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Low-mass lepton pairs from hadron and neutrino beams

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A simple model based on meson annihilation is proposed to explain low-mass production of lepton pairs for both hadron and neutrino beams. Resonances, such as the ρ and ω, are naturally taken into account and are shown to give rise to a large but unexpected contribution at very low pair masses. Numerical results are given together with comparisons to data. For the most part, the normalizations are taken from other experimental data.

I. INTRODUCTION

The simplicity and appeal of the Drell-Yan (DY) model[1] for lepton pair production in hadronic scattering has fostered its application well outside the kinematic regimes for which it was originally justified. Indeed, as Drell and Yan stressed, the approximations made in the parton model [in the impulse approximation or the hard-scattering expansion[2] (HSE)] are not valid at large pair transverse momenta nor at low pair masses.

The HSE is illustrated in Fig. 1(a). In the sum over states labeled by a, b, and d, one must ensure that the individual terms are incoherent as well as gauge invariant. This requires that partons a and b have a small transverse momentum k_T. The DY model analyzes only the subprocess $q\bar{q}\to l^+l^-$, and since there is no recoil d parton, the pair must also have a small p_T.

Recently, a more general analysis of this expansion (but without the small effects due to quantum-chromodynamic radiative corrections, for example) has been carried out.[3] By summing over parton types and independent incoherent subprocesses [including, for example, meson-quark, Fig. 1(b), and diquark-antiquark, Fig. 1(c)] significant additional physical information is included; it is simple to achieve a gauge-invariant extension of the DY model to large-p_T pairs. The experimental results are quite well reproduced.

In this paper we wish to extend the discussion to low-mass pairs at low p_T and explicitly include resonance effects. We argue that at low mass and low p_T, the most important intermediate states are those which have small mass but are strongly interacting, that is, those states that determine the large-distance structure of the hadron. Since the most important of these are expected to be light mesons, we shall study here the contribution of the subprocess $M+M'\to$ "γ" $\to l^+l^-$.[4]

As the pair mass and/or transverse momentum increases, one expects in this picture to probe smaller and smaller distances, and the most important subprocess should change smoothly from meson-meson to meson-quark and diquark-antiquark, and finally to quark-gluon and quark-antiquark processes.

The general HSE formula for the fully differential pair production cross section is written as

$$Q^4\frac{d\sigma}{d^4Q}(AB\to l^+l^-X)=\sum_{a,b,c}\int dx\, d^2k_T dy\, d^2l_T P_{a/A}(x,k_T)P_{b/B}(y,l_T)Q^4\frac{d\sigma}{d^4Q}(ab\to l^+l^-d;s',t',u')\,, \tag{1}$$

where, for sufficiently small k_T and l_T,

$$\begin{aligned} s'&=xys\,,\\ t'&=xt+(1-x)Q^2\,,\\ u'&=yu+(1-y)Q^2\,. \end{aligned} \tag{2}$$

Some trivial kinematic factors which tend to unity as a and b approach their mass shells have been omitted in the above. The function $P_{a/A}$ is the probability function of finding constituent a in hadron A with (infinite) momentum fraction x and transverse momentum k_T.

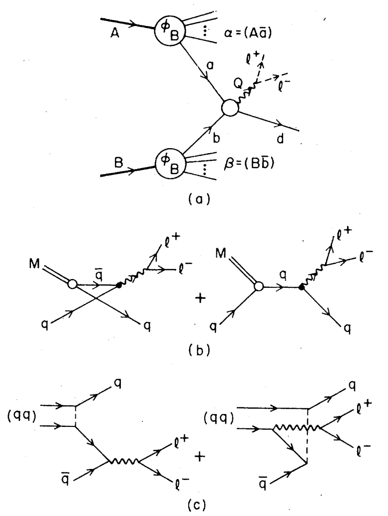

Fig. 1. (a) A general contribution to the hard-scattering expansion. (b) The meson-quark basic subprocess. (c) The diquark-antiquark basic subprocess.

