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Nonlinear photons in the universe

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A nonlinear theory of electrodynamics is generated by nonminimal coupling with gravitation. As a result, the photon acquires a mass which depends on the value of the scalar of curvature. A homogeneous isotropic/anisotropic universe filled with such nonlinear photons is presented.

I. INTRODUCTION

One of the most remarkable consequences of the theory of general relativity was the prediction of the bending of light in a gravitational field. This proves that the photon has a passive gravitational energy and, consequently, by means of the equivalence principle, it must have an active gravitational energy, too, which should be responsible for the curvature of space-time generated by photons. Such an influence of gravitation on the behavior of light was shown by Eddington in a memorable exposition.[1] Since then, it has been the subject of a large number of experimental observations. However, the set of equations which governs the coupled system of electrodynamics and gravitation may still be in an incomplete form: We do not really know the behavior of electric and magnetic fields in a strong gravitational field.

In order to arrive at a reasonable theoretical framework, one starts by assuming the so-called minimal coupling principle. Owing to this principle, the equations of motion of electrodynamics in a curved space can be obtained from Maxwell's equations in a flat Minkowski universe without ambiguity. Additional terms which contain the curvature of space-time are usually disregarded for *a priori* reasons, such as the difficulty of compatibility of such terms with charge conservation, for instance, or the high degree of arbitrariness contained in such a coupling.

Irrespective of these arguments, we start here an exhaustive and systematic analysis of nonminimal coupling between vector W_μ and tensor $g_{\mu\nu}$ fields. The main reason which induced us to undertake such a study is linked to the subject of nonlinearities in electrodynamics. These nonlinearities are contained naturally in such nonminimal coupling.

Traditionally, nonlinearities in electrodynamics are introduced either by an *ad hoc* assumption (e.g., Born and Infeld) or by introduction of quantum effects (Euler and Heisenberg; see also Akhiezer and Berestetski[2] for a more detailed discussion). The general belief was that the nonlinear theory could change drastically the properties of the field in the neighborhood of an electron, and so could provide a successful electron model.

Other models, however, have been proposed with different leitmotives. It seems worthwhile to recall here a recent interesting suggestion which adds to the usual Maxwell equations a term derived from a Lagrangian of the type $L=\lambda(W_\mu W^\mu)^2$, where λ is a constant. Such a term, which further breaks gauge invariance, can give rise to an inelastic photon-photon interaction. This term has been used tentatively to give an alternative explanation to galactic red-shift anomalies.[3,4] The theory we will explore here breaks the gauge invariance, too. To first order in the curvature, we can have two possibilities, either

$$L_{\mathrm{I}}=\sqrt{-g}\,R\,W_\mu W_\nu g^{\mu\nu}$$

or

$$L_{\mathrm{II}}=\sqrt{-g}\,R_{\mu\nu}W^\mu W^\nu .$$

In the present paper we will limit our discussion to the case in which the Lagrangian L_{I} has to be added to Maxwell's electrodynamics.

The effects of adding such a term to the equation of motion can be interpreted as giving to the photon a mass proportional to the scalar of curvature. As a consequence, the theory will also break conformal invariance. The discussion on massive electrodynamics has an extensive bibliography (see Ref. 4 for an up-to-date review). However, as we will see in this paper, the mass $m_\gamma \sim R^{1/2}$ introduces some very peculiar properties not contained in the usual models.

II. THE MODEL

The equations of motions are obtained from the Lagrangian L, which consists of three parts,

$$L=L_A+L_B+L_C, \tag{1}$$

where

$$L_A = \frac{1}{k}\sqrt{-g}\,(1+\lambda W_\mu W^\mu)R,$$

$$L_B = -\tfrac{1}{2}\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu},$$

$$L_C = L_{\text{matter}},$$

in which $F_{\mu\nu} = W_{\mu\|\nu} - W_{\nu\|\mu}$, λ is a constant with the same dimensionality as Einstein's coupling constant k, that is $(\text{energy})^{-1}\times(\text{length})$, and R is the scalar curvature defined by

$$R = R_{\mu\nu\alpha\beta}\,g^{\nu\beta}g^{\mu\alpha};$$

the double bar represents a covariant derivative. From the Lagrangian (1), by variation of $g_{\mu\nu}$, we obtain the equations of motion

