

Weak exchange degeneracy in the reactions
 $\pi^+ p \rightarrow K^+ \Sigma^+$ and $K^- p \rightarrow \pi^- \Sigma^+$ at 4, 7, 11.5, and 70 GeV/c

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(Received 26 February 1979)

The existing data on the line-reversed reactions $\pi^+ p \rightarrow K^+ \Sigma^+$ and $K^- p \rightarrow \pi^- \Sigma^+$ near 4, 7, and 11.5 GeV/c are discussed from the point of view of weak exchange degeneracy (WEXD). It is noted that a smooth and simple generalization of the model of Navelet and Stevens is able to describe adequately, in the Regge region, the data at 7 and 11.5 GeV/c. (Recall that the Navelet and Stevens model appears to fit the pronounced violation of WEXD for leading K^* and K^{**} exchanges in the data near 4 GeV/c whereas the data at 7 and 11.5 GeV/c are in agreement with this particular WEXD.) A prediction for 70 GeV/c (namely, essentially that WEXD for leading K^* and K^{**} exchanges should hold true) is given. Further, an effort is made to indicate how this particular generalization of the model of Navelet and Stevens may arise in a dual multiperipheral bootstrap model of the general Lee-Veneziano-Chan-Paton variety as formulated by Balázs.

I. INTRODUCTION

Sometime ago, it was pointed out by Gilman¹ that, if K^* and K^{**} Regge exchanges were indeed weak exchange degenerate, then the line-reversed reactions

$$\pi^+ p \rightarrow K^+ \Sigma^+, \quad (1)$$

$$K^- p \rightarrow \pi^- \Sigma^+, \quad (2)$$

when dominated by these two exchanges, should have equal differential cross sections $d\sigma/dt$ and mirror-symmetric polarizations, where t is the momentum transfer squared. (The K^* and K^{**} exchanges are said to be weak exchange degenerate if their trajectories are equal.) According to the lore, the kinematic region in which the two exchanges should dominate would be the region of high s and relatively low $|t|$, where s is the squared center-of-momentum energy. Thus, it has come to pass that a number of experiments²⁻⁵ have been done which probe the region

$$s \geq 6 \text{ (GeV/c)}^2, \quad |t| \leq 1 \text{ (GeV/c)}^2.$$

We should mention that the ideas of weak exchange degeneracy also make similar predictions about other line-reversed pairs of reactions and that we, by focusing on $\pi^+ p \rightarrow K^+ \Sigma^+$ and $K^- p \rightarrow \pi^- \Sigma^+$, in no way mean to imply that the other reactions are less significant. Rather, we take the view that all of the predictions of the (weak) exchange degeneracy idea are important and that, in order to have a complete picture of the data in relation to the idea, one should evidently consider all of the relevant predictions in relation to observation. For a more complete picture of the various predictions, we refer the reader to Ref. 1. Our work must be viewed in this broader context.

The single most interesting aspect of the data on

the two line-reversed reactions (1) and (2) is that the lower-energy data, near a laboratory momentum of 4 GeV/c, exhibit marked violations of weak exchange degeneracy (WEXD) for K^* and K^{**} leading trajectories whereas the higher-energy data of Baker *et al.*⁴ at 7- and 11.5-GeV/c laboratory momenta are in general agreement with WEXD.⁶ The agreement appears to be better at the higher laboratory momentum. Further, we should also mention that the data of Berglund *et al.*,³ taken at 7.0- and 10.1-GeV/c laboratory momenta, are in general agreement with the data of Baker *et al.*,⁴ and, hence, are also in general agreement with the expectations of WEXD. Thus, for the sake of simplicity we shall work with the data of Baker *et al.*,⁴ with the understanding that, at the level of our discussions, everything we say about the data of Ref. 4 will apply in general also to the data of Ref. 3. A more complete discussion of both sets of data may be appropriate at a later time. In other words, here we simply take the data of Baker *et al.*⁴ as representative of the level of agreement of WEXD with observation at the respective type of laboratory momenta, i.e., 7.0 and 11.5 GeV/c.

Now, Navelet and Stevens,⁷ for example, have shown that the data at laboratory momenta ~ 4 GeV/c, although in disagreement with the expectations of WEXD, nonetheless can be fitted with a flip amplitude, H_F , which is essentially weak exchange degenerate and a nonflip amplitude, H_{NF} , which is the sum of an amplitude (a) that is essentially what one expects from weak-exchange-degenerate K^* and K^{**} exchange and an amplitude (b) which represents the effect of Regge cuts. The latter amplitude (b) is taken as a sum of effective Regge poles, giving two distinct trajectories for each signature. Now, if we ignore the issue of summing the diagrams responsible for generating

the cut to all orders, it is to be expected that a cut can be represented as a pole only in a limited kinematic region. For, there are logarithms which accompany such a cut. Thus, it is not surprising that the fit of Navelet and Stevens may fail at some higher energy.

Of course, it can happen that the logarithms which are well known to characterize the true effect of a Regge cut at the one-loop level, when all loops are summed, become a Regge pole with, perhaps, a trajectory which is quite different from the one-loop effective cut trajectory. Depending on the manner in which the two differ, a given phenomenologist may arrive at an effective representation of the data in one kinematic region which does not appear to bear a simple dynamical relationship to the representation of the data in another kinematic region, although all data are in the Regge region. It is from this particular point of view that we shall discuss, in this paper, the data on reactions (1) and (2) in the range $6 \lesssim s \lesssim 23$ (GeV/c)².

