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Singular classical solutions of Euclidean field theories

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The classical solutions of the equations of motion are studied in some Euclidean field theory models either conformal invariant or noninvariant. In the conformal-invariant models a virial theorem for merons is derived which is of the same form as the known one for instantons. Some examples of singular solutions are discussed. An interesting relation seems to hold between the local symmetry properties of the singular solutions and the degree of divergence of the Euclidean action.

I. INTRODUCTION

Classical solutions of equations of motion in quantum field models may help us understand nonperturbative quantum properties. Although the emphasis has been on regular and sourceless solutions with finite energy (solitons) or Euclidean action (instantons), solutions with logarithmically divergent action (merons) have also been considered interesting in the approximate evaluation of the Feynman path integral as well as solutions of gauge fields in the presence of sources in studying the dynamics of heavy quarks. Thus it seems desirable to understand the properties of the general classical solution in Euclidean space.[1]

In Sec. II we derive virial-type theorems[2] for meronlike solutions. As in the case of instantons the theorem is useful in excluding solutions for some models in certain space-time dimensions or in finding exact or approximate solutions when they exist. Virial theorems which may have a similar use are also easily derived for higher moments of the energy-momentum tensor. We discuss the properties of meron solutions in two models: a scalar self-interacting multiplet and the CP^{N-1} model. In Sec. III we indicate that some of the meron properties are lost by the singular solutions of nonconformal models. This is illustrated by two examples in $\mathbf{R}^3$: a scalar self-interacting massless multiplet and a similar multiplet interacting with a Yang-Mills field.

Finally let us remark that there seems to be a connection between the symmetry properties of a classical solution and the degree of divergence of its Euclidean action. It is well known[8,9] that in the conformal ϕ^4 model in Euclidean $\mathbf{R}^4$, the only solution invariant under O(5), the instanton, has a finite action, while the meron solution, invariant under $O(4)\times O(2)$ only, has a logarithmically divergent action.

Moreover, the general solution with O(4) symmetry only[10] which is computed in terms of Jacobi elliptic functions has an action at least logarithmically divergent. Similar features are observed in pure Yang-Mills theory by using the ansatz[11] that maps solutions of the conformal ϕ^4 model into solutions of Yang-Mills theory. Very few solutions are known[12] which have less than O(4) invariance, and they have an action divergent as a power of the cutoff.

It may be useful to stress that the symmetry property of the classical solutions that we are discussing is a local symmetry property of the solution in a region where the physical densities are subtanstially different from zero, and we do not refer to the global symmetry property which may be less significant. For instance, by use of conformal symmetry a meron-meron pair solution that has one center at the origin and the other at infinity [and therefore $O(4)\times O(2)$ global symmetry] may be converted into a solution with both centers at finite points, thus reducing the global symmetry property of the solution, but not its local properties around the centers.

II. MERON SOLUTIONS IN CONFORMAL MODELS

Let us recall the Laue theorem[5,6] in its simplest form which is useful for regular time-independent solutions (static solitons and instantons). Let $\mathcal{L}[\phi, x]$ denote the Lagrangian of a set of fields ϕ in Euclidean $\mathbf{R}^n$ space and $T_{\mu\nu}[\phi, x]$ the energy-momentum tensor (it may be the canonical, Belinfante, or the "improved" one). The energy-momentum conservation law

$$\partial_\mu T_{\mu\nu}[\phi, x] = 0 \tag{2.1}$$

implies

$$\partial_\alpha (x_\mu T_{\alpha\nu}) = T_{\mu\nu}\,. \tag{2.2}$$

If (2.2) is integrated over all Euclidean $\mathbf{R}^n$ space and if the regularity and asymptotic properties of the fields allow one to neglect the surface contribution of the current $J^{\alpha\mu\nu} \equiv x^\mu T^{\alpha\nu}$, one obtains

$$\int d^n x\, T_{\mu\nu}[\phi, x] = 0 \quad (\mu, \nu = 1, \ldots, n)\,. \tag{2.3}$$

The usual form of the virial theorem[2] follows by taking the trace of Eq. (2.3),

$$\int d^n x\, T_{\mu\mu}[\phi, x] = 0 \ , \tag{2.4}$$

and the more detailed theorems obtained by non-isotropic scale transformations[4] correspond to linear combinations of Eq. (2.3).