In Sec. II, the above model is applied to hadron beams and numerical results are given for pion-nucleon scattering using a $P_{\pi/N}(x)$ distribution function consistent with inclusive production of large-p_T mesons. In Sec. III, the model is applied to neutrino scattering by computing the probability of finding a meson in a neutrino, $P_{M/\nu}(x)$. We do this in two stages. Using the (presumed) exact lepton-current–quark-current coupling, we first calculate $P_{q/\nu}(x)$ (in a manner analogous to the equivalent-particle calculations of Chen and Zerwas.[5] We then convolute this with $P_{M/q}(x)$ (inferred from experimental results in e^+e^- annihilation into hadronic jets[6]) to obtain $P_{M/\nu}(x)$. For the case $M=\pi$, we obtain a (common) lower bound for the ratio of *trimuon* to single-muon cross sections in charged-current (CC) neutrino-hadron scattering and of dimuon to muonless cross sections in neutral-current (NC) scattering. We find that this lower bound is consistent with the experimentally observed ratio in the measured CC case indicating that there may be no other significant contributions beyond π-π annihilation in that case. Finally, we indicate some tests of our model and summarize our conclusions in Sec. IV.

II. HADRON BEAMS

The primary goal of this section is to give a theoretical treatment of low-mass pair production based on the physical picture described in the Introduction. Since there is no requirement of any large transverse momentum in the process, the expansion in Eq. (1) is properly defined (there are no subtle coherence problems), and the structure functions are properly restricted, i.e., the intermediate particles are only slightly off shell and enforcing gauge invariance is no problem.

The contribution of meson-meson, M-M', annihilation is given by

$$Q^4\frac{d\sigma}{dQ^2d\xi}=\frac{4\pi\alpha^2}{3}\sum_{M\neq M'}|F_{MM'}(Q^2)|^2dxdy\,\delta(\xi-x+y)xy\,\delta(xy-Q^2/s)[P_{M/A}(x)P_{M'/B}(y)+P_{M'/A}(x)P_{M/B}(y)], \qquad (3)$$

where off-shell effects in the form factor have been neglected.

These subprocesses automatically included multiple soft scattering corrections to the Drell-Yan process in which soft particles accompany the quarks in the form of correlated pairs (the mesons). What we have picked out here is a computable and physically sensible subset of all possible processes which, as we shall see, seem to be dominant at low p_T and for small pair masses.

The main contribution to the above sum is anticipated to be charged-pion annihilation. For this process there are direct measurements of the relevant form factor in electron-positron annihilation.[7] There are other interesting contributions such as pion-"ρ" annihilation which contains the ω resonance, etc. (It should be pointed out that these resonance effects will also play some role in the Drell-Yan process, quark-antiquark annihilation, but this is a small fraction of the cross section at these low masses in our model.) From uncertainty-principle arguments, such as given in the Introduction, these contributions are expected to be unimportant at low masses and small momentum transfers, and numerically they are small.

The probability functions $P_{M/A}(x)$ are not well studied, but an analysis[8] of large-transverse-momentum scattering yielded the approximate form for a proton target which is consistent with the spectator counting rules[9]

$$xP_{M/p}(x)=P_0(1-x)^2F_2(x)\,,$$

where $F_2(x)$ is the proton structure function. P_0 is a normalization constant chosen so that this probability behaves as

$$xP_{M/p}(x)\sim C_{M/p}(1-x)^5\,, \qquad (4)$$

where $C_{M/p}$ is a constant fitted to the data

($C_{M/P} \sim 0.2$). The above form and value of $C_{M/P}$ are consistent with the large momentum transfer scattering results.[10] For pion beams, we also need

$$xP_{M/\pi}(x) = x\delta(1-x) + C_{M/\pi}(1-x)^3 ,$$

where $C_{M/\pi} \sim 0.2$. One expects that the probability function for pions, ρ's, and ω's will be of the same order of magnitude and that all are described by the above approximate forms (omitting the δ function when $M \neq \pi$, of course).

The formula for the cross section, Eq. (3), will be evaluated and compared with data below. Before doing this, however, it is amusing to note several properties of the model. If Eq. (3) is evaluated at $\xi \sim 0$, and for $x = (Q^2/s)^{1/2}$, the result can be written in terms of the Drell-Yan amplitude for point quarks

$$Q^4 \frac{d\sigma}{d^2Q d\xi}\bigg|_{\xi=0} \simeq Q^4 \frac{d\sigma}{d^2Q d\xi}\bigg|_{\xi=0}^{\mathrm{DY}} \sum_{M,M'} |F_{MM'}(Q^2)|^2 C_{MM'} , \quad (5)$$

where $C_{MM'}$ contains the relative normalization factors. The DY process itself, can be included in the sum by setting $F_{q\bar{q}} = 1$ and $C_{q\bar{q}} = \frac{1}{3}$ (for color). It will be shown to be small for pair masses below 1 GeV. The crossover between the q-$\bar{q}$ and M-M' annihilation contributions occurs for masses between 1.0 and 1.5 GeV.