$$(1+\lambda W^2)G_{\mu\nu} - \lambda\Box W^2 g_{\mu\nu} + \lambda W^2{}_{|\mu\|\nu} + \lambda R W_\mu W_\nu = -kE_{\mu\nu} - kT^*_{\mu\nu}, \quad (2)$$

in which $T^*_{\mu\nu}$ represents the stress-energy tensor of the matter, W^2 is the norm $W_\mu W^\mu$, and $E_{\mu\nu}$ is Maxwell's tensor

$$E_{\mu\nu} = F_{\mu\alpha}F^{\alpha}{}_{\nu} + \tfrac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}. \quad (3)$$

By variation of W^μ in (1) we obtain

$$F^{\mu\nu}{}_{\|\nu} = -\frac{\lambda}{k}RW^\mu + J^\mu, \quad (4)$$

in which we have added an extra current J^μ. Taking the divergence of expression (4) yields the modified law of charge conservation

$$J^\mu{}_{\|\mu} - \frac{\lambda}{k}(RW^\mu)_{\|\mu} = 0. \quad (5)$$

At this point we can follow two distinct procedures: Either we assume that charge is conserved, and thus impose the constraint $(RW^\mu)_{\|\mu}=0$, or we allow for charge creation by the gravitational field. In this latter case the number of created particles depends on the value of the scalar of curvature through Eq. (5). Note that creation of charge in our model can occur only at a region of curved space-time where the scalar of curvature is not null. This condition, of course, is not a sufficient condition for particle creation, but it is a necessary one.

The effect of a breakdown of charge conservation on a cosmological scale was analyzed, some years ago, by Lyttleton and Bondi[5] and criticized by Hoyle.[6] The essential idea of the Lyttleton-Bondi (LB) analysis rests on the observation that a slight difference in the magnitude of electric charges of the proton and the electron could give rise to a repulsive force. On a cosmic scale, the result could be an alternative explanation to the observed expansion of the universe. The modification suggested by LB consists in adding a mass term $\epsilon W_\mu W^\mu$ to Maxwell's Lagrangian, allowing for a non-null divergence of the potential vector W^μ. Then they construct a cosmological solution of a universe filled with such massive photons. The result is a steady-state (de Sitter-type) cosmological configuration.

Hoyle[6] in a subsequent paper has shown that the Lyttleton-Bondi model is equivalent to the introduction of a fluid with negative energy that could be constructed with a scalar field. As a consequence, the equation of motion which gives the behavior of LB electrodynamics in an expanding steady-state homogeneous and isotropic universe is similar to the equation of Hoyle's C field, which is responsible for matter creation. Thus, the effect of the proposed modification of electrodynamics through the Lyttleton-Bondi hypothesis is indistinguishable—with respect to cosmic effects—from Hoyle's model of continuous creation of matter.

Although there is a point of contact with the Lyttleton-Bondi scheme of modified electrodynamics, the model we investigate here is very distinct from their proposal. The crucial differences is contained in the introduction of nonlinearities through the dependence of the mass term on the scalar of curvature. Actually, many new features appear in our model which have no equivalent in LB's. For instance, as we will show next, our model does not admit a cosmological steady-state configuration. Such a solution, which is a typical property of the Lyttleton-Bondi model, is indeed the main point of contact of the LB model and Hoyle's version of continuous creation of matter.

Let us come back now to our Eq. (2). Taking the trace of this equation, we find

$$R = kT^* - 3\lambda\Box W^2, \quad (6)$$

where T^* is the trace of the stress-energy tensor. Thus, we obtain from Eq. (4)

$$F^{\mu\nu}{}_{\|\nu} = \frac{3\lambda^2}{k}(\Box W^2)\,W^\mu - \lambda T^* W^\mu + J^\mu, \quad (7)$$

which exhibits explicitly the nonlinear of our model. It seems worthwhile to remark here that such a a type of nonlinearity behavior of our model can be introduced in an equivalent way without making an appeal to nonminimal coupling with gravitation. Indeed, if we consider a Lagrangian of the form

$$\mathcal{L}_N = \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + E\,(W^\mu W_{\mu\|\alpha})^2,$$

a straightforward calculation shows that the equation of motion obtained from such $\mathcal{L}_N$ is precisely Eq. (7) (without the trace term, of course).

