parameters of the model at values fixed earlier<sup>1-3</sup> and look only at a qualitative agreement or disagreement with the data. At the present level of understanding of the details of the dynamics of hadronic collisions, fits of the model to a particular set of data make little sense. More instructive are just the qualitative features of the model and of the data.

The paper is organized as follows. At the end of this section we shall briefly describe the Monte Carlo quark-parton model which we are using here (for details see I, II, and III). Then in Sec. II we shall compare the model with the data on  $dQ/dv$ (the distribution of charge in rapidity). Section  $\text{III}$ contains a discussion of charge fluctuations. The distribution of lengths of gaps is taken up in Sec. IV. Section V contains comments and concluding remarks to the present series of papers.

The model<sup>1-3</sup> has much in common with the earlier work on quark models of multiparticle production, in particular the work of Anisovich and cion, in particular the work of Amsovich and<br>Shekhter<sup>5</sup> and Bjorken and Farrar.<sup>6</sup> It is based on the assumption that hadrons in the final state result from recombination of quarks  $(Q's)$  and antiquarks  $(\overline{Q}'s)$  created (mostly) during the collision, and that valence quarks keep large momentum fractions (this means that the collision is initiated by the interaction of wee partons'). We do not try to describe the whole evolution of a hadronic collision, but instead we make a Monte Carlo model of the exclusive probability distribution of  $Q'$ s and  $\overline{Q}'$ s just prior to the recombination. In, for example, a  $\pi N$  collision, the probability of having three valence quarks (proton), valence Q and  $\overline{Q}$  (pion), and  $2n$  additional Q's and  $\overline{Q}$ 's is given as

$$
dP_N(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2}, \dots, y_N, \vec{p}_{TN}) \sim W_{id} G^{n\delta} \left( \sum_{1}^{N} \vec{p}_i \right) \delta \left( W - \sum_{1}^{N} E_i \right)
$$
  
 
$$
\times \exp \left( - \sum_{1}^{N} p_{T1}^{2} / R^2 \right) V(x_1, x_2, x_3) V(x_4, x_5) \prod_{1}^{N} dy_1 d^2 p_{T1}.
$$
 (1)

Here  $N = 2n + 5$  denotes the total number of Q's and  $\overline{Q}$ 's in the system before the recombination. The

factor  $W_{id}$  stands for the effects due to the identity of partons (this term denotes hereafter  $Q$ 's and

699

20

 $\overline{Q}$ 's), G regulates the average multiplicity of partons (and consequently also of final-state particles), factors for  $p<sub>r</sub>$  cutoff, for energy-momentum conservation, and for phase space are self-explanatory. The factors  $V(x_1, x_2, x_3)$  and  $V(x_4, x_5)$  give larger weights to configurations in which valence  $Q$ 's and  $\overline{Q}$ 's keep large momentum fractions x,. We have used a simple Kuti-Weisskopf $^{\text{8}}$  ansat  $V(x_1, x_2, x_3) = |x_1x_2x_3|^{1/2}$  and  $V(x_4, x_5) = |x_4x_5|^{1/2}$  for proton and pion valence partons. It is also assumed that valence partons keep their direction of motion in the c.m. system of the hadronic collision. In considering the proton-proton collision, we just replace the factor  $|x_4x_5|^{1/2}$  by  $|x_4x_5x_6|^{1/2}$ 

Equation (1) is probably the simplest ansatz which takes into account energy-momentum conservation,  $p_T$  cutoff, and the tendency of valence partons to keep large momentum fractions. In the next step the rapidity neighbors are recombined (see I) to hadrons. The unstable hadrons then decay and one obtains the particles observed in the final state. The model contains three free parameters  $G$ ,  $R$ , and  $\lambda$  (the suppression of the production of strange  $Q\overline{Q}$  pairs during the collision). These parameters were fixed at values following

from our previous studies of other aspects of multiparticle production.<sup>1-3</sup>