The ρ resonance will be a dominant feature of $\pi^+\pi^-$ annihilation (as well as $K\bar{K}$, etc.) and data exist for the relevant form factor.[7] The ω resonance will likewise be a dominant feature of π-ρ annihilation but the relevant experimental form factor is not directly available. One can, however, proceed by direct analogy with the π-π case by using the ω width in the relevant form factor and normalizing to the (narrow) ω peak. This neglects the small effects of ρ-ω mixing but is adequate for our purposes.

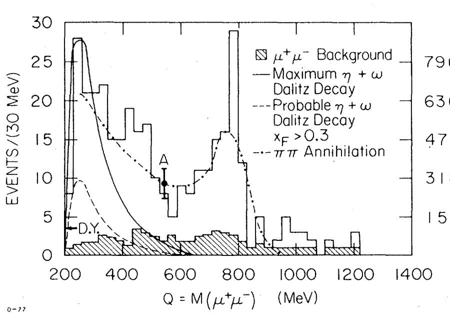

FIG. 2. Numerical results of the model, Eq. (3), compared with the data of Ref. 12. The solid and dashed curves are limits from the Dalitz decay of the ω and η.

For numerical calculation, we use a fit to the pion form factor as determined at Orsay[11]

$$|F_{\pi\pi}(Q^2)|^2 = F_0{}^2 M_\rho{}^2 \Gamma_\rho{}^2 [(M_\rho{}^2 - Q^2)^2 + M_\rho{}^2 \Gamma_\rho{}^2 (p/p_0)^6 (M_\rho/Q)^2]^{-1} , \quad (6)$$

where p is the pion momentum and $M_\rho = 0.775$ GeV, $\Gamma_\rho = 0.15$ GeV, $F_0 = 5.83$, and $p_0 = 0.36$ GeV/c.

The numerical results for $\pi + p \rightarrow \mu^+\mu^-$ are given in Fig. 2 for an energy of $E = 16$ GeV and compared to the data of Bunnel et al.[12] The sharp rise near the π-pair threshold can be explained from the Dalitz decay of ω and η. The normalization is achieved by fitting with the value[10] $C_{\pi/p} = 0.2$.and assuming only π-π annihilation in the region of the ρ. Because of the very fine resolution of this streamer-chamber experiment, an additional ω peak due to π-ρ annihilation is also clearly visible. We have not explicitly included these contributions in our calculation since they involve a term with independent normalization and form factor, do not check our model in any way, and do not contribute significantly to the total cross section.

Using the same value of $C_{\pi/p}$ as above, we find that the calculated cross section is consistent with the high-energy π-p data, e.g., Anderson et al.,[13] after taking scaling violations into account. In such higher-energy experiments, the pair-mass resolution is not sufficient to resolve the ω contribution.

These results lend credence to the idea that the meson-meson mechanism dominates low-mass μ production, and fixes the hadronic distribution functions that we need in the next section.

III. NEUTRINO BEAMS

We now turn to the discussion of low-mass μ-pair production in neutrino scattering. For charged-current (CC) events this means a three-muon final state with its attendant difficulty of separating the identification of the pair and the "leading" muon. For neutral-current (NC) events, the situation is entirely analogous to hadron-hadron-scattering μ-pair production as discussed above.

In order to parallel the preceding calculations and discussions, we need to compute the probability of finding a meson in a neutrino as a function of momentum fraction x. This is analogous to the equivalent photon and electron calculations of Chen and Zerwas.[5] The calculation here is somewhat more involved, since the fundamental coupling (in the approximation appropriate to present energies) generates a one-to-three-body transition ($\nu \rightarrow \mu q\bar{q}$) rather than a one-to-one-body transi-

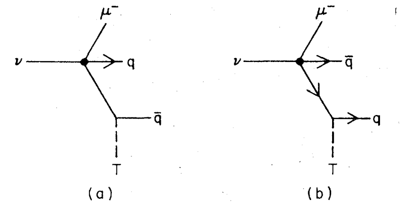

FIG. 3. Dominant diagrams for q-$\bar{q}$ production by neutrino beams on the target T.

tion ($\gamma \to e\bar{e}$ or $e \to \gamma e$). We will make things easier by working in the finite-momentum frame (FMF) and combining two of the three bodies into a single effective object.