Of course one may easily obtain infinitely many other relations of the virial type by partial integration,

$$\int d^n x\, T_{\mu\nu} \partial_\mu f_\alpha(x_1, \ldots, x_n) = 0 \ , \tag{2.5}$$

provided that $f_\alpha(x_1, \ldots, x_n)$ is a set of smooth functions and the integrals converge. Equation (2.5) contains both the local property (2.1) and the asymptotic properties of the energy-momentum density tensor.

If the set f_α is chosen to be a complete set of orthonormal functions in $\mathbf{R}^n$, Eqs. (2.5) are well defined and fully equivalent to Eq. (2.1). Sometimes the first few equations are helpful in the search for approximate solutions which do not have simple symmetry properties.[13]

We shall now extend this theorem to the singular solutions often called merons. Such solutions have only been studied in some conformal models: nonlinear σ models in $\mathbf{R}^2$,[14] massless ϕ^4 theory in $\mathbf{R}^4$,[9] pure Yang-Mills theory in $\mathbf{R}^4$,[9,15] Yang-Mills field coupled with a scalar multiplet in $\mathbf{R}^4$,[16] massless scalars interacting with massless fermions in $\mathbf{R}^4$.[17] Multimeron solutions in these models have also been studied.[18]

Most of the single-meron-pair solutions here mentioned have the following properties:

(i) The Lagrangian density is regular and has a finite integral over any region of Euclidean space-time excluding the neighborhoods of two points which we call centers of the meron pair. These two regions give logarithmically divergent contributions.

(ii) In non-Abelian gauge models the gauge field is proportional to a pure gauge; in some models the topological charge density is concentrated at the centers of the meron pair.

(iii) The meron solutions have a large group of symmetry. For instance, in models in Euclidean $\mathbf{R}^4$ space invariant under the conformal group O(5, 1) (generated by $M_{\mu\nu}$, P_μ, K_μ, and D), while the instanton solutions are invariant under the subgroup O(5) generated by $M_{\mu\nu}$ and $R_\mu = P_\mu + K_\mu$, the meron solutions are invariant under the subgroup generated by $M_{\mu\nu}$ and D.

Yet no single such property holds for all the above-mentioned solutions (in fact no general characterization of meron solutions seems to exist).

Property (ii) is probably the less appropriate not only because one may be interested in classical solutions (regular or otherwise) in models that have no topological number but mainly because it was shown that for a singular solution the topological number may be changed (by a singular gauge transformation) without affecting the type of singular behavior of the solution[15] (this does not happen for regular solutions). One may also notice that properties (iii) and (i) seems to be closely related. We then adopt a definition based on property (i) and, furthermore, we require a specific *local* behavior of any symmetric traceless energy-momentum tensor $T^{\mu\nu}$, in the regions where the solution is singular. Such local behavior is suggested by properties (i) and (iii) and indeed it holds for all known solutions where (i) and/or (iii) hold. However, it is more convenient than the above properties in looking for exact or approximate solutions in less simple cases (for instance multimerons). Specifically we shall here consider the sourceless solutions of conformal field theories in the Euclidean $\mathbf{R}^n$ space everywhere regular except for two points, the centers of the meron pair. In the neighborhoods of each center, say $x_\mu \simeq a_\mu$, we assume that the energy-momentum tensor is

$$T_{\mu\nu}[\phi, x] \simeq \left[\delta_{\mu\nu} - n\,\frac{(x_\mu - a_\mu)(x_\nu - a_\nu)}{(x_\alpha - a_\alpha)^2}\right] \times f((x_\alpha - a_\alpha)^2) \, . \tag{2.6}$$

The conservation of the energy-momentum tensor implies that

$$f((x_\alpha - a_\alpha)^2) = \frac{c_a}{[(x_\alpha - a_\alpha)^2]^{n/2}} \ , \tag{2.7}$$

c_a being a constant. One might check for example that the solutions of the conformally invariant ϕ^4 theory in $\mathbf{R}^4$ which have O(4) symmetry[10] (but not a higher one) do not allow the local representation [(2.6) and (2.7)] even in the cases where the action is minimally (i.e., logarithmically) divergent.