It will happen that we shall be able, strictly from the phenomenological point of view, to predict $d\sigma/dt$ and the polarization P for reactions (1) and (2) at even higher energy, in particular, at a laboratory momentum of 70 GeV/c where an upcoming result from Fermilab^{5,8} will be a direct check. And, indeed, a comparison of our generalized version of the Navelet and Stevens model with the preliminary results for reaction (1) alone at 70 GeV/c from the Fermilab experiment will be presented already in this paper.

What we shall do is the following. First, we shall, by looking at the data at 4-, 7-, and 11.5-GeV/c laboratory momenta arrive at a simple phenomenological description. This description will be a simple generalization of the work of Navelet and Stevens. Then, we shall attempt to illustrate how such a generalization might arise theoretically by examining a simple toy model, the dual multiperipheral bootstrap advocated by Lee,⁹ Veneziano,¹⁰ and Chan and Paton,¹¹ as formulated by Balázs,¹² to be specific. We would like to emphasize that we take this type of model no more seriously than do its authors. It will be necessary to extend the model, in a certain way,

beyond the region investigated by Balázs. This extension is such that we feel the conclusions which we arrive at may have something to do with nature. For, as one can see from the work of Refs. 9–12, the dual multiperipheral model does have some experimental support.

Our work proceeds as follows. In Sec. II, we give our phenomenological generalization of the work of Navelet and Stevens and discuss our predictions for a laboratory momentum of 70 GeV/c. In Sec. III, we analyze the dual multiperipheral bootstrap to see if our phenomenological results can be a reasonable prediction of such a theory. Finally, Sec. IV contains some concluding remarks.

II. THE PHENOMENOLOGICAL MODEL AND THE DATA

Two experimentally accessible quantities for reactions (1) and (2) which are immediately relevant to the weak exchange degeneracy idea are $d\sigma/dt$ and the polarization, P , of the Σ^+ . Specifically, one may write, following Ref. 7, for each reaction

$$\frac{d\sigma}{dt} = |H_{\text{NF}}|^2 + |H_{\text{F}}|^2, \quad (3)$$

$$P \frac{d\sigma}{dt} = -2 \text{Im}(H_{\text{NF}} H_{\text{F}}^*), \quad (4)$$

where H_{NF} is the s -channel nonflip amplitude, H_{F} is the s -channel flip amplitude, * denotes complex conjugation, and Im denotes the imaginary part. As we indicated in Sec. I, t is the four-momentum transfer squared and s is the squared center-of-momentum energy. Our metric is that of Bjorken and Drell.¹³ Thus, in order to distinguish the two amplitudes in each reaction, for $\pi^+ p \rightarrow K^+ \Sigma^+$, reaction (1), we shall use the obvious superscripts on H_{NF} and H_{F} : $H_{\text{NF}}^{(1)}$ and $H_{\text{F}}^{(1)}$. Similarly, for $K^- p \rightarrow \pi^- \Sigma^+$, reaction (2), we shall use the obvious notation $H_{\text{NF}}^{(2)}$ and $H_{\text{F}}^{(2)}$ for the nonflip and flip amplitudes, respectively.

As one can see from Ref. 7, Navelet and Stevens are able to achieve a reasonable description of the data near 4-GeV/c laboratory momentum for $d\sigma/dt$ and P by taking $H_{\text{NF}}^{(l)}$ and $H_{\text{F}}^{(l)}$ in Regge form as follows ($l=1, 2$):

$$\begin{aligned} H_{\text{NF}}^{(l)} = & \kappa \{ (-1)^{l+1} \gamma_{K^*, \text{NF}} [-1 + \exp(-i\pi \alpha_{K^*}(t))] s^{\alpha_{K^*}(t)} \exp(a_{K^*, \text{NF}} t) \\ & + \gamma_{K^{**}, \text{NF}} [1 + \exp(-i\pi \alpha_{K^{**}}(t))] s^{\alpha_{K^{**}}(t)} \exp(a_{K^{**}, \text{NF}} t) \\ & + (-1)^{l+1} \gamma_{K^*, \text{NF}} \gamma_{K^*}^c i \exp(-i\pi \alpha_{K^*}^c(t)/2) s^{\alpha_{K^*}^c(t)} \exp(a_{K^*}^c t) \\ & - \gamma_{K^{**}, \text{NF}} \gamma_{K^{**}}^c \exp(-i\pi \alpha_{K^{**}}^c(t)/2) s^{\alpha_{K^{**}}^c(t)} \exp(a_{K^{**}}^c t) \}, \end{aligned} \quad (5)$$

$$\begin{aligned} H_{\text{F}}^{(l)} = & \kappa \sqrt{-t}^l \{ (-1)^{l+1} \gamma_{K^*, \text{F}} [-1 + \exp(-i\pi \alpha_{K^*}(t))] s^{\alpha_{K^*}(t)} \exp(a_{K^*, \text{F}} t) \\ & + \gamma_{K^{**}, \text{F}} [1 + \exp(-i\pi \alpha_{K^{**}}(t))] s^{\alpha_{K^{**}}(t)} \exp(a_{K^{**}, \text{F}} t) \}. \end{aligned} \quad (6)$$