In analogy with the equivalent photon calculations, we analyze ν-T scattering into μ-q-$\bar{q}$ (see Fig. 3) where an intermediate, off-shell $\bar{q}'$ (q') strikes the massless scalar target T to make the final-state-on-shell $\bar{q}$ (q). The target T is chosen for calculational convenience; since we will lose all spin information when we convert the derived quark distribution into a pion distribution, we calculate only a scalar structure function, rather than the full tensor object. We do *not* wish to calculate the whole of the Feynman diagrams of Fig. 3, but rather, in the sense of the hard-scattering expansion (HSE),[2] only the part where the $\bar{q}'$ (q') are "almost" on shell so that we may write, approximately,

$$\sigma_{\nu T \to \mu q \bar{q}} = \int_0^1 dx[P_{q/\nu}(x)\sigma(q'T \to q) + P_{\bar{q}/\nu}(x)\sigma(\bar{q}T \to \bar{q})] \tag{7}$$

and identify the function $P_{q/\nu}$ and $P_{\bar{q}/\nu}$ for further use.

We use the standard $V-A$ charged-current quark-lepton coupling and parametrize the momenta in the FMF as [for Fig. 3(b)]

$$\begin{aligned}
\nu &= (P, \vec{0}_T, P)\,, \\
q' &= \left(xP - \frac{L^2+L_T^2}{4(1-x)P}, -\vec{L}_T, xp + \frac{L^2+L_T^2}{4(1-x)P}\right), \\
L &= \left((1-x)P + \frac{L^2+L_T^2}{4(1-x)P}, \vec{L}_T, (1-x)p - \frac{L^2+L_T^2}{4(1-x)P}\right), \\
T &= (P, \vec{0}_T, -P)\,,
\end{aligned} \tag{8}$$

where $L = \mu + \bar{q}$. Straightforward calculation then yields

$$\sigma_{\nu T \to \mu \bar{q} q} \simeq 3\frac{2G_F^2}{\pi^6}\int d^4\mu\, d^4\bar{q}\,\delta(\bar{q}^2 - m_q^2)\delta(\mu^2 - m_\mu^2)\frac{q'\cdot T\,\nu\cdot\bar{q}[\mu\cdot(2T+q')]}{\nu\cdot T(q'^2 - m_q^2)^2}\left[\frac{\pi}{2q'\cdot T}\int\frac{d_q^3}{2E_q}\delta^4(T+q'-q)g^2 m_q^2\right], \tag{9}$$

where the factor of 3 counts quark colors, the last factor in large square brackets is $\sigma(q' + T \to q)$, and for simplicity we have used the symbol for each particle to represent also the corresponding four-momentum. Note that $q' = \nu - L$. Defining $l = (\mu - \bar{q})/2$, we replace the $d^4\mu\, d^4\bar{q}$ integration by $d^4L\, d^4l$ integrations, and, after completing the d^4l integral, identify from Eq. (9)

$$\int dx\, P_{q/\nu}(x) \simeq 3\frac{sG_F^2}{16\pi^4}\int dx\, dL^2 dL_T^2\frac{x(1-x)(2L_T^2+L^2)}{[L_T^2+xL^2+m_q^2(1-x)]^2}\,,$$

where we have also used

$$\int d^4L = (\pi/2)\int dL^2 dL_T^2 dx/(1-x)\,,$$

and

$$q'^2 = -(L_T^2 + xL^2)/(1-x)\,.$$

In equivalent photon calculations, such as those of Chen and Zerwas,[5] the corresponding formula would not have L^2, L_T^2 dependence in the numerator function of the integrand, and the denominator function would force the dominant contribution to be from low q'^2 automatically. Here we must recognize that *only* the small-q'^2 part of the above expression is sensible in the HSE picture that we are using; the factorization of Eq. (9) into the form of a term of Eq. (7) and treatment of $\sigma(T + q' \to q)$ as a constant is justified *only* for q'^2 less than or on the order of a natural hadronic (mass)^2 scale. We take this scale to be $\langle m^2\rangle \sim 1$ GeV^2. Thus, although the integrand above does not force this restriction, the physics of the HSE tells us that only in the $L^2, L_T^2 \lesssim 1$ GeV^2 region do we have a consistent picture. Note that this restriction is just the standard one limiting the validity of the equivalent particle approximation to low-mass objects (but here, composed of a pair). We, therefore, cut off the integration at that value and keep only that part of the calculated cross section; this gives us lower bounds on $P_{q/\nu}(x)$. The omitted region is present in a higher-order term in the HSE, where it is consistently treated.