### II. DISTRIBUTION OF CHARGE IN RAPIDITY

The results of our model for the charge distribution  $dQ/dy$  in  $\pi$  p collisions at 360, 147, and 40 GeV/c are compared with the data in Figs.  $1(a)-1(c)$ . On a qualitative level the agreement is quite reasonable at 360 GeV/ $c$ , somewhat worse but still acceptable at 147 GeV/ $c$ , and there is a clear disagreement with the data at 40 GeV/ $c$ . The situation is shown also in Table I where we give the forward-moving charge  $Q_F = \int_{0}^{b} \frac{1}{2} dQ/dy dy$  in the center-of-mass  $(c.m.)$  system. In our model,  $Q_{\vec{r}}$  in  $\pi \vec{p}$  collisions is different from  $-1$  because of the short-range effects of recombination and resonance decays which lead to some charge transfer from the proton to the pion hemispheres. Apart from these effects, the charge in the forward (backward) hemisphere should be equal (in our model) to that of the pion (proton). The reason is simple. Quantum numbers of the initial hadrons are carried by their valence quarks. In our model it is assumed that the valence quarks take no part



FIG. 1. (a) The comparison of data (Ref. 9) (solid line) on  $dQ/dy$  in  $\pi^*p$  collisions at 360 GeV/c with the results following from the present model (dotted line). Protons with  $p_{1ab}$  below 1.4 GeV/c were excluded from both the data and model calculations. (b) Calculated  $dQ/dy$  in  $\pi p$  collisions at 147 GeV/c (dotted line) compared with the data (Ref. 10) (solid line). (c)  $dQ/dy$  in  $\pi p$  collisions at 40 GeV/c. Data (Ref. 11) are represented by the solid line, the calculations by the dotted one. (d) Comparison of data (Ref. 11) on  $dQ/dy$  in  $\pi p$  at 40 GeV/c, with a model calculation (dotted line) permitting the occurrence of valence quarks in the other hemisphere.

system for  $\pi$ <sup>-</sup>p collisions.  $p_{1ab}$  (GeV/c)  $Q_F$ , calcul  $Q_F$ , exp 40 147 360  $-0.75$  $-0.85$  $-0.86$  $-0.5$ —0.72  $-0.80$ 

TABLE I. The forward-moving charge  $Q_F$  in the c.m.

in the first part of the collision (the interaction of wee partons), and that they roughly conserve their momentum and, as a consequence, they move in the same direction after the collision as before it (in the c.m system). Because of that, hadrons produced by recombinations of valence quarks most likely remain in the corresponding hemisphere. Since the central region is neutral, it does mt contribute to the net charge transfer between the hemispheres. Resonance decays spoil this picture a little; however, the expected  $Q_F$  is not far from the charge of  $\pi$ <sup>-</sup> (see the column  $Q_{\bm{F}, \text{cal}}$  in Table I). On the experimental side some complications are caused by a possible particle misidentification. If the proton in the final state is taken as a pion, it is sometimes assigned to the wrong hemisphere in the c.m. system. 'This effect may be rather large; in our simulations of this misidentification we found that this can increase the value of  $Q_{F,\exp}$  by about -0.1 (e.g., from  $Q_{F,\exp} = -0.5$  to  $Q_{F,\exp} = -0.6$ ). Because of that we do not consider the differences between  $Q_{F, \text{cal}}$ and  $Q_{F, \text{exp}}$  at 147 and 360 GeV/c as really significant. The situation is different at 40 GeV/ $c$  where the discrepancy cannot be explained that way. Here, however, a much simpler and more natura<br>explanation is possible.<sup>11</sup> At this energy, valence explanation is possible. $^{11}$  At this energy, valence quarks can rather frequently participate in the first part of the collision, and in such cases the charge of the valence quarks can be partly transferred between the hemispheres. In order to get a feeling of how much these effects can influence the distribution of charge in rapidity, we have made a simple modification of the model. In the standard version discussed above, valence quarks have to remain in their own c.m. hemispheres and the weight of the configuration includes factors  $|x_i|^{1/2}$  for each valence quark. In the modification we have permitted the occurrence of the valence quarks also in "the other" hemisphere, and instead of  $x^{1/2}\theta(x)$  we have used  $(m_{\tau}e^{y})^{1/2}$  as the factor in the weight of the configuration. Both expressions are rather similar for positive values of x and differ only for  $x<0$ , where one of them is very small and the other vanishes. The results obtained for  $dQ/dy$  in  $\pi p$  collisions at 40 GeV/c are shown in Fig. 1(d). It is seen that the agreement with the data is considerably improved. The