Here,

$$\begin{aligned}
 \alpha_{K^*}(t) &= 0.375 + 0.678t, & \alpha_{K^{**}}(t) &= 0.322 + 0.678t, \\
 \gamma_{K^*, \text{NF}} &= 10.34, & \gamma_{K^{**}, \text{NF}} &= 11.15, \\
 \gamma_{K^*, \text{F}} &= -9.862, & \gamma_{K^{**}, \text{F}} &= -7.114, \\
 a_{K^*, \text{NF}} &= 2.414, & a_{K^{**}, \text{NF}} &= 1.331, \\
 a_{K^*, \text{F}} &= 1.895, & a_{K^{**}, \text{F}} &= 1.895, \\
 \alpha_{K^*}^c(t) &= 0.099 + 0.532t, & \alpha_{K^{**}}^c(t) &= 0.127 + 0.532t, \\
 \gamma_{K^*}^c &= 1.01, & \gamma_{K^{**}}^c &= 1.44, \\
 a_{K^*}^c &= 1.134, & \text{and } a_{K^{**}}^c &= 0.091.
 \end{aligned} \tag{7}$$

Also,

$$t' = t - t_{\min} \tag{8}$$

and

$$\kappa = \frac{0.38935 M_p^2}{16\pi p_i^2 s} \text{ mb}/(\text{GeV}/c)^2, \tag{9}$$

where p_i is the incident momentum in the center-of-momentum frame and M_p is the proton rest mass. Thus, all invariants and momenta are to be expressed in GeV/c units. At this point, let us emphasize that we take the fit described by (5), (6), and (7) no more seriously than do its authors. For our purposes, it is a convenient representation of the lower-energy data which is not unreasonable from the point of view of the standard Regge lore.

Indeed, the form of the trajectory functions $\alpha_{K^*}(t)$ and $\alpha_{K^{**}}(t)$ in (5) is reasonably consistent with the expectations of weak exchange degeneracy for the K^* and K^{**} vector and tensor exchanges. Thus, interpreting the odd- and even-signatured terms as due to K^* and K^{**} exchange, respectively, as our notation indicates, we see that the flip amplitudes are essentially weak exchange degenerate. However, the nonflip amplitudes, containing, as they do, the contributions of the effective poles $\alpha_{K^*}^c$ and $\alpha_{K^{**}}^c$ in addition to approximately weak-exchange-degenerate K^* and K^{**} exchange contributions, exhibit pronounced violations from what one expects from the weak-exchange-degenerate view of K^* and K^{**} leading exchanges. We remind the reader that, by weak exchange degeneracy, we should have $\alpha_{K^*}(t) = \alpha_{K^{**}}(t)$.

If one compares the prediction of (5) and (6) for the polarization P , for example, with the data of Baker *et al.*⁴ at 11.5 GeV/c, one sees that the parametrization of Navelet and Stevens appears to fail at such laboratory momenta (see Fig. 1). Thus, we are invited by this observation to modify the model described by (5) and (6).

We should like to do this entirely phenomenologically, at first. For, in this way, we shall probably create the most interesting theoretical chal-

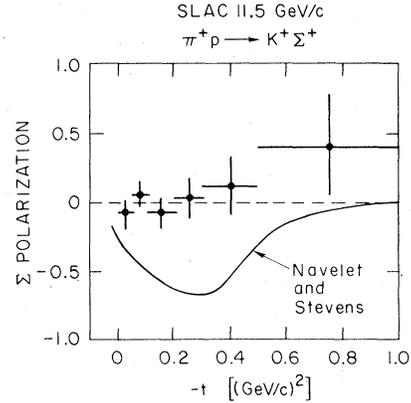


FIG. 1. Comparison of the Navelet and Stevens model with the polarization data of Ref. 4 for $\pi^+ p \rightarrow K^+ \Sigma^+$. This figure is taken from Ref. 8.

lenge. Specifically, the main characteristic of the data of Baker *et al.*,⁴ from our point of view, is the agreement with the expectations from weak exchange degeneracy for the K^* and K^{**} exchanges, assuming they are the leading Regge exchanges in reactions (1) and (2). Recall this agreement appears to be better at $p_{\text{lab}} = 11.5$ GeV/c than at $p_{\text{lab}} = 7$ GeV/c. Here, p_{lab} is the laboratory incoming momentum. Hence, in the simplest situation, which is the one we shall discuss, we need nonflip amplitudes $H_{\text{NF}}^{(j)}$ which extrapolate smoothly from Eq. (5) at $p_{\text{lab}} \cong 4$ GeV/c to essentially weak-exchange-degenerate K^* and K^{**} exchange at $p_{\text{lab}} = 7$ GeV/c. The flip amplitudes $H_{\text{F}}^{(i)}$ in Eqs. (6) are already essentially weak exchange degenerate. So, we shall leave them as they are given in (6). Further, we wish to stay close to the Regge form (5). Thus, we replace, for simplicity, the effective poles $\alpha_{K^*}^c(t)$, $\alpha_{K^{**}}^c(t)$ with $\alpha_{K^*}^c(t) - n$, $\alpha_{K^{**}}^c(t) - n$, respectively, for some integer n . If we require that the phase of the effective pole amplitudes be unchanged at $p_{\text{lab}} = 4$ GeV/c, we shall assure that the asymmetric polarization predictions of (5) and (6) are unchanged at this momentum, provided we correct $\gamma_{K^*}^c$ and $\gamma_{K^{**}}^c$ in (7) by an appropriate power of