Completing the integration, we find

$$P_{q/\nu}(x) \simeq \frac{G_F^2\langle m^2\rangle}{16\pi^4}(1-x)\,. \tag{10}$$

Equation (10) is not the actual calculated form, which is quite complicated, but is a simple interpolating function, accurate to ±30% over the range of x. This form was chosen in accord with spectator counting rules.[9]

Note that the hard vertex has given rise to the typical weak interaction $sG_F{}^2$ dependence and that there is no x^{-1} factor. We have assumed that s is sufficiently small so that the effects of the intermediate-vector-boson propagator do not need to be included. If s were much larger, the calculation would then resemble that for finding say a positron in an electron,[5] by convoluting the probability of finding a quark (e^+) in a W boson (eventually with negligible mass) (γ), with that of finding a W boson (γ) in a neutrino (e^-).

If we imagine the T in Fig. 3 to be a hadron, it become apparent that Eq. (7), with $P_{q/\nu}$ as given by Eq. (10), describes a portion of the total CC ν-hadron cross section. This allows us a consistency check on the normalization of Eq. (10) as we must have

$$2\int dx P_{q/\nu}\sigma(q+\text{hadron}) \ll \sigma(\nu+\text{hadron}) \tag{11}$$

since we have not included the entire contribution of all of the relevant Feynman diagrams. The factor of 2 accounts for the contribution of quarks and antiquarks [Figs 3(a) and 3(b)] since $P_{\bar{q}/\nu} \simeq P_{q/\nu}$. This last point argues that the inequality in (11) should be well satisfied: A lack of distinction between q and $\bar{q}$ is characteristic of "wee" quark effects which are known to contribute $\lesssim 15\%$ of the total cross section. In fact, estimating the left-hand side of (11), we find

$$\begin{aligned}\frac{sG_F{}^2\langle m^2\rangle}{8\pi^4}\left[\tfrac{1}{2}\sigma(\pi\text{-hadron})\right] &\simeq (0.006\text{ mb GeV}^2)sG_F{}^2\\ &\lesssim (0.075\text{ mb GeV}^2)sG_F{}^2,\end{aligned} \tag{12}$$

where the right-hand side describes the experimental value.

The calculation for $P_{\bar{q}/\nu}$ is very similar to that above. The result for *neutral*-current neutrino scattering is similar in form, although reduced in scale by the relative strength of the weak neutral current. That is, the *detailed* distribution function is *process-dependent*.[14] However, since our procedure is approximate, rather than an exact Feynman-diagram calculation, we shall below uniformly use a common approximate form

$$P_{q/\nu}(x) \simeq P_{\bar{q}/\nu}(x) \simeq \frac{sG_F{}^2\langle m^2\rangle}{16\pi^4}(1-x) \tag{13}$$

for CC scatterings, and $\sigma(\text{NC})/\sigma(\text{CC})$ times this for neutral-current reactions. Recall that this already includes a factor of 3 for color.

Equation (13) is sufficient for us to be able to calculate *massive*-lepton-pair production via $\bar{q}$-q annihilation in neutrino-hadron scattering. However, the quark-annihilation process is certainly inadequate for *low-mass* pairs even though it may also have resonance effects present. Its normalization is small at small masses. As we have done in hadron-hadron scattering, this can be handled by using a meson-meson annihilation picture for the low-pair-mass regime. Fortunately, it is quite simple to compute $P_{M/\nu}$ from Eq. (10) since we know that[15]

$$P_{M/q}(y) \simeq P_{M/\bar{q}}(y) \simeq 0.5(1-y)/y \tag{14}$$

for any given meson M; we need only convolute the two distributions

$$\begin{aligned}P_{M/\nu}(x) &\simeq P_{M/\bar{\nu}}(x)\\ &= 2\int_0^1 dy\,dw\,\delta(x-wy)P_{M/q}(w)P_{q/\nu}(y),\end{aligned} \tag{15}$$

where the factor of 2 accounts for M coming from a q or a $\bar{q}$. As in Eq. (10), we again rewrite the result in Eq. (15) as a simple interpolating function good to 30% accuracy, where the form is as suggested by the spectator counting rules[9]

$$P_{M/\nu}(x) \simeq \frac{sG_F{}^2\langle m^2\rangle}{32\pi^4}\frac{(1-x)^{3.5}}{x}. \tag{16}$$

We will use this result in our calculations below. Note the appearance of the $1/x$ factor.