The wave equation for the potential vector W^μ is given by

$$\Box W^\mu + R^\mu{}_\alpha W^\alpha - (W^\nu{}_{\|\nu})^{|\mu} = \frac{3\lambda^2}{k}(\Box W^2)\,W^\mu \quad (8)$$

in the absence of currents and matter. The first two terms of this equation are nothing but de Rahm's wave operator in curved space. The third term is proportional to the gradient of the variation of the scalar of curvature in the W^μ direction.

A case of particular interest occurs when we can neglect the second and the third terms and simplify Eq. (8) to the expression

$$\Box W^\mu - \frac{3\lambda^2}{k}(\Box W^2)\, W^\mu = 0. \tag{9}$$

This equation has some very interesting properties of its own. Let us rewrite it in a Gaussian system of coordinates, where

$$ds^2 = dt^2 - g_{ij}(x)dx^i dx^j.$$

We have

$$\Box W^\mu - \epsilon\left(\frac{\partial^2}{\partial t^2} W^2\right) W^\mu = -\epsilon g^{ij}(W^2)_{|i\|j}\, W^\mu, \tag{10}$$

in which ϵ is a constant, $\epsilon \equiv 3\lambda^2/k$.

The left-hand side has a striking analogy to the equation that governs the electric field inside a nonlinear dielectric, due, for instance, to the dependence of the dielectric constant on light intensity. One may interpret Eq. (10) as having given origin to a generalized relativistic Kerr effect.

Let us give here, for completeness, the equation of motion obtained from the Lagrangian

$$L = \sqrt{-g}\left(R + \frac{\epsilon}{k} W^\mu W^\nu R_{\mu\nu}\right) + \mathcal{L}_{\text{Maxwell}}.$$

From the variational principle we obtain

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{2}R g_{\mu\nu} = \tfrac{1}{2}\epsilon\,(& W^\alpha W^\beta)_{|\alpha\|\beta} g_{\mu\nu} - \Box(W_\mu W_\nu) \\ & + (W^\beta W_{(\nu})_{\|\mu)\|\beta} + 2R_{\alpha(\mu} W_{\nu)} W^\alpha \\ & - R_{\alpha\beta} W^\alpha W^\beta g_{\mu\nu} \\ & = -kT^*_{\mu\nu} \end{aligned} \tag{11}$$

and

$$F^{\mu\nu}{}_{\|\nu} = -\frac{\epsilon}{k} R^\mu{}_\nu W^\nu. \tag{12}$$

It seems worthwhile to remark here that even in the case of a homogeneous and isotropic universe the equations of motion obtained from Lagrangian $L_{\rm I} = \sqrt{-g}\, R\, W_\mu W^\mu$ are distinct from those given by $L_{\rm II} = \sqrt{-g} R_{\mu\nu} W^\mu W^\nu$. Indeed, the set of Eqs. (11) and (12), unlike the set (2) and (4), does not admit a Friedmann-type universe. This can be shown by the following arguments. Owing to the isotropy of the universe, the electric and magnetic vectors are null. Thus, from Eq. (12), $R_{\mu\nu} W^\nu$ must vanish.

However, the unique non-null component of the potential vector W^α is for $\alpha = 0$. Thus, we obtain the equation $R^\mu{}_0 = 0$. Owing to the symmetry conditions of such a universe, $R^i{}_0$ is identically null and remains the unique equation

$$R^0{}_0 = 0.$$

Now it is straightforward to verify that the above equation is not compatible with the remaining set of equations (11).

III. NONLINEAR EQUATION

The scalar equation associated with (9) has the form

$$\Box\phi + \tfrac{1}{2}\mu^2(\Box\phi^2)\phi = 0, \tag{13}$$

where μ is constant. This equation has an interest of its own. Let us examine some properties of it under two special circumstances:

(i) The background geometry is flat (Minkowskian).

(ii) The geometry represents an expanding homogeneous and isotropic Friedmann universe.

