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Quantum-chromodynamic effects in polarized electroproduction

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The quantum-chromodynamics (QCD) higher-order effects (next to the leading order in the running coupling constant) are calculated for the moments of structure functions in polarized deep-inelastic electroproduction. The QCD correction to the Bjorken sum rule is obtained and compared with the existing data.

In quantum chromodynamics (QCD) the theoretical formulation of deep-inelastic lepton-hadron scatterings is well established in the framework of operator-product expansions and the renormalization-group equations.[1] The moments of structure functions are related to the coefficients of the lowest-twist operators appearing in the light-cone expansion of the product of currents. The Q^2 dependence of these coefficient functions is governed by the renormalization-group equations.

The coefficient functions in unpolarized electroproduction in the leading order of the running coupling constant $\bar{g}$ is well known.[2] The $\bar{g}^2$ corrections beyond the leading order were calculated by several authors.[3-7] Recently, the calculations of the full $\bar{g}^2$ corrections including the two-loop anomalous dimensions have been carried out.[8-10] Similar calculations in the case of polarized electroproduction are straightforward. In fact the coefficient functions in the leading order have already been calculated.[11-13] The $\bar{g}^2$ correction to the leading order, however, has been totally unknown in polarized electroproduction. In this paper we shall present the results of our calculations for this $\bar{g}^2$ correction.

The polarized electroproduction is described by two structure functions[14,15] G_1 and G_2 in addition to the ordinary unpolarized structure functions W_1 and W_2. In the following it is more convenient to use g_1 and g_2 which are defined[15] by $g_1 = Mp\cdot qG_1$ and $g_2 = (p\cdot q)^2G_2/M$, where p and M are the momentum and mass of the target particle and q is the momentum of the virtual photon with $Q^2 = -q^2$. The moments of $g_1(x, Q^2)$ and $g_2(x, Q^2)$ with $x = Q^2/2p\cdot q$ satisfy the relations

$$\int_0^1 dx\, x^{n-1}g_1(x, Q^2) = a_n C_{1n}(Q^2/\mu^2, g)\,, \tag{1}$$

$$\int_0^1 dx\, x^{n-1}g_2(x, Q^2) = -[(n-1)/n]a_n C_{1n}(Q^2/\mu^2, g) - \tfrac{1}{2}d_n C_{2n}(Q^2/\mu^2, g)\,, \tag{2}$$

where μ is the scale parameter, g is the QCD coupling (renormalized), a_n and d_n are the nucleon matrix elements of composite operators,[11,16] and C_{1n} and C_{2n} are the coefficient functions in the light-cone expansion of the product of electromagnetic currents.[16] We consider, for simplicity, only the nonsinglet combination of the structure functions where no operator mixing takes place. The extension to the case of singlet combinations is straightforward.

The functions C_{in} $(i=1,2)$ are given by solving the renormalization-group equations

$$C_{in}(Q^2/\mu^2, g) = C_{in}(1, \bar{g})\exp\left[-\int_0^t \gamma_{in}(\bar{g}(t', g))dt'\right], \tag{3}$$

where $t = \frac{1}{2}\ln(Q^2/\mu^2)$ and $\bar{g}$ is the effective coupling constant with $\bar{g}(t=0, g) = g$, and γ_{in} the anomalous dimensions of the relevant quark composite operators. For later convenience, we expand here $\gamma_{in}(\bar{g})$ and $C_{in}(1, \bar{g})$ in powers of $\bar{g}$:

$$\gamma_{in}(\bar{g}) = \gamma_{in}^0\bar{g}^2 + \gamma_{in}^1\bar{g}^4 + O(\bar{g}^6)\,, \tag{4}$$

$$C_{in}(1, \bar{g}) = 1 + c_{in}\bar{g}^2 + O(\bar{g}^4)\,. \tag{5}$$

We have performed calculations of c_{in} $(i=1,2)$ both in the conventional renormalization method with momentum cutoff[5] and in the minimal-subtraction scheme of 't Hooft using dimensional regularization.[9] In the former, since we work with zero-mass quarks, the subtraction has been made at off-mass-shell points corresponding to off-shell momentum of external lines.[17] In the latter, we calculate the current correlation function of quarks through the order g^2 [Fig. 1(a)], subtract from it the operator matrix element [Fig. 1(b)], and obtain c_{in}. Here we present our results obtained by the latter scheme of the minimal subtraction in the dimensional regularization.