By using conformal symmetry, one of the centers of the action density can be shifted to the origin and the second to infinity. Then it is plausible that the assumed local behavior (2.6) holds globally:

$$T_{\mu\nu}[\phi, x] = \left(\delta_{\mu\nu} - n\,\frac{x_\mu x_\nu}{r^2}\right)\frac{c}{r^n} \ . \tag{2.8}$$

Indeed, Eq. (2.8) holds for every known meron solution, but it will not be needed in the derivation of the virial theorem.

We shall now easily prove the virial theorem for merons. Let us call a_μ and b_μ the centers of the meron pair and σ_a and σ_b two small spheres with centers in a_μ and b_μ. By integrating Eq. (2.2) over all $\mathbf{R}^n$ space and using the divergence theorem we have

$$\int d^n x\, T_{\mu\nu} = \int_{S_\infty} d\sigma_\alpha J_{\alpha\mu\nu} + \int_{\sigma_a} d\sigma_\alpha J_{\alpha\mu\nu} + \int_{\sigma_b} d\sigma_\alpha J_{\alpha\mu\nu}. \tag{2.9}$$

The integral over the surface at infinity S_∞ vanishes because of the meron property that the action density is integrable over any domain that excludes the centers of the meron pair. By using (2.6) and (2.7) we obtain

$$\begin{aligned}\int d^n x\, T_{\mu\nu} &= (1-n)c_a \int d\Omega_n \frac{y_\nu(y_\mu - a_\mu)}{y^2} \\ &\quad + (1-n)c_b \int d\Omega_n \frac{(y_\mu - b_\mu)y_\nu}{y^2} \\ &= \frac{(1-n)}{n}\Omega_n \delta_{\mu\nu}(c_a + c_b),\end{aligned} \tag{2.10}$$

where y_μ is the coordinate with respect to one of the centers of the meron pair and Ω_n is the total solid angle in $\mathbf{R}^n$. Now by taking the trace of Eq. (2.10), conformal symmetry implies $c_a = -c_b$, therefore

$$\int d^n x\, T_{\mu\nu}[\phi, x] = 0 \quad (\mu, \nu = 1, \ldots, n). \tag{2.11}$$

The virial theorem for merons then has the same form as for instantons. Of course it will not be useful here to consider the trace relation, as in Eq. (2.4), because by conformal symmetry this is a trivial identity.

As an example of the use of this theorem, let us consider a set of massless scalar fields ϕ_a $(a = 1, \ldots, 4)$, which transforms as a vector under the internal-symmetry group O(4) in the Euclidean $\mathbf{R}^4$ space with the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \phi_a \partial_\mu \phi^a + \tfrac{1}{4}\lambda(\phi_a\phi^a)^2. \tag{2.12}$$

The proper energy-momentum tensor here is the improved one,[19]

$$T_{\mu\nu} = \partial_\mu\phi_a\partial_\nu\phi^a - \delta_{\mu\nu}\mathcal{L} + \tfrac{1}{6}(\delta_{\mu\nu}\Box - \partial_\mu\partial_\nu)(\phi_a\phi^a) \tag{2.13}$$

because we need $T_{\mu\mu} = 0$, but the improvement gives no contribution after integration. Then Eq. (2.11) yields

$$\begin{aligned}\int d^4x(\partial_x\phi_a)^2 &= \int d^4x(\partial_y\phi_a)^2 = \int d^4x(\partial_z\phi_a)^2 \\ &= \int d^4x(\partial_t\phi_a)^2 = -\tfrac{1}{4}\lambda \int d^4x(\phi_a\phi^a)^2\end{aligned} \tag{2.14}$$

and

$$\int d^4x\, \partial_\mu\phi_a\partial_\nu\phi^a = 0 \quad (\mu \neq \nu). \tag{2.15}$$