modification was, however, very crude and ad hoc and the problem of  $dQ/dv$  at energies below 100 GeV/ $c$  requires a more detailed study.

#### III. CHARGE FLUCTUATIONS

The charge distribution  $dQ/dy$  in  $\pi p$  and  $p p$  collisions discussed in the preceding section is essentially given by the distribution of charges of the valence quarks modified by the short-range effects of recombination of partons to hadrons and of resonance decays. Consequently,  $dQ/dy$  is not very sensitive to the behavior of partons in the central region. However, the dynamics of partons in this region is rather interesting since this region contains not only the sea  $Q\overline{Q}$  pairs of the initial hadrons but also the  $Q\overline{Q}$  pairs created durin<br>the collision.<sup>1,12</sup> As pointed out in Ref. 13, charge the collision. $^{1,12}$  As pointed out in Ref.  $13\mathrm{,}$  charg fluctuations may be sensitive to the details of the dynamics of the central region. As can be seen from Eq. (1), the nonvalence  $Q'$ s and  $\overline{Q}'$ 's are distributed in the present model statistically within the cylindrical phase-space region left by the valence partons (by constraints following from energy-momentum conservation). This assumption is probably oversimplified. We have used it here partly because it was used in I, II, and III and partly because we wish to learn whether the data really require some form of local compensation of quantum numbers of sea partons. '4

In Fig. 2 we compare our results for the charge fluctuation  $\langle (\Delta Q)^2 \rangle$  across  $y = 0$  with the data<sup>15</sup> in the Fermilab energy range. Here  $\Delta Q$  is defined as  $\Delta Q = (Q_R)_{\text{final}} - (Q_R)_{\text{initial}}$ , and  $(Q_R)_{\text{final}}$  is the charge in the right-hand hemisphere in the final state and  $(Q_R)$ <sub>initial</sub> is the charge of the rightmoving hadron in the initial state. Figure 3 shows the probability distribution of charge transfer  $P(\Delta Q)$  for pp collisions<sup>16</sup> at 102, 205, and 405 GeV/c. Both Fig. 2 and Fig. 3 indicate that in our model



FIG. 2. The energy dependence of the charge fluctuation across  $y = 0$  in  $pp$  collisions data (Ref. 15) is denote by full circles, calculations by crosses.



FIG. 3. Energy dependence of the probabilities  $P(\Delta Q)$ of the charge transfer  $\Delta Q$  across  $y = 0$  in pp collisions. Data (Ref. 16) are represented by the histogram, our results by full circles.

we somewhat overestimate the large fluctuations. This could be remedied by introducing local compensation of the charge of nonvalence partons. Such calculations would require some additional parameters and the available data could hardly fix them in an unambiguous way. The point is simple; in models such as the present one, the number of central partons increases like ln(s). If these partons can fluctuate in the central region,  $\langle (\Delta \mathcal{Q})^2 \rangle$ will also increase as  $ln(s)$  (for a qualitative discussion see Ref. 13). In models with a local charge compensation of nonvalence partons, the energy dependence of  $\langle (\Delta Q)^2 \rangle$  should eventually level off. The data in the Fermilab energy range are (see Fig. 2), however, still consistent with a logarithmic rise of  $\langle (\Delta Q)^2 \rangle$ . More detailed and quantitative insight into the problem can thus come only from data at higher energies.