The contribution of meson annihilation to the differential cross section for low-mass muon pairs in neutrino scattering on a nucleon N is therefore

$$\frac{d^2\sigma}{dQ^2 d\xi} = \frac{4\pi\alpha^2}{3}\sum_{MM'}\int dx\,dy\,\delta(\xi-x+y)\delta(xy-Q^2/s)\left(\frac{Q^2}{s}\right)\left(\frac{1}{Q^4}\right)|F_{MM'}(Q^2)|^2 P_{M/\nu}(x)P_{M'/N}(y), \tag{17}$$

where Q is the pair mass, ξ is the longitudinal-momentum fraction of the pair (along the ν beam) in the center-of-momentum frame, and $F_{MM'}(Q^2)$ is the transition form factor for M-M' annihilation into an off-shell photon of mass Q^2 ($F=0$ if $M+M'$ is a state of nonzero charge, etc.)

A reliable lower bound in (17) may be found by restricting MM' to the states $\pi^\pm\pi^\mp$ so that $F=F_\pi$,

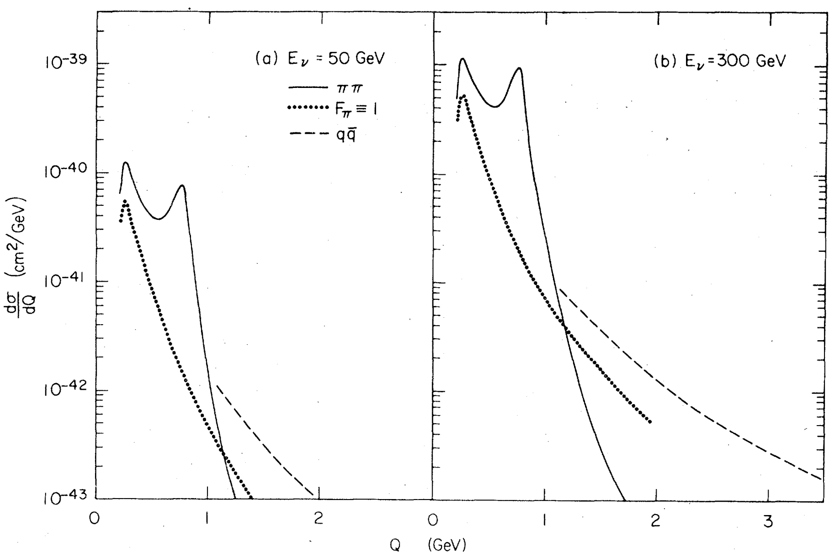

FIG. 4. Muon pairs produced by the $\pi\pi$ and $q\bar{q}$ (Drell-Yan) processes from neutrino beams. The dotted curve assumes a point pion ($F_\pi = 1$), and Q = pair mass. (a) 50-GeV incident neutrino beam. (b) 300-GeV incident neutrino beam.

the pion form factor with timelike argument. Using $P_{\pi/N}$ from Eq. (4) and $|F_\pi|^2$ as *parametrized* in Eq. (6), we have calculated this lower bound by numerical integration. We display the results as singly differential cross section $d\sigma/dQ^2$ in Fig. 4 for $E_\nu = 50$ and 300 GeV with $|F_\pi|^2$ set equal to unity in the dotted curves; the $\bar{q}q$ contribution at high Q^2 is shown by the dashed curves. Note that there is a crossover between these two contributions in the region of 1–1.5 GeV. This is the same behavior found in Sec. II for π-induced pairs. In Fig. 5, we show $d\sigma/d\xi$ with the same labeling conventions