$$s_l = m_l^2 + M_p^2 + 2M_p [(4 \text{ GeV}/c)^2 + m_l^2]^{1/2}$$

in $H_{\text{NF}}^{(l)}$, $l = 1, 2$, where m_1 is the π^+ rest mass and m_2 is the K^+ rest mass; clearly, the correct power is s_l^n in $H_{\text{NF}}^{(l)}$. Thus, we require

$$-i\pi(-n)/2 = 2m\pi i \tag{10}$$

or

$$n = 4m, \tag{11}$$

for some integer m . Notice that, since $\gamma^c \sim 1$, in order for the effective poles to be at most a few percent of the remaining approximately weak-

exchange-degenerate parts of $H_{\text{NF}}^{(i)}$ at $p_{\text{lab}} = 7 \text{ GeV}/c$, we must have

$$(s_i/s)^n \leq \text{a few percent} \quad (12)$$

at $p_{\text{lab}} = 7 \text{ GeV}/c$. In other words, we need

$$(4/7)^n \leq \text{a few percent}. \quad (13)$$

For the sake of discussion we take $m = 1$ in (11), giving $n = 4$:

$$(4/7)^4 \sim 10\%.$$

The data of Ref. 4 are expected to be accurate to about this level.

Now, clearly, if we had extremely accurate data, one might wish to try replacing the effective poles

$\alpha_{K^*}^c, \alpha_{K^{**}}^c$ with the sum of a series of poles

$$\begin{aligned} & \{ \alpha_{K^*}^c - n \mid n = 0, 1, 2, \dots \}, \\ & \{ \alpha_{K^{**}}^c - n \mid n = 0, 1, 2, \dots \}, \end{aligned}$$

each with

$$\gamma_{K^*}^c(n) = a_n s_i^n \gamma_{K^*}^c, \quad \gamma_{K^{**}}^c(n) = b_n s_i^n \gamma_{K^{**}}^c \quad \text{in } H_{\text{NF}}^{(1)},$$

and require

$$\sum_{n=0}^{\infty} a_n e^{-i\pi n/2} = 1, \quad \sum_{n=0}^{\infty} b_n e^{-i\pi n/2} = 1. \quad (14)$$

In other words, for example, one might wish to carry out a fit to the detailed t and s dependence of the data by replacing, in (5),

$$(-1)^{l+1} \gamma_{K^*, \text{NF}} \gamma_{K^*}^c i \exp[-i\pi \alpha_{K^*}^c(t)/2] s^{\alpha_{K^*}^c(t)} \exp(a_{K^*}^c t) \quad (15)$$

with

$$(-1)^{l+1} \gamma_{K^*, \text{NF}} \gamma_{K^*}^c \sum_{n=0}^{\infty} a_n s_i^n i \exp\{-i\pi [\alpha_{K^*}^c(t) - n]/2\} s^{\alpha_{K^*}^c(t) - n} \exp(a_{K^*}^c t), \quad (16)$$

and by replacing

$$-\gamma_{K^{**}, \text{NF}} \gamma_{K^{**}}^c \exp[-i\pi \alpha_{K^{**}}^c(t)/2] s^{\alpha_{K^{**}}^c(t)} \exp(a_{K^{**}}^c t) \quad (17)$$

with

$$-\gamma_{K^{**}, \text{NF}} \gamma_{K^{**}}^c \sum_{n=0}^{\infty} b_n s_i^n \exp[-i\pi (\alpha_{K^{**}}^c(t) - n)/2] s^{\alpha_{K^{**}}^c(t) - n} \exp(a_{K^{**}}^c t), \quad (18)$$

subject to (14).

In the notation of (16) and (18), the choice $n = 4$ in (11) corresponds to

$$a_4 = 1, \quad a_n = 0 \text{ for } n \neq 4, \quad b_4 = 1, \quad b_n = 0 \text{ for } n \neq 4. \quad (19)$$

We now have

$$\begin{aligned} H_{\text{NF}}^{(m)} = & \kappa \{ (-1)^{m+1} \gamma_{K^*, \text{NF}} [-1 + \exp(-i\pi \alpha_{K^*}^c(t))] s^{\alpha_{K^*}^c(t)} \exp(a_{K^*, \text{NF}} t) \\ & + \gamma_{K^*, \text{NF}} [1 + \exp(-i\pi \alpha_{K^*}^c(t))] s^{\alpha_{K^*}^c(t)} \exp(a_{K^*, \text{NF}} t) \\ & + (-1)^{m+1} \gamma_{K^*, \text{NF}} \gamma_{K^*}^c i \exp(-i\pi \alpha_{K^*}^c(t)/2) s^{\alpha_{K^*}^c(t)} (s_m/s)^4 \exp(a_{K^*}^c t) \\ & - \gamma_{K^{**}, \text{NF}} \gamma_{K^{**}}^c \exp(-i\pi \alpha_{K^{**}}^c(t)/2) s^{\alpha_{K^{**}}^c(t)} (s_m/s)^4 \exp(a_{K^{**}}^c t) \} , \quad (20) \end{aligned}$$

where all parameters are given by (7) and (9). The comparisons of (20) and (6) with experiment at $p_{\text{lab}} = 7$ and $11.5 \text{ GeV}/c$ are shown in Fig. 2. Figures 3 and 4 show the predictions of (20) and (6) for $p_{\text{lab}} = 70 \text{ GeV}/c$, where only the Fermilab data on $\pi^+ p \rightarrow K^+ \Sigma^+$ are available. The data on $K^- p \rightarrow \pi^- \Sigma^+$ should be available soon. One sees that the agreement between the amplitudes (20), (6) and the data are rather reasonable in the strict Regge region.¹⁴