Let us look for a stationary solution, where ϕ depends only on one variable, say χ. Equation (13) turns into

$$\phi''(1 + \mu^2\phi^2) + \mu^2\phi\phi'^2 = 0, \tag{14}$$

in which

$$\phi' \equiv d\phi/d\chi.$$

A solution of this equation can be found under the form

$$mx + n = \operatorname{arc\,sinh}(\mu\phi) + \phi(1 + \mu^2\phi^2)^{1/2}, \tag{15}$$

where m and n are arbitrary constants. Let us now consider the case of Friedmann geometry. The infinitesimal length is given in a coordinate system (t, χ, θ, ϕ) by

$$ds^2 = dt^2 - a^2(t)[d\chi^2 + \sigma^2(\chi)(d\theta^2 + \sin^2\theta\, d\sigma^2)].$$

The function $\sigma(\chi)$ may assume the values χ, $\sin\chi$, or $\sinh\chi$, in which cases the 3-geometry has a constant curvature which is flat, positive, or negative, respectively.

We set $\phi = \phi(t)$. Then Eq. (13) assumes the form

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3\frac{d\phi}{dt}\right) + \frac{\mu^2}{a^3}\phi\frac{d}{dt}\left(a^3\phi\frac{d\phi}{dt}\right) = 0. \tag{16}$$

A straightforward integration yields

$$\operatorname{arc\,sinh}(m\phi) + \phi(1 + \mu^2\phi^2)^{1/2} = B\int a^{-3}dt, \tag{17}$$

in which β is an arbitrary constant. This last expression can be directly integrated for different Friedmann models, once the function $a(t)$ is given.

IV. THE ENERGY BALANCE

Let us return now to the original set of equations (2.4). In general the divergence of the current does not vanish. Thus, it gives a contribution to the balance of energy which we will now evaluate. Taking the divergence of Eq. (2), one obtains

$$W^2{}_{|\nu}G^{\mu\nu}-(\Box W^2)_{|\nu}\,g^{\mu\nu}+(W^2)_{|\epsilon\|\nu\|\lambda}\,g^{\epsilon\mu}g^{\nu\lambda}+(R\,W^\mu W^\nu)_{\|\nu} = -\frac{k}{\lambda}E^{\mu\nu}{}_{\|\nu}-\frac{k}{\lambda}\,T^{*\mu\nu}{}_{\|\nu}. \quad (18)$$

We have

$$E_\mu{}^\nu{}_{\|\nu}=F_{\mu\alpha}F^{\alpha\nu}{}_{\|\nu}+F_{\mu\alpha\|\beta}\,F^{\alpha\beta}+\tfrac{1}{2}F^{\alpha\beta}F_{\alpha\beta\|\mu}.$$

or, using the antisymmetry of $F_{\mu\nu}$,

$$E^{\mu\nu}{}_{\|\nu}=F^\mu{}_\alpha F^{\alpha\nu}{}_{\|\nu}. \quad (19)$$

From Eq. (4) we obtain, as in Maxwell's electrodynamics, the relation

$$E^{\mu\nu}{}_{\|\nu}=F^\mu{}_\alpha J^{*\alpha}, \quad (20)$$

in which the total current $J^{*\alpha}$ is defined by the expression

$$J^{*\alpha}=J^\alpha-\frac{\lambda}{k}R\,W^\alpha. \quad (21)$$

We have

$$(\Box\phi)_{|\mu}=\Box(\phi_{|\mu})+R_\mu{}^\epsilon\phi_{|\epsilon}.$$

Thus, using the above relations we obtain

$$\tfrac{1}{2}(W^2)_{|\nu}\,g^{\mu\nu}R-\frac{K}{\lambda}J^\alpha{}_{\|\alpha}W^\mu-R\,W^\nu W^\mu{}_{\|\nu} = \frac{k}{\lambda}F^{\mu\alpha}J^*_\alpha+\frac{k}{\lambda}\,T^{*\mu\nu}{}_{\|\nu} \quad (22)$$

or finally, after some simplifications,

$$T^{*\mu\nu}{}_{\|\nu}=-F^{\mu\nu}J_\nu-J^\alpha{}_{\|\alpha}W^\mu. \quad (23)$$

The first term on the right-hand side give the rate of work of the field; the second term is the contribution to the energy due to the created particles.