Hence meron solutions may only exist for negative λ. If we look for a meron solution with a center in the origin and the other at infinity, the global assumption (2.8) yields

$$\begin{aligned}\left(\delta_{\mu\nu} - \frac{4x_\mu x_\nu}{r^2}\right)\frac{c}{r^4} &= \partial_\mu\phi_a\partial_\nu\phi^a - \partial_{\mu\nu}\mathcal{L} \\ &\quad + \tfrac{1}{6}(\Box\delta_{\mu\nu} - \partial_\mu\partial_\nu)(\phi_a\phi^a)^2.\end{aligned} \tag{2.16}$$

Analogous equations may be obtained for Yang-Mills systems and they are of first order.

Two simple (sourceless) solutions are obtained from (2.12) and (2.16):

$$\phi_a = \frac{\delta_{a1}}{\sqrt{-\lambda}}\frac{1}{r} \quad (a = 1, \ldots, 4), \tag{2.17}$$

which is well known,[9] and

$$\phi_a = \pm\frac{2}{\sqrt{-\lambda}}\frac{x_a}{r^2} \quad (a = 1, \ldots, 4). \tag{2.18}$$

The topological charge of a Higgs multiplet ϕ_a is usually calculated from the Kronecker index of the normalized vectorial field $\hat{\phi}_a = \phi_a/(\phi_b\phi^b)^{1/2}$. This, however, requires regularity properties for the modulus $(\phi_a\phi^a)^{1/2}$ which do not hold in the solution (2.18). Every regularization of (2.18) would produce a unit topological charge.

The meron solution (2.17) is invariant under an $O(4)\times O(2)$ group, where O(4) is generated by the four-dimensional rotation operators and O(2) is generated by $D = x_\mu\partial_\mu + 1$. In the case of meron solution (2.18), since space and internal indices are mixed, the space-time rotations have to be supplemented by similar transformations in the internal space, so that (2.18) is still invariant under an $O(4)\times O(2)$ group where now O(4) corresponds to the "complete" (i.e., space plus internal) four-dimensional rotations and O(2) is generated by $D = x_\mu\partial_\mu + 1$. As another example of a meron solution in conformal models, we consider the two-dimensional Euclidean CP^{N-1} nonlinear σ models[20] with Lagrangian density

$$\mathcal{L} = \partial_\mu\bar{z}_\alpha\partial_\mu z^\alpha + (\bar{z}_\alpha\partial_\mu z^\alpha)^2, \tag{2.19}$$

where $z^\alpha(x)$ $(\alpha = 1, \ldots, N)$ is a complex N-component field satisfying the constraint $\bar{z}_\alpha z^\alpha = 1$.

A meron solution which is a simple generalization of that of the O(3) σ model[14] is

$$z_\alpha(x) = \frac{1}{\sqrt{2}}(e^{-i\theta/2}u_\alpha + e^{i\theta/2}v_\alpha), \tag{2.20}$$

where $\theta = \arg(x_1 + ix_2)$ and $\bar{u}_\alpha u^\alpha = \bar{v}_\alpha v^\alpha = 1$, $\bar{u}_\alpha v^\alpha = 0$.