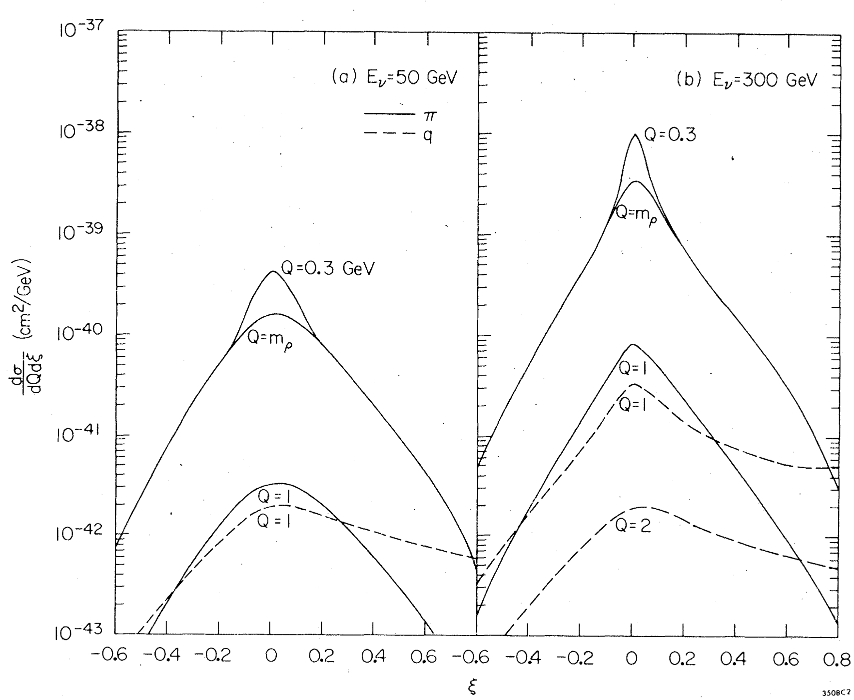

FIG. 5. The ξ distribution for muon pairs at selected pair masses Q. (a) 50-GeV incident neutrino beam. (b) 300-GeV incident neutrino beam.

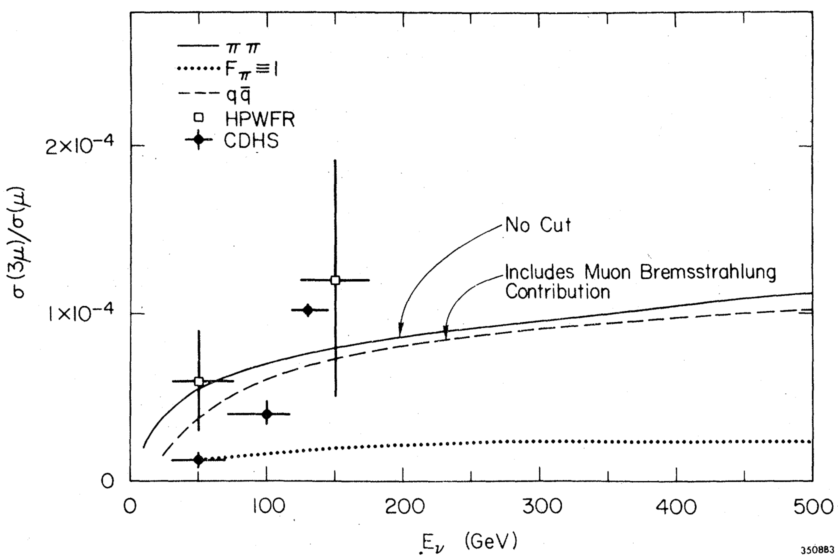

FIG. 6. Comparison of experimental data (Ref. 16) with theoretical calculations for $(3\mu/1\mu)$ ratio of total cross sections.

as in Fig. 4. Note the enhanced forward "throw" effect of the relatively hard $P_{q/\nu}$ distribution in the $\bar{q}q$ case.

When we integrate to form a total muon-pair cross section, we find the high-mass contribution negligible and a total value for the cross section which corresponds to a trimuon-to-charged-current cross-section ratio of

$$\frac{\sigma(\nu N\to \mu^-\mu^+\mu^- x)}{\sigma(\nu N\to \mu^- x)} \geqslant 10^{-4} \tag{18}$$

at very high energies. This is about a factor of 5 larger than would be the case with $|F_\pi|^2=1$ (no ρ form-factor effect). This result should be reduced in any comparison with uncorrected experimental values since our calculations have *not* included the effects of experimental cuts on the individual muon momenta. Our detailed results as a function of s are shown in Fig. 6, along with some recent data.[16]