In Fig. 4 we compare our results on the charge fluctuations across  $y = 0$  at fixed topologies with the data<sup>17</sup> for pp collisions at 205 GeV/c. The agreement is quite good. The fluctuation of charge across a given value of c.m. rapidity is defined



FIG. 4. The charge fluctuation  $\langle (\Delta Q)^2 \rangle$  across y = 0 in  $pp$  collisions at 205 GeV/c as a function of the number of negative particles in the final state. Data {Hef. 17) denoted by full circles, our results by crosses.



FIG. 5. The charge fluctuation  $D(y)$  across a given c.m. rapidity <sup>y</sup> compared with the density of charged particles  $dn/dy$  in the final state. Data (Ref. 17) on  $D(y)$  are represented by full circles, our results for  $D(y)$  by open squares, and the histogram gives our results on  $dn/dy$  multiplied by 0.6.

for  $bb$  collisions as

$$
D(y) = \langle (\Delta Q(y))^2 \rangle - \langle \Delta Q(y) \rangle^2,
$$

where

$$
\Delta Q(y) = \frac{1}{2} \left[ \sum Q_i \theta(y - y_i) - \sum Q_i \theta(y_i - y) \right].
$$

 $Q_i$  is the charge of the hadron at  $y_i$ , and  $\theta(x) = 1$ for  $x \ge 0$ ,  $\theta(x) = 0$  for  $x < 0$ . The data<sup>17,18</sup> show that  $D(y)$  is roughly proportional to the average density of charged particles  $dn/dv$  in the final state. This result has been successfully explained<sup>19</sup> by the cluster model and interpreted as a strong support of that model. In Fig. 5 we compare the results of our calculations with the data<sup>17</sup> on  $pb$  collisions at 205 GeV/ $c$ . It is seen that the quark-parton model reproduces well the data on  $D(y)$  and that



FIG. 6. The distribution of rapidity gaps between charged secondaries (upper part) and between negative secondaries (lower part). Our results are given by histograms, the data (Ref. 20) by dots. Since arbitrary units are used in Ref. 20 we have made our results coincide with the data at small rapidity gaps.

the model calculations satisfy the approximate relationship

$$
D(y) \approx 0.6 (dn/dy).
$$

The average density of charged particles  $dn/dy$ was also calculated within the model. As was shown in II, the calculations of quantities such as  $dn/dy$  are in a reasonable agreement with the data. This shows that the proportionality between  $dn/dy$  and  $D(y)$  is not a specific feature of the cluster model but can be obtained quite naturally also in the quark-parton model.

#### IV. GAP-LENGTH DISTRIBUTIONS

In Ref. 20 Pirilä, Quigg, and Thomas argued that the probability distribution  $P(r)$  of gap widths  $r = \Delta y$  between charged particles in the final state gives direct evidence for the existence of clusters. Let  $y_1 < y_2 < \cdots < y_n$  and  $Q(y_i)$ ,  $i = 1, 2, \dots, n$  be the rapidities and charges of charged particles in a particular event. The length of the  $i$ th rapidity gap is  $r_i = y_{i+1} - y_i$ . The symbols  $P_{ch}(r)$  and  $P_{neg}(r)$ denote the distributions of gap lengths between charged and negative particles respectively. The ith gap is said to carry the charge

$$
\Delta Q = \frac{1}{2} \left[ \sum_{k=i+1}^{n} Q(y_k) - \sum_{k=1}^{i} Q(y_k) \right].
$$

 $P_{\Delta\Omega}(r)$  denotes the distribution of lengths of gaps carrying a specified charge  $\Delta Q$ . In Figs. 6 and 7 we compare our results with the data<sup>20</sup> on  $P_{\infty}$ 7 we compare our results with the data<sup>20</sup> on  $P_{\text{ch}}(r)$  $P_{\text{neg}}(r)$ , and  $P_{\Delta Q}(r)$  obtained in pp collisions at  $205 \text{ GeV}/c$ . In general, our results have a smaller slope than the data. Quigg  $et$   $al.^{20}$  have shown that the independent-cluster model predicts

 $P(r) = \rho e^{-\rho r}$ 

for larger values of r, say  $r>1$ , where  $\rho$  is the rapidity density of clusters. The data seem to agree with this relationship with  $\rho \approx 1$  (this means that the cluster decays in the average to about three particles, one of them being neutral).

