New limit on independence of charge and velocity

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Suppose that the charge of a slow electron (or proton) varies as $q = e(1 + kv^2/c^2)$, where $|k| \ll 1$ and v is the velocity of the moving charge. A strict limit on $|k|$ may be inferred from the observed neutrality of atoms. In particular this neutrality persists despite the moderately fast speeds of the protons in light nuclei. Using King's data on the neutrality of He and D_2, we determine that $|k| < 8 \times 10^{-19}$. This limit represents a factor of 250 improvement from a previous result.

Recently, Bartlett and Ward[1] have examined the hypothesis that the charge of a moving proton (or electron) is not constant but rather varies as

$$q = e(1 + kv^2/c^2)\,. \tag{1}$$

Here v is the velocity of the moving charge and k is a small constant. Various experiments that could set a limit on k were investigated. The most stringent limit was imposed by the observed neutrality of atoms. Specifically, an experiment of King[2] has shown that the charges of both molecular hydrogen and atomic helium are less than $10^{-20}e$.

Both He and H_2 are bound systems of two electrons. Since the motion of the electrons is quite different in the two systems, Bartlett and Ward concluded that $|k| < 2 \times 10^{-16}$ for *electrons* bound by Coulombic forces.

In this addendum we wish to show that a better limit on k may be set by considering the differing motion of the *protons* in light nuclei. This superiority arises because the Fermi motion of the protons in light nuclei is faster than the orbital motion of electrons in light atoms. Applying Eq. (1) to the case of a general atom, we have

$$Q(\text{atom}) = Z\Delta q_0 + Nq_n + Zke\,c^{-2}(v^2_{pA} - v^2_{eA})\,. \tag{2}$$

Here v^2_{pA} and v^2_{eA} are the mean square speed of protons and electrons in the atom, respectively; Δq_0 is a possible small difference between the static charge of the proton and that of the electron; and q_n is a possible small neutron charge.

Evidently the possibility that either Δq_0 or $q_n \neq 0$ makes it impossible to set a stringent limit on k from the observed neutrality of a single atom. However, by finding several atoms to be neutral, one can eliminate the possibility of a nonzero neutron charge or proton-electron charge difference. Particularly convenient is a comparison of the charge of the helium atom with that of the deuterium molecule. Both He and D_2 are bound systems consisting of two electrons, two protons, and two neutrons. Thus the difference between the charges of these two systems is independent of Δq_0 or q_n.

Furthermore, for both these systems $v^2_{eA} \ll v^2_{pA}$. Applying Eq. (1) to both He and D_2 and solving for k, we then have

$$k = \frac{Q(\text{He}) - Q(D_2)}{2e(v^2_{p\text{He}} - v^2_{p\text{D}})c^{-2}}\,. \tag{3}$$

King has made sensitive measurements of both $Q(\text{He})$ and $Q(\text{D}_2)$ using the gas-efflux technique. In this method the charge inside a metallic container of gas is monitored as the contents are exhausted. The charge is thus measured by Gauss's law, a method most compatible with Purcell's definition of a moving charge.[3] King[4] finds

$$Q(\text{He}) = (-0.7 \pm 4.7) \times 10^{-20}e$$

and

$$|Q(\text{D}_2)| < 3 \times 10^{-20}e\,.$$

Since the helium nucleus is much more tightly bound than is the deuterium nucleus, one would expect that $v^2_{p\text{He}} > v^2_{p\text{D}}$. These velocities may be estimated from the uncertainty principle. If the uncertainty in the momentum of the proton is Δp and the uncertainty in its position is Δx, then $\Delta p = \hbar/\Delta x$. Identifying Δp with mv and Δx with the radius of the nucleus r, we have

$$v^2 \approx \hbar^2/m^2r^2\,, \tag{4}$$

where m is the mass of the proton. Evaluating this expression for the mean square velocity of the proton in the helium nucleus ($r = 1.64$ fm) and in the deuteron ($r = 2.095$ fm) gives $v^2/c^2 = 0.016$ and 0.010, respectively.[6] Substituting these values into Eq. (3) we find a crude limit for k:

$$k \leq 3\times 10^{-20}/2(0.016-0.010) = 2.5\times 10^{-18}.$$

To obtain a better estimate of v^2 (and hence of k), we have evaluated the expectation value of the kinetic energy for the proton relative to the residual core of either nucleus. The proton-core wave function is assumed to be of the Hulthén form:

$$\varphi(r) = \frac{[2\alpha\beta(\alpha+\beta)]^{1/2}}{\beta-\alpha}\,\frac{e^{-\alpha r}-e^{-\beta r}}{r}, \tag{5}$$

where $\alpha = 0.86$ fm^{-1} and $\beta = 0.93$ fm^{-1} for a proton in the He nucleus. Similarly, $\alpha = 0.23$ and $\beta = 1.61$ for a proton in the deuterium nucleus. These values are determined by simultaneously requiring the correct long-range form of the wave function and the correct rms charge radius (see, e.g., Ref. 6).

The operator for kinetic energy is $T = -(\hbar^2/2\mu)\nabla^2$, where μ is the reduced mass of the proton-core system. Thus

$$\langle T\rangle = \frac{-\hbar^2}{2\mu}\,\frac{2\alpha\beta(\alpha+\beta)}{(\beta-\alpha)^2} \times \int_0^\infty (e^{-\alpha r}-e^{-\beta r})(\alpha^2 e^{-\alpha r}-\beta^2 e^{-\beta r})\,dr. \tag{6}$$

The integral may be evaluated explicitly, giving (after some cancellation)

$$\langle T\rangle = \frac{\hbar^2\alpha\beta}{2\mu}. \tag{7}$$

Now $\langle T\rangle$ is the sum of the expectation value of the kinetic energy of the proton $\langle T_p\rangle$ and that of the residual core $\langle T_c\rangle$. Thus

$$\langle T_p\rangle = \langle T\rangle - \langle T_c\rangle = \frac{\mu}{m_p}\langle T\rangle = \frac{\hbar^2\alpha\beta}{2m_p}. \tag{8}$$

Finally the velocity of the proton is given by

$$v^2/c^2 = (2/m_p c^2)\langle T_p\rangle = \frac{\hbar^2\alpha\beta}{m_p^2 c^2}. \tag{9}$$

Thus for each of the two protons in the helium nucleus we have $v^2_{p\mathrm{He}}/c^2 = 0.036$, whereas, for the protons in the deuterium nucleus $v^2_{p\mathrm{D}}/c^2 = 0.017$. Substituting these values into Eq. (3) we find that

$$k \leq 3\times 10^{-20}/2(0.036-0.017) = 8\times 10^{-19}e.$$

This limit is a factor of 250 more stringent than that obtained in Ref. 1. It should be emphasized, however, that this limit applies only to protons *bound* by strong forces in the nucleus. It is conceivable that in this binding a renormalization of charge can occur which would cancel the effect of a v^2/c^2 term.

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[1]D. F. Bartlett and B. F. L. Ward, Phys. Rev. D 16, 3453 (1977). Note there is a misprint in the abstract of this article. The most stringent limit on k is 2×10^{-16} not 2×10^{-15}.

[2]J. G. King, Phys. Rev. Lett. 5, 562 (1960), with improved experimental result as quoted in H. F. Dylla and J. G. King, Phys. Rev. A 7, 1224 (1973).

[3]E. M. Purcell, *Electricity and Magnetism* (McGraw-Hill, New York, 1963), pp. 151–154.

[4]The charge of helium as quoted in Dylla and King, Ref. 2; the charge of deuterium as quoted in C. G. Shull, K. W. Billman, and F. A. Wedgwood, Phys. Rev. 153, 1415 (1967).

[5]For the radii of d and α, see F. Boehm, At. Data Nucl. Data Tables 14, 479 (1974).

[6]J. R. Shepard, W. R. Zimmerman, and J. J. Kraushaar, Nucl. Phys. A275, 189 (1977).