Having achieved a phenomenological description of the data at 7 and $11.5 \text{ GeV}/c$, we turn in the next section to the obvious theoretical question, "From where could such an amplitude as (20) come?" For, at this stage, one must consider the forms (16) and (18) completely *ad hoc* so that

the amplitude (20) is a special case of an *ad hoc*, empirical analysis.

III. A TOY MODEL

The amplitude (20) and the more general forms (16) and (18) have the structure that might be associated with the daughter trajectories of the effective pole trajectories $\alpha_{K^*}^c(t), \alpha_{K^{**}}^c(t)$. But, the latter poles are actually supposed to be approximations to Regge cuts. Thus, we are looking for a model which might have "daughters" of the cut trajectories. For simplicity, we will ignore the difference between $\alpha_{K^*}^c(t)$ and $\alpha_{K^{**}}^c(t)$ and consider ourselves to have the cut trajectory

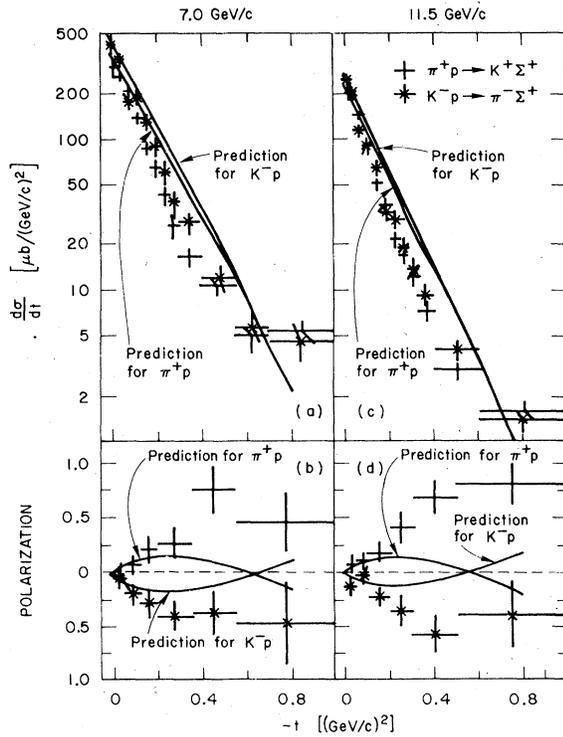


FIG. 2. Comparison of the amplitudes (20) and (6) with the data of Ref. 4.

$$\alpha^c(t) \approx 0.1 + 0.5t, \quad (21)$$

which is taken to be weak exchange degenerate, i.e., which gives contributions of both odd and

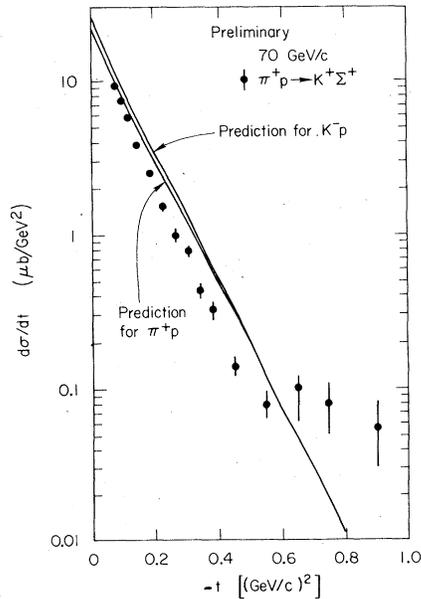


FIG. 3. Comparison of the amplitudes (20) and (6) with the data of Ref. 5 on $d\sigma/dt$ for $\pi^+p \rightarrow K^+\Sigma^+$. The prediction for $K^-p \rightarrow \pi^-\Sigma^+$ is also shown.

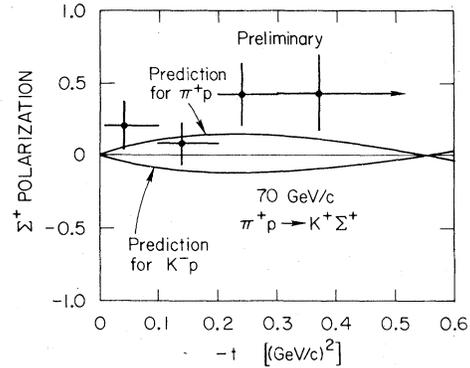


FIG. 4. Comparison of the amplitudes (20) and (6) with the data of Ref. 5 on the Σ^+ polarization for $\pi^+p \rightarrow K^+\Sigma^+$. The prediction for $K^-p \rightarrow \pi^-\Sigma^+$ is also shown.

even signature in the same way as weak-exchange-degenerate leading trajectories would. A theoretical framework which is set up to consider the effects of cuts to all orders in Regge exchange is the dual multiperipheral bootstrap of Lee,⁹ Veneziano,¹⁰ and Chan and Paton¹¹ as represented by Balázs.¹² We will work in the planar approximation, for we are not interested in the Pomeron. Balázs' representation is particularly appropriate to our needs because it pays special attention to certain threshold effects.