Our results have a somewhat smaller slope of gap distributions. 'This is a bit surprising since in our model the role of clusters is played by resonances and by stable (directly produced) hadrons, and one could expect that our "density of clusters" should be a bit larger than that of the authors of Ref. 20. Unfortunately, a model of independent clusters is a rather crude approximation and it is impossible to use it in order to learn what went wrong. In fact, the quantities  $P_{ch}(r)$ ,  $P_{neg}(r)$ , and  $P_{\Lambda 0}(r)$  are given by a rather complicated interplay of three factors: the distribution of  $Q$ 's and  $\overline{Q}$ 's before the recombination, the prescription' for the recombination, and the resonance decays. It

is therefore rather difficult to say which of these is responsible for the observed disagreement with the data. For future possible applications of quark-parton models to multiparticle production, it is useful to keep in mind that the distribution of rapidity gap lengths may be a sensitive quantity for discriminating between the models.



FIG. 7. The distribution  $P_{\Delta Q}(r)$  of lengths (r) of rapidity gaps carrying the charge  $\Delta Q$  in pp collisions at 205 GeV/ $c$ . Our results are given by histograms, the data (Ref. 20) by dots. The same normalization convention as in Fig. 6 is used: (a)  $\Delta Q=0$ , (b)  $|\Delta Q|=1$ , (c)  $|\Delta Q|$  = 2.

## V. COMMENTS AND CONCLUSIONS

The model used here and in the preceding papers of this series is probably the simplest way to construct a quark-parton model of multiparticle production with

(i) final-state hadrons being originated by the recombination of quarks and antiquarks,

(ii) the  $p<sub>r</sub>$  cutoff,

(iii) energy-momentum conservation, and

(iv) valence quarks keeping rather large momentum fractions during the collision.

At the price of implementing these items into a Monte Carlo program, the results following from the model can be compared, in principle, with any characteristics of the experimental data. In I, II, III, and here we have shown that the model gives a reasonable qualitative description of the following data: particle multiplicities and their energy dependence, rapidity distribution of final- state dependence, rapidity distribution of final-sta<br>particles,  $2^1$   $p_T$  spectra of stable particles and particles,  $\frac{p}{r}$  spectra of stable particles and particles and fluctuations in the Fermilab energy range. The same model with some simple additional assumptions $23$ has given<sup>3</sup> a qualitative description of the quark fragmentation functions and the multiparticle production in  $e^+e^-$  annihilation (the latter, of course, only for the part not caused by the production of charmed quarks and heavy leptons in the first stage of the annihilation). In an approach closely related to that of <sup>I</sup>—III, we have also studied the production of low-mass dimuons in hadron-hadron production of low-mass dimuons in hadron-hadro<br>and lepton-hadron collisions.<sup>24</sup> The idea<sup>12,24</sup> was rather simple. We assumed that the  $Q$ 's and  $\overline{Q}$ 's created during the collision can, during their short time of existence, either recombine to hadrons or annihilate to dilepton pairs. The constraints following from the space-time evolution $14$ of the collision allow only annihilations of  $Q$ 's

and  $\overline{Q}$ 's separated by small rapidity gaps (only such  $Q$ 's and  $\overline{Q}$ 's are excited simultaneously). As a result thereof, the rapidity and  $p<sub>r</sub>$  spectra of low-mass dimuons are rather similar to the spectra of directly produced hadrons. The comparison with the data<sup>25</sup> on low-mass dimuon production was, on a qualitative level, quite favorable. As pointed out already in I, the model has several deficiencies. It works with probabilities and not with amplitudes. As a consequence, it cannot describe the diffractive dissociation and the Bose-Einstein effects. Furthermore, when we started to work on the model, there was much less experimental information about the resonance production than available today. Still, we did not want to modify the model continuously, and we have rather kept the same values of resonance-stablehadrons recombination probabilities throughout.