The results for c_{in} $(i=1,2)$ are[18,19]

$$c_{1n}=\frac{C_2(R)}{16\pi^2}\left(-9+\frac{1}{n}+\frac{2}{n+1}+\frac{2}{n^2}+3\sum_{j=1}^{n}\frac{1}{j}-4\sum_{j=1}^{n}\frac{1}{j^2}-\frac{2}{n(n+1)}\sum_{j=1}^{n}\frac{1}{j}+4\sum_{s=1}^{n}\frac{1}{s}\sum_{j=1}^{s}\frac{1}{j}\right)\quad (n=1,3,\ldots)\,, \tag{6}$$

$$c_{2n}=c_{1n}+\frac{C_2(R)}{16\pi^2}\frac{n}{n-1}\left[5+\frac{1}{n}-\frac{4}{n+1}-\frac{2}{n^2}-2\sum_{j=1}^{n}\frac{1}{j}+2\sum_{j=1}^{n}\frac{1}{j^2}+2\left(\frac{1}{n}-\frac{2}{n+1}\right)\sum_{j=1}^{n}\frac{1}{j}-2\sum_{s=1}^{n}\frac{1}{s}\sum_{j=1}^{s}\frac{1}{j}\right]\quad (n=3,5,\ldots)\,, \tag{7}$$

where $C_2(R)=\frac{4}{3}$ for SU(3) color.

In calculating c_{1n} and c_{2n} we also obtain one-loop anomalous dimensions for nonsinglet operators. Corresponding to two independent operators R_1 and R_2, defined in Ref. 11, we find[20]

$$\gamma_{1n}^{0}=\frac{C_2(R)}{8\pi^2}\left[1-\frac{2}{n(n+1)}+4\sum_{j=2}^{n}\frac{1}{j}\right], \tag{8}$$

$$\gamma_{2n}^{0}=\frac{C_2(R)}{8\pi^2}(n-2)\left(\frac{1}{n}+\frac{2}{n-1}\sum_{j=2}^{n}\frac{1}{j}\right), \tag{9}$$

where n in the above Eqs. (8) and (9) corresponds to $n+1$ and $n+2$ in Eqs. (2.18) and (2.23) of Ref. 11, respectively. With this substitution our result Eq. (8) agrees with those of Refs. 11, 12, and 13.[21]

It can be shown that the anomalous dimension

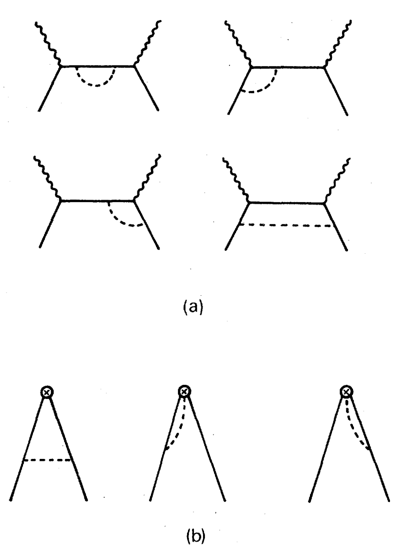

FIG. 1. Diagrams contributing in order g^2 to (a) the current correlation function of quarks and (b) the operator matrix element. The wavy line denotes the virtual photon and the dashed line the gluon.

γ_{1n} relevant to $C_{1n}(Q^2/\mu^2,g)$ is of the same form as that of the nonsinglet operator in the unpolarized case. This statement, in fact, can be seen to be valid for the one-loop anomalous dimension γ_{1n}^0 in Eq. (8). For the two-loop anomalous dimension γ_{1n}^1 we may use the results for the nonsinglet operator in unpolarized leptoproductions which were already obtained in Ref. 8. Thus by combining their results for γ_{1n}^1 with Eq. (6) we can immediately carry out a phenomenological analysis of polarized electroproduction. The statistics of existing data on polarized electroproduction are not yet high enough to allow for the moment analysis of structure functions and we do not pursue any further in this direction.