Specifically, in this scheme the amplitude for the production of clusters is given by the standard multiperipheral diagrams illustrated in Fig. 5. In our case, we will be interested in generating Reggeons actually, so that the vertical lines are clusters and the horizontal lines are linear combinations of exchange-degenerate sets of Regge exchanges with Regge propagators

$$R = e^{-i\pi\alpha_e(t)} s^{\alpha_e(t)}. \quad (22)$$

We are, of course, interested in the case of exchange-degenerate K^* and K^{**} exchange. The diagrams illustrated in Fig. 5 give the two-body absorptive part, via multiparticle unitarity, as the sum of ladders as illustrated in Fig. 6. Since we are only interested in generating Reggeons, only the (planar) quark-duality diagrams shown in Fig. 7 are relevant.

The model has the following additional constraint. The vertical lines in Fig. 6 are dual to Regge behavior in the sense of a finite-energy



FIG. 5. Typical type of diagram in the multiperipheral model for cluster production.

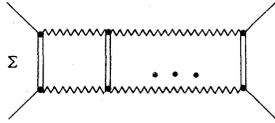


FIG. 6. Absorptive part generated by the diagrams illustrated in Fig. 5.

sum rule (FESR), as illustrated in Fig. 8. Specifically, we have (we will use the notation of Ref. 12 wherever possible)

$$\Gamma(t_1, t_2, t'_1, t'_2, t) = F(t) g_1(t_1, t'_1, t) g_2(t_2, t'_2, t), \tag{23}$$

where Γ represents the Reggeon-Reggeon-cluster-Reggeon-Reggeon coupling in Fig. 8, g_1 and g_2 are triple-Regge couplings, and F is a kinematic factor. In the model, it can be argued that F is independent of t_i and t'_i , $i=1, 2$, so that Γ factorizes; this permits a simple solution of the model.

Specifically, we assume the absorptive part $A(t, s)$ of the process $12 \rightarrow 1'2'$ is given by Fig. 9, wherein only one narrow-resonance cluster of type a is produced, with the exception of the end clusters R and Q . A more realistic assumption might be to sum Fig. 9 over several such clusters a . We take the squared mass of a , s_a , to be $s_a \approx 0.5 \text{ GeV}^2$, corresponding to ignoring the difference between the squared masses of the ρ and K^* . A better choice might be $s_a \approx 0.65 = (m_\rho^2 + m_{K^*}^2)/2$, where $m_a = \text{mass of } a, a = \rho, K^*$. Clearly in each loop in Fig. 9, one of the exchanged Reggeons carries strangeness, the other does not. However, since the empirical result in (7)

$$\alpha_{K^*}(t) = 0.375 + 0.678t, \tag{24}$$

is not too far from the canonical form

$$\alpha_V(t) = 0.5 + t \tag{25}$$

for the leading vector trajectory, we shall, for simplicity, consider that we have a single exchange trajectory function $\alpha_e(t) = \alpha_e^0 + \alpha_e' t$. We then require that the cut trajectory $\alpha^c(t)$ in (21) be associated with α_e in the usual way

$$\begin{aligned} 0.1 + 0.5t = \alpha^c(t) &= 2\alpha_e(\frac{1}{2}t) - 1 \\ &= 2\alpha_e^0 + \frac{1}{2}\alpha_e' t - 1. \end{aligned} \tag{26}$$

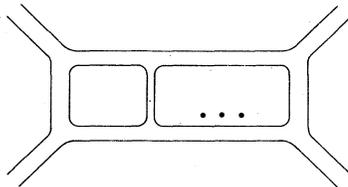


FIG. 7. Planar quark-duality diagram relevant to the generation of Reggeons.

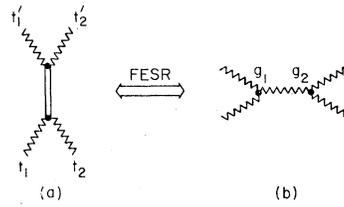


FIG. 8. Cluster (a) and Reggeon (b) average duality.

This gives

$$\alpha_e = 0.55 + t, \tag{27}$$

which is not too different from (24) and (25). Thus, in our toy calculation, we will simply use $\alpha_e = \alpha_V$.

We shall take $R = a$ and Q will play the role of an appropriate strange or nonstrange baryon. In other words, if particle $1'$ is K^+ , particle $2'$ is Σ^+ so, if the last Reggeon coupled to $2'$ is strange, we could take Q to be the nucleon. If this Reggeon were nonstrange, we must take Q to be a baryon with $S \neq 0$. In our toy calculation, we simply choose Q to have mass squared $s_Q \approx 1 \text{ GeV}^2$.

Let us now describe the actual computation of $A(t, s)$. One first notes that in the narrow-resonance approximation the graph in Fig. 8(a), out of which $A(t, s)$ is constructed, gives

$$V(t, s) = \Gamma_a \delta(s - s_a). \tag{28}$$

Therefore, from (23) the Mellin transform of V is

$$V(t, j) = g(t_1, t_2, t) g(t'_1, t'_2, t) F_a(t) s_a^{-j-1}, \tag{29}$$

where

$$V(t, j) = \int_0^\infty ds s^{-j-1} A(t, s) \tag{30}$$

and g is the $\alpha_e - \alpha_e - K^{**}$ triple-Regge coupling.