The conclusions following from the results of the model are simple and probably not very encouraging. It seems to us that the multiparticle production in hadron-hadron and lepton-hadron collisions is to a large extent regulated just by the four constraints listed at the beginning of this section. This follows from the fact that our simple model gave a qualitatively reasonable description of the data we have studied.<sup>26,27</sup> One apparently has to look very carefully at the data to be able to select those which are really sensitive to the details of the multiparticle dynamics. We think that further studies of this type need also deeper theoretical input. This concerns in particular the introduction of color degrees of freedom,<sup>28</sup> a mor  $introduction$  of  $color$  degrees of freedom, $^{28}$  a more deeply motivated description of the distribution of  $Q$ 's and  $\overline{Q}$ 's in the intermediate stages of the collision, and a deeper understanding of the dynamics of the recombination, or more generally, of the process responsible for the formation of the finalstate hadrons in multiparticle production.

- $\,$  <sup>1</sup>V. Cerný, P. Lichard, and J. Pisút, Phys. Rev. D 16, 2822 (1977).
- $v^2$ V. Černý, P. Lichard, and J. Pisút, Phys. Rev. D 18, 2409 (1978).
- $3V.$  Černý, et al., Phys. Rev. D<sub>18</sub>, 4052 (1978).
- <sup>4</sup>T. T. Chou and C. N. Yang, Phys. Rev. D 7, 1425 (1973); C. Quigg and G. H. Thomas,  $ibid. 7$ , 2752 (1973); C. Quigg, *ibid.* 12, 834 (1975); A. Bial/as et al., Acta. Phys. Pol. 86, 59 (1975) and references cited therein.
- 5V. V. Anisovich and V. M. Shekhter, Nucl. Phys. 855, 455 (1973); <u>B55</u>, 474 (1973); V. N. Guman and V. M. Shekhter,  $ibid.$   $B99, 523$  (1975).
- $6J.$  D. Bjorken and G. R. Farrar, Phys. Rev. D 9, 1449 (1974).
- ${}^{7}R$ , P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972).
- ${}^{8}$ J. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1970).
- $\rm ^9V.$  Barnes, in Proceedings of the Symposium on Multiparticle Dynamics, Kaysersberg, 1977, edited by P. Schubelin (CBN, Strasbourg, 1977) (results on  $dQ/dy$  in Barnes's contribution are based on a private communication from W. D. Shepard).
- $^{10}$ D. Fong et al., Phys. Lett. 61B, 99 (1976).
- $^{11}E$ . N. Kladnitskaya, V. M. Shekhter, and L. M. Shcheglova, Yad. Fiz. 26, <sup>337</sup> (1977) [Sov. J. Nucl. Phys. 26, 176 (1977)].
- <sup>12</sup>J. D. Bjorken and H. Weisberg, Phys. Rev. D 13, 1405 (1976).
- 13V. Černý, and J. Pisut, Acta. Phys. Pol. B8, 469  $(1977)$ .
- <sup>14</sup>Such local compensations are expected in models incorporating the constraints imposed by the picture of

the space-time evolution of hadronic collisions proposed by Bjorken and'Gribov. [J. D. Bjorken, in Proceedings of the Summer Institute on Particle Physics, edited by M. Zipf (SLAC, Stanford, 1973); V. N. Gribov, in Elementary Particles, First ITEP School on Theoretical Physics (Atomizdat, Moscow, 1973), Vol. I, p. 65].