It has a topological charge density

$$q(x) \equiv \frac{1}{2\pi i}\,\partial_\mu(\bar{z}_\alpha \epsilon_{\mu\nu}\partial_\nu z^\alpha) = \tfrac{1}{2}\delta^2(x)\,; \tag{2.21}$$

the canonical energy-momentum tensor is

$$T_{\mu\nu} = \left(\delta_{\mu\nu} - 2\,\frac{x_\mu x_\nu}{x^2}\right)\frac{1}{4x^2} \tag{2.22}$$

and the Lagrangian density is

$$\mathcal{L}(x) = \frac{1}{2x^2}\,. \tag{2.23}$$

III. SINGULAR SOLUTIONS IN NONCONFORMAL MODELS

It is clear that the singular solutions of equations of motion of nonconformal model lack most of the properties of meron solutions in conformal models. However, nonconformal models also are interesting and the singular solutions that we describe in this section are of obvious relevance in the study of the general solution of the classical equations of motion.

First we consider a scalar massless multiplet ϕ_a $(a=1,\ldots,n)$ which transforms as a vector under the internal-symmetry group O(n) in the Euclidean $\mathbf{R}^n$ space described by the Lagrangian density

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi_a\partial_\mu\phi^a + \tfrac{1}{4}\lambda(\phi_a\phi^a)^2\,. \tag{3.1}$$

With a radial ansatz $\phi_a = (x_a/r)f(r)$, $r=(x_\mu x_\mu)^{1/2}$, one has the equation of motion

$$\frac{d^2f}{dr^2} + \frac{(n-1)}{r}\,\frac{df}{dr} - \frac{(n-1)}{r^2}\,f - \lambda f^3 = 0\,. \tag{3.2}$$

By a change of variables $g=rf$, $r=e^z$ one obtains the autonomous equation

$$\frac{d^2g}{dz^2} + (n-4)\,\frac{dg}{dz} - 2(n-2)g - \lambda g^3 = 0\,. \tag{3.3}$$

The associated autonomous system of first-order differential equations

$$\begin{aligned} \frac{dg}{dz} &= y\,,\\ \frac{dy}{dz} &= (4-n)\,y + 2(n-2)g + \lambda g^3 \end{aligned} \tag{3.4}$$

has three singular points $(dg/dz = dy/dz = 0)$:

$$g=0,\quad g=\pm\left[\frac{2(2-n)}{\lambda}\right]^{1/2}, \tag{3.5}$$

which yield the singular solution

$$\phi_a = \pm\left[\frac{2(2-n)}{\lambda}\right]^{1/2}\frac{x_a}{r^2}\,. \tag{3.6}$$

For $n=4$ the model is conformal invariant and the singular solution is the meron already mentioned in the preceding section. For $n=3$, Eq. (3.3) is essentially the equation discussed by Wu and Yang[21] that describes static solutions of a pure Yang-Mills system in $\mathbf{R}^3$. We note some obvious properties of the solution (3.6) in $\mathbf{R}^3$:

(a) It is invariant under the group $O(3)\times O(2)$ generated by the usual "complete" operator $M_{\mu\nu}$ and $\tilde{D}$ (where $\tilde{D} = x_\mu\partial_\mu + 1$ while the canonical dimension of ϕ_a in $\mathbf{R}^3$ is $\frac{1}{2}$).

(b) The density of the canonical energy-momentum tensor vanishes.

(c) The Euclidean Lagrangian density is regular everywhere in $\mathbf{R}^3$ except at the origin; it diverges linearly when integrated in a domain that includes the origin.

(d) The same comment made on the topological charge of the solution (2.18) holds here.

As a second example of singular solution for a nonconformal theory, we consider the SU(2) Yang-Mills field coupled with an SU(2) Higgs field in Euclidean $\mathbf{R}^3$ space. For convenience, as in the previous example, the scalar multiplet is taken to be massless. With the usual radial ansatz

$$\begin{aligned} A_i^a &= \epsilon_{aij}x_j\,\frac{1-K(r)}{er^2}\,,\\ \phi_a &= x_a\,\frac{H(r)}{er^2}\,, \end{aligned} \tag{3.7}$$

the Euclidean action is

$$A = -\frac{4\pi}{e^2}\int_0^\infty dr\left[(K')^2 + \frac{(K^2-1)^2}{2r^2} + \frac{H^2K^2}{r^2} + \frac{(rH'-H)^2}{2r^2} + \frac{\lambda H^4}{4e^2r^2}\right], \tag{3.8}$$

and the field equations are

$$\begin{aligned} r^2K'' &= K(K^2-1) + KH^2\,,\\ r^2H'' &= 2HK^2 + \frac{\lambda}{e^2}H^3\,. \end{aligned} \tag{3.9}$$