The moment sum rules (1) and (2) for the case of $n=1$ deserve particular attention. Equation (2) for $n=1$ reduces to the Burkhardt-Cottingham sum rule,[22] and it has been shown that there is no $\bar{g}^2$ correction.[13] The sum rule (1) with $n=1$ is especially interesting. γ_{1n} must vanish for $n=1$ since it is the anomalous dimension of the axial-vector current. Setting $n=1$ in Eqs. (1), (3), and (6) we obtain the $\bar{g}^2$ correction to the Bjorken sum rule[14]

$$\int_0^1 dx\,(g_1^p-g_1^n)=\frac{1}{6}\frac{G_A}{G_V}\left(1-\frac{\bar{g}^2}{4\pi^2}\right), \tag{10}$$

where $\bar{g}^2/4\pi^2=12/[(33-2f)\ln Q^2/\Lambda^2]$ with f the number of flavors. In terms of the quantities A_1F_2 which may be more familiar to experimentalists,[23] Eq. (10) can be rewritten as

$$\int_0^1 \frac{dx}{x}(A_1^pF_2^p-A_1^nF_2^n)=\frac{1}{3}\frac{G_A}{G_V}\left(1-\frac{12}{33-2f}\frac{1}{\ln Q^2/\Lambda^2}\right). \tag{11}$$

Assuming four flavors $(f=4)$, we plot in Fig. 2 our prediction based on Eq. (11) with $G_A/3G_V=0.417$ (Ref. 24) for $\Lambda=0.5$ and 0.7 GeV as a func-

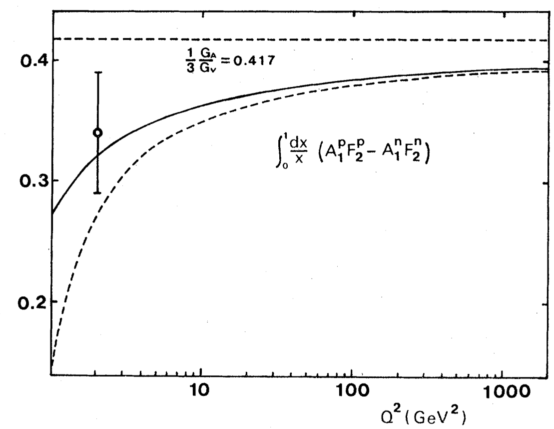

FIG. 2. Comparison of the $\bar{g}^2$ correction to the Bjorken sum rule Eq. (11) with the experimental data from Ref. 23. The solid (dashed) line corresponds to our prediction with $\Lambda = 0.5$ GeV (0.7 GeV).

tion of Q^2. We see that the $\bar{g}^2$ correction to the Bjorken sum rule is quite large in the wide range of Q^2.[25] The data point shown in Fig. 2 is the only existing experimental value measured at SLAC.[23] Our prediction is consistent with the data. Here one should note that in obtaining this experimental value, A_1^n was assumed to be zero and an extrapolation of the data to the low-x region was made.[23] We expect that future measurements of both A_1^p and A_1^n for a wider range of x at several points of Q^2 may serve as a clean test of QCD.

Finally we wish to make a remark on Crewther's relation[26] in which the factor K appearing on the right-hand side of the Bjorken sum rule and the R ratio in e^+e^- annihilations are related to the Adler's anomalous constant S. We find that the QCD correction to K is compensated by that to R (Ref. 27), resulting in S being free from the QCD correction.[28]

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[16] a_n, d_n, C_{1n}, and C_{2n} in this paper correspond to a_{n-1}, d_{n-2}, $\tilde{E}_{i,1}^{n-1}$, and $\tilde{E}_{i,2}^{n-2}$ in Ref. 11, respectively.

[17] The detailed arguments in the conventional renormalization scheme including Calvo's approach with massive quarks will be presented elsewhere.

[18] We have dropped the term proportional to γ_{in}^0 since this term can be absorbed by redefinition of the renormalization scale μ.

[19] Actually c_{1n} turns out to be of the same form as the coefficient function corresponding to F_3 in neutrino reactions. Compare Eq. (6) with the result in Ref. 9.

[20] We worked with massless quarks and kept external lines off the mass shell. One may also work with massive quarks and keep external lines on the mass shell as discussed in Calvo's paper (Ref. 4). In this case, however, one should be careful in extracting the correct anomalous dimensions of R_2. Otherwise one would pick up a finite contribution from the quark mass term which should actually vanish as $Q^2 \to \infty$.

[21] The result (9) is slightly different from Eq. (2.23) of Ref. 11.

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