- $^{15}$ C. Bromberg et al., Phys. Rev. D  $12$ , 1224 (1975).
- $^{16}$ C. Bromberg *et al.*, Phys. Rev. D  $\overline{9}$ , 1864 (1974); T. Ferbel, University of Rochester Report No. UB-500 (unpublished). For compilation see G. Levman et al., Phys. Rev. D 14, 711 (1976); E. N. Argyres and C. S. Lam,  $ibid. 16, 114$  (1977).
- $1^{7}$ T. Kafka et  $\overline{al}$ ., Phys. Rev. Lett. 34, 687 (1976).
- <sup>18</sup>P. Lauscher et al., Nucl. Phys.  $\overline{B106}$ , 31 (1976);
- U. Idschok et al., ibid. B67, 73 (1973); P. Bosetti et  $al., ibid. B62, 46 (1973).$
- $^{19}$ A. W. Chao and C. Quigg, Phys. Rev. D 9, 2016 (1974); C. Quigg and G. H. Thomas, Ref. 4; A. Bialas et al., Ref. 4; A. Bialas, in Proceedings of the IVth International Symposium on Multiparticle Hadrodynamics, Pavia, 1973, edited by F. Duimio, A. Giovannini, and S. Batti (INFN, Pavia, 1973), p. 93; R. Baier and F. Bopp, Nucl. Phys. 879, 344 (1974).
- <sup>20</sup>C. Quigg, P. Pirila, and G. H. Thomas, Phys. Rev. Lett. 34, 290 (1975); P. Pirilä, G. H. Thomas, and C. Quigg, Phys. Bev. D 12, 92 (1975).
- $^{21}$ Owing to the limited statistics we were unable to make a meaningful comparison with single-particle spectra at large values of Feynman  $x$ . This part of single-particle spectra has been reasonably described by other quark-parton models with recombination. See, e.g., W. Ochs, Nucl. Phys. B118, 397 (1977); K. P. Das and B. Hwa, Phys. Lett. 688, 459 (1977): D. W. Duke and F. E. Taylor, Phys. Bev. <sup>D</sup> 17, 1788 (1978); L. Vari Hove, CERN Report No. Bef. TH.2580, 1978 (unpub-

lished) and references cited therein.

- <sup>22</sup>The  $p_T$  spectra of Q's and  $\overline{Q}$ 's before the recombination are described (an input to the calculation) as  $\exp(-b p^2)$ . The resonances formed directly by the recombination of Q's and  $\overline{Q}$ 's have the same shape of the spectrum. The pion spectrum results partly from resonance decays and partly from pions formed directly. As shown in II, the calculated  $\stackrel{\_}{p}_T$  spectrum of pions agrees with the data.
- $^{23}$ It was assumed that the Q and  $\overline{Q}$  produced in the first stage of the  $e^+e^-$  collision behave in a way similar to the valence quarks in a hadronic collision.
- $^{24}V$ . Černý, P. Lichard, and J. Pisut, Phys. Lett. 70B, 61 (1977); Acta. Phys. Pol. 89, 269 (1978); ibid. 810, (1978); V. Cerny, Phys. Rev. <sup>D</sup> 18, <sup>4345</sup> (1978); J. Pis6t, Acta. Phys. Pol. 89, <sup>275</sup> (1978).
- $^{25}$ K. J. Anderson et al., Phys. Rev. Lett. 37, 799 (1976); 37, 803 (1976).
- ${}^{26}$ There are also exceptions such as the production of  $\Lambda$ discussed in I and II. Our model overestimates it by a factor of about 3. As pointed out by L. M. Shcheglova and V. M. Shekhter, Leningrad Nuclear Physics Institute report, 1978 (unpublished), this may be caused by the fact that the production of resonances from higher SU(6) multiplets was neglected. Another disagreement with the data was found in gap-length distributions (see Sec. IV here).
- $2^{7}$ It is amusing to note that we have started with this model with the intention of understanding the rapidity correlations of secondaries in the multiparticle production. It turned out, however, that one would need to study these correlations at fixed topologies and that was too computer-time consuming.
- $^{28}$ One of the authors (J. P.) is indebted to J. Nyiri (JINR, Dubna) and V. M. Shekhter (LNPI, Leningrad) for an instructive discussion on that point.