After the change of variable $r=e^z$ we have the autonomous system

$$\begin{aligned} \frac{dK}{dz} &= Y\,,\\ \frac{dY}{dz} &= K(K^2-1) + KH^2\,,\\ \frac{dH}{dz} &= W\,,\\ \frac{dW}{dz} &= 2HK^2 + \frac{\lambda}{e^2}H^3\,. \end{aligned} \tag{3.10}$$

There are two sets of singular points $(dK/dz = dY/dz = dH/dz = dW/dz = 0)$:

$$H=0, \quad K=0, \pm 1, \tag{3.11}$$

$$H^2=\frac{2e^2}{2e^2-\lambda}, \quad K^2=\frac{-\lambda}{2e^2-\lambda}. \tag{3.12}$$

The first set implies the vanishing of the Higgs fields and then yields the singular points of the self-coupled Yang-Mills system discussed by Wu and Yang.[21] We just recall that $K^2=1$ implies $F^a_{ij}=0$, while $K=0$ provides the singular solution

$$H^a_l \equiv \tfrac{1}{2}\epsilon_{ijl}F^a_{ij}=-\frac{x_a x_l}{er^4}. \tag{3.13}$$

The second set provides, for negative λ, singular solutions that have interesting properties:

(a) The non-Abelian gauge field is proportional to a pure gauge:

$$A_i \equiv e\frac{A^a_i\sigma_a}{2i}=\frac{1-K}{2}U^{-1}\partial_i U,$$

where $U=i\sigma_k x_k/r$.

(b) The solution is invariant under the group $O(3)\times O(2)$ generated by the "complete" rotation operators and by $\tilde{D}=x_K\partial_K+1$.

(c) The density of the symmetric (Belinfante) energy-momentum tensor vanishes.

(d) The Euclidean Lagrangian density is everywhere regular except at the origin; it diverges linearly if it is integrated in a domain that includes the origin.

(e) The same comment made on the topological charge of the solution (2.18) holds here.

(f) This solution, like the solution of the previous example, is closely related to the Wu-Yang solutions[21] of static Yang-Mills fields and has the same source problem. It is easily seen that the radial equations have a source proportional to $\delta^3(r)$ and therefore the solution (3.12) actually solves the equation of motion of the field A^a_i in the presence of a source proportional to $J^a_i=\epsilon_{aij}(x_j/r)\delta^3(r)$ which is obviously ill-defined.[22] One may note that by taking $K=1$, $H\neq 0$, the gauge fields A^a_i vanish so the previous example is recovered.

As a last example we consider the same system allowing for massive Higgs fields, restricting to the simpler configuration with $K=0$. Then the only equation is

$$r^2H''=\frac{\lambda}{e^2}H^3-\mu^2Hr^2, \tag{3.14}$$

where μ is the Higgs mass.

The powerlike solution $H(r)=\pm(e\mu/\sqrt{\lambda})r$ yields $\phi_a(x)$ with spherical and scale symmetry (although with noncanonical scale dimension). Now the Euclidean action is divergent both at the origin and at infinity.

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[1]We use a metric $g_{\mu\nu}=\delta_{\mu\nu}$.

[2]Virial theorems were derived by isotropic (see Ref. 3) or nonisotropic (see Ref. 4) scale transformations or as simple consequences of the conservation of the energy-momentum tensor (see Refs. 5 and 6). Other theorems that prove the nonexistence of time-dependent nondissipative lumps in certain models are in Ref. 7.

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