For comparison with the result (18), we recall that Smith and Vermaseren,[17] and Barger, Gottschalk, and Phillips[18] find a cross-section ratio of $(5-7)\times10^{-5}$ (see Fig. 6) using a quark and muon bremsstrahlung calculation which also has a steeper mass dependence than our result at very low pair masses. (The muon contribution is clearly present in the data at about the right level.) Those calculations include the entire contribution of (quark) Feynman diagrams and so give larger results than Drell-Yan-type quark calculations. Nonetheless, they do not include the mesonic contributions that we have calculated, and since Fig. 4 showed that the quark and pionic annihilation contributions occur in complementary regions, the two curves may be added to give the total trimuon ratio. Associated charm production followed by semimuonic decay of *both* charmed particles yields a ratio of ~10^{-6} (see Ref. 18); this mechanism is suppressed both in production and by the small semimuonic charm decay branching ratios.

Since (18) is a lower bound, we conclude that the mechanism of meson annihilation explains a large fraction of the observed trimuon event rate in neutrino-nucleon scattering. This mechanism is best tested by comparing the data with our predictions for ξ and Q^2 dependence of the differential cross sections. Finally, we note that in a ratio such as (18) the absolute cross-section normalization cancels out. Thus we predict that the dimuon to no-muon ratio in neutral-current scattering is

$$\frac{\sigma(\nu N\to \nu\mu^+\mu^- x)}{\sigma(\nu N\to \nu x)} \gtrsim 10^{-4} \tag{19}$$

from the assumed mechanism. Unfortunately, charged-current charm production, occurring at a level of several percent, will completely swamp this source. The approximately 1% of observed $\mu^+\mu^-$ events which are due to the mechanism in Eq. (19) are, however, characterized by a low μ-pair mass ($\lesssim M_\rho$).

On the other hand, since $P_{q/\nu}$ given in Eq. (13) is not sensitive to whether we start with ν or $\bar{\nu}$, the smaller absolute $\bar{\nu}$ CC cross section imples that the $(3\mu/1\mu)$ ratio in $\bar{\nu}$ CC events will be ~2 times larger than for ν CC scattering.

IV. CONCLUSION

We have shown that the $\pi^+\pi^-$ parton-annihilation mechanism provides the largest single contribution to the production cross section in hadronic and neutrino-hadron scattering for low-mass low-p_T lepton pairs, and is consistent with the experimental data. In particular, the neutrino-induced charged-current trimuon–single-muon ratio of ~10^{-4} which was once of such great concern is well explained by our mechanism. It also predicts approximately the same for the neutral-current dimuon-muonless ratio and a factor of $\sigma^{\nu}(\mathrm{CC}/\sigma^{\bar{\nu}}\mathrm{CC})$ larger antineutrino-induced trimuon ratio.

The validity of the application of this mechanism may be further tested by comparing the ξ and Q^2 distributions predicted with other experiments. In particular, we note the following salient features: (1) The μ-pair mass distribution is very similar to that in π-p scattering. More than half of the events should have pair masses below 1 GeV and ~25% should be in the ρ mass region. (2) In the $W^{\pm}N$ center-of-momentum frame, the mean $Q_{\perp}$ (momentum transverse to the W direction) of the μ pair should rise to a limiting value of ~1.2 GeV/c (with a slight energy dependence) as the μ-pair masses increase above ~3 GeV, since this has been shown to occur for the corresponding case in pion-induced pair production based on the same meson annihilation mechanism. (3) The μ pairs from meson-meson annihilation tend to follow the hadronic-shower direction. In addition, of course, there are bremsstrahlung μ pairs associated with the leading muon direction.

Also, it should be noted that, except at resonances, the $\pi^+\pi^-$ annihilation mechanism predicts a $(1-\cos^2\theta)$ decay angular distribution for the virtual photon, which should be contrasted with the $(1+\cos^2\theta)$ distributions for $q\bar{q}$ annihilation.[19] Except on resonances, we expect $A(Q^2)$, where $(1+A\cos^2\theta)$ is fitted to the decay angular distribution, to fall from $\simeq +1$ in the high-mass continuum to a negative value (although possibly small in absolute value) for low ($\leq$1-GeV) pair masses.

Finally we note that if charm production occurs at the ~10% level in ν interactions, and since charmed mesons have ~10% semileptonic branching ratios, our mechanism for trileptons also predicts tetraleptons at the ~10^{-6} level.[20]

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