Hence, if one takes the Mellin transform of $A(t, s)$,

$$A(t, j) = \int_0^\infty ds s^{-j-1} A(t, s), \tag{31}$$

then, with the standard high-energy approximation one obtains the result of Balázs¹²:

$$\begin{aligned} A(t, j) &= \gamma_{11'K^{**}}(t) s_a^{-j-1} F_a(t) B(t, j) \\ &\quad \times F_Q(t) s_Q^{-j-1} \gamma_{22'K^{**}}(t). \end{aligned} \tag{32}$$

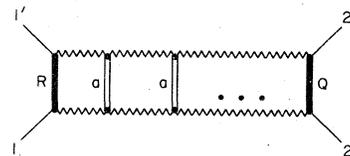


FIG. 9. Absorptive part A corresponding to the production of a clusters.



FIG. 10. Series of graphs corresponding to $B(t, j)$ in (32).

Here, $\gamma_{11'K^{**}}, \gamma_{22'K^{**}}$ are the $11'K^{**}, 22'K^{**}$ Regge couplings, respectively, and $B(t, j)$ is given by

$$B(t, j) = K(t, j) / D(t, j), \tag{33}$$

where $K(t, j)$ has the general structure

$$K(t, j) = \int g RRg \tag{34}$$

and

$$D(t, j) = 1 - s_a^{-j-1} F_a(t) K(t, j). \tag{35}$$

Thus, B corresponds to summing the series of graphs shown in Fig. 10; K is evidently just the one-loop graph in this series. In other words, one can write B as

$$B(t, j) = K(t, j) + K(t, j) F_a(t) s_a^{-j-1} K(t, j) + \dots \tag{36}$$

The function $K(t, j)$, whose treatment by Balázs represents the central point of his form of the dual multiperipheral planar bootstrap, is the function which we wish to concentrate on in this toy model of ours for the hypercharge-exchange reactions (1) and (2). Specifically, for the case of π - π scattering, Balázs solved for the output f trajectory $\alpha = 0.5 + t$ with s_a taken to be the same as in our toy model. Working to first order in t , from Balázs' work we know that if one takes

$$K(t, j) = \frac{k_0(1 + \frac{1}{2}\pi\tau t)}{j - \alpha^c} (x_0 + x_1 t)^{\alpha^c - j} \tag{37}$$

and require $D(t, \alpha_v(t)) = 0$ so that $A(t, j)$ has a pole at $j = \alpha_v$, then a conventional first-moment finite-energy sum rule will give, approximately,

$$k_0 = 0.36, \quad \tau = 1.39, \quad x_0 = 1.34, \quad x_1 = 1.76x_0, \tag{38}$$

completely solving the model. [See Ref. 12 for more details on the derivation of the results (38). We have used the FESR

$$F_a(t) \cong (s_a + \frac{1}{2}t)^{-1} (2s_a + \frac{1}{2}t)^{\alpha_v + 2} (\alpha_v + 2)^{-1}$$

$$\rho(t, \xi) = \frac{2}{8\pi^2} \int_{-\infty}^0 dt' \int_{-\infty}^0 dt'' \delta(\xi + 1 - \alpha(t') - \alpha(t'')) \Delta(t, t', t'') C(t') C(t'') \left[\cot\left(\frac{\pi\alpha(t')}{2}\right) \cot\left(\frac{\pi\alpha(t'')}{2}\right) + 1 \right] \tag{44}$$

with

$$\Delta(t, t', t'') = \frac{\theta(-t^2 - t'^2 - t''^2 + 2tt' + 2tt'' + 2t't'')}{(-t^2 - t'^2 - t''^2 + 2tt' + 2tt'' + 2t't'')^{1/2}}. \tag{45}$$

for simplicity.] The choice (37) is motivated by two things: (1) $K(t, j)$ is known to have a logarithmic singularity at $j = \alpha^c$, which is approximated by the pole in (37). (It is expected that this approximation fails for $j < \alpha^c$.) (2) $K(t, s)$ should have a threshold factor at $s = x(t) \pm x_0 + x_1 t$. Thus, we are invited to consider a more realistic choice for $K(t, j)$.

Specifically, since we know that in (37) $1/(j - \alpha^c)$ should be $\sim \ln[1/(j - \alpha^c)]$, we could simply replace $1/(j - \alpha^c)$ by the logarithm. This we do as follows: We take

$$K(t, j) = k_0' (1 + \frac{1}{2}\pi\tau t) (x_0 + x_1 t)^{\alpha^c - j} \ln \left| \frac{j - \alpha_m}{j - \alpha^c} \right|, \tag{39}$$

where k_0' and α_m are constrained by the requirement that

$$k_0' \ln \left| \frac{\alpha_v - \alpha_m}{\alpha_v - \alpha^c} \right| = \frac{k_0}{\alpha_v - \alpha^c}. \tag{40}$$

For simplicity, we work with (40) to zeroth order in t only and find at $t = 0$

$$k_0' \ln |1 - 2\alpha_m| = 2k_0. \tag{41}$$

The condition (40) ensures that $D(t, j)$ will have a pole at $j = \alpha_v$ for the values of k_0, τ, x_0 , and x_1 in (38).

In case the reader is wondering about the origin of α_m , let us remark that it is simply an approximation to the lower support limit $\xi_m(t)$ of the Amati-Stanghellini-Fubini-Bertocchi-Tonin¹⁵ Regge weight function $\rho(t, s)$ in their multiperipheral theory of K ; namely, following their notation, the first iteration of a Regge pole

$$T(s, t) = C(t) s^{\alpha(t)} \left[-\cot\left(\frac{\pi\alpha(t)}{2}\right) + i \right] \tag{42}$$

gives the asymptotic result

$$T(s, t) = \int_{\xi_m(t)}^{\xi_M(t)} s^\xi \rho(t, \xi) \left[-\cot\left(\frac{\pi\xi}{2}\right) + i \right] d\xi, \tag{43}$$

where

The functions $\xi_M(t)$ and $\xi_m(t)$ are the upper and lower support limits of $\rho(t, \xi)$ and to make contact with our work, we should point out that

$$\xi_M(t) = \alpha^c(t), \tag{46}$$

$$\zeta_m(t) = \alpha_m(t). \quad (47)$$

An advantage of (39) is that, unlike (37), it may be valid for $j < \alpha^c$. In other words, the toy model for $A(t, j)$ does not obviously fail for $j < \alpha^c$ if we use (39), whereas the model does appear to fail for $j < \alpha^c$ if we use (37). Indeed, using (39) we have, again at $t=0$, that, for example, there is a pole in $A(t, j)$ also at $j \pm \alpha^c - 4$ if

$$1 = (s_a x_0)^{0.5+4} \ln \left| \frac{-4 - \alpha_m}{-4} \right| / \ln \left| \frac{0.5 - \alpha_m}{0.5} \right| \quad (48)$$

or

$$\alpha_m(0) = 0. \quad (49)$$

This value of $\alpha_m(0)$ is not inconsistent with $\alpha^c(0)$, although we would naively expect that $\alpha_m(0) < 0$. That $\alpha_m(0) = 0$ is probably just an artifact of our approximations; in a more complete treatment, more of the smooth dependence of $\rho(t, \zeta)$ in ζ would have to be taken into account. It is clear, however, that when properly extended to $j < \alpha^c$, models like the dual multiperipheral planar bootstrap may indeed contain further Regge poles at places like $j = \alpha^c - 4$, i.e., at places like the empirical extra trajectories that appear to be required by the data on the hypercharge exchange reactions (1) and (2).

IV. DISCUSSION

This paper has attempted to do two things. First, it has attempted to present an empirical fit to the apparently weak-exchange-degenerate data on the hypercharge-exchange reactions $\pi^+ p \rightarrow K^+ \Sigma^+$ and $K^- p \rightarrow \pi^- \Sigma^+$ at laboratory momenta like 7, 11.5 GeV/c which, at the same time, incorporates apparent violations of weak exchange degeneracy at laboratory momenta of order 4 GeV/c. The type of fit arrived at requires a trajectory at $\alpha^c - 4$, where α^c may be identified with the cut trajectory of the leading K^*, K^{**} trajectory exchanges. Thus, the second part of the paper was devoted to a toy model calculation, which by the way is nothing but a simple version of the Chew-Goldberger-Low¹⁶ model applied to cluster production. The result of the model calculation is that if the j -plane struc-

ture of the two-Regge exchange graph is properly extended to the region $j < \alpha^c$, one may indeed find trajectories like $\alpha^c - 4$. Thus, the empirical fit in Sec. II may not be without theoretical support.

There is one more thing we should like to emphasize. This is that, from our empirical analysis, the trajectory $\alpha^c - 4$, if it were the only trajectory present in the reactions $\pi^+ p \rightarrow K^+ \Sigma^+$ and $K^- p \rightarrow \pi^- \Sigma^+$, would give essentially weak-exchange-degenerate predictions. Thus, in some sense, the notion that each trajectory function contributes both odd- and even-signatured contributions in the weak-exchange-degeneracy sense is still true. What differs from the naive weak-exchange-degenerate idea is that at lower laboratory momenta, two different trajectories, each by itself respecting weak exchange degeneracy, nonetheless are both important and interfere to give apparent violations of the predictions one would make if one only had to consider a single weak-exchange-degenerate trajectory.

Finally, there is an interesting coincidence to which we wish to call attention. This is that, if one follows the work of Balázs¹⁷ on threshold effects, one finds that the D^* trajectory is, approximately,

$$\begin{aligned} \alpha_{D^*}(t) &\doteq -0.75 + 0.433t \\ &\approx \alpha^c - 1, \end{aligned} \quad (50)$$

where α^c is given by (21) for example. At this time, we will simply leave this as a coincidence.

ACKNOWLEDGMENTS

This paper is the result of an investigation carried out at the suggestion of and with the encouragement of Group BC at SLAC—especially of its director, Professor J. Ballam. The author is grateful to Professor Ballam and his Group. The hospitality of Professor S. D. Drell of the SLAC Theory Group is also appreciated. The author further acknowledges a helpful conversation with Professor F. J. Gilman. This work was supported by the Department of Energy under Contract No. EY-76-C-03-0515.

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