V. CONFORMAL TRANSFORMATION

It seems worthwhile to call attention to the fact that the present model is not equivalent to a local rescaling of all existing masses in the universe, leaving the photon mass to be null. The ultimate reason for this is the behavior of the theory under a conformal transformation.

The situation is very different as, for instance, in scalar-tensor theories. Indeed, many authors have shown that scalar-tensor theories of the Jordan type (with a cosmic variation of the gravitational coupling constant) are conformally related to theories with continuous creation of matter (either steady state or not). This is a consequence of the existence of a unique free function for both theories and of the behavior of these models under a conformal map. Harrison[7] has shown that a conformal transformation $g_{\mu\nu}\to\tilde{g}_{\mu\nu}=\phi^s g_{\mu\nu}$ coupled to the change of the scalar field $\phi\to\tilde{\phi}=\phi^{-\mu}$ induces the equality

$$\delta\int\sqrt{-g}\,(\tilde{R}\,\tilde{\phi}^c+\tilde{\omega}\tilde{\phi}_{|\alpha}\tilde{\phi}_{|\beta}\,\tilde{g}^{\alpha\beta}\,\tilde{\phi}^{c-2}+k\tilde{L}_m\tilde{\phi}^D)\,d^4x \doteq \delta\int\sqrt{-g}\,(R\,\phi^A+\omega\,\phi_{|\alpha}\phi_{|\beta}\,g^{\alpha\beta}\,\phi^{A-2}+k\,L_m\phi^B)\,d^\phi\chi, \quad (24)$$

where the symbol $\doteq$ means equality of the Euler-Lagrangian equations (that is, up to a divergence term). A direct calculation can relate the values of the constants $\tilde{\omega}$, $\tilde{C}$, and $\tilde{D}$ in terms of ω, μ, A, and S. Thus, all scalar-tensor theories are conformally equivalent. However, this is not the situation in vector-tensor theories. The point is that if we try to generate a mapping which brings our model (with a space-time–dependent photon mass) to a theory in which all other masses in the universe are altered (retaining Maxwell's electrodynamics), then we inevitably introduce nonlinearities into the equation of motion of the vector field. The reason is simple, and is elucidated in the above analysis of the conformal map. In order to eliminate the nonminimal coupling term we are obliged to set the conformal function to be proportional to some power of W^2. This will give origin to the nonlinear terms, as can be easily seen. Thus, our model cannot be reduced to a rescaling of masses in Maxwell's electrodynamics. Actually, this seems to be a generic situation for nonminimal coupling with gravitation for non-null spin fields.

VI. THE COSMIC SOLUTION

In this section we will give a cosmic solution of our set of equations obtained by the nonminimal coupling of electrodynamics and gravitation. We will try here to answer the following question: Assuming the existence of a universe filled with such nonlinear photons, what are the global properties of such a cosmos? As we will show, there is a solution of our set of Eqs. (2) and (4) which represents a homogeneous and isotropic universe. However, contrary to the usual Friedmann cosmologies in which an explicit function for the radius of the universe with time is not in general available, our solution has a simple explicit form, as we will see.

As there is no privileged direction in space, in which the electric and the magnetic vectors could point, we conclude that both vectors must vanish. From Eq. (4), the scalar of curvature must vanish, too,

$$R=0. \tag{25}$$

As a consequence, charge is conserved. Equation (25) may be written equivalently,

$$\Box W^2=0. \tag{26}$$

Let us define a function $\Omega \equiv 1+\lambda W^2$. Then the set of Eqs. (2) and (4) can be written in the form

$$R_{\mu\nu}=-\frac{\Omega_{|\mu||\nu}}{\Omega}, \tag{27}$$

$$\Box\Omega=0. \tag{28}$$

We look for a solution of this set of equations in which the infinitesimal element of length has the form

$$ds^2=dt^2-a^2(t)[d\chi^2+\sigma^2(\chi)(d\sigma^2+\sin^2\sigma\, d\sigma^2)]. \tag{29}$$

After some simple calculations we obtain the equations for $a(t)$ and $\Omega(t)$.

The values of the curvature are

$$\begin{aligned} R^0{}_0&=3\frac{\ddot a}{a},\\ R^1{}_1&=-\frac{\ddot a}{a}-2\frac{\dot a}{a^2}+\frac{2}{a^2}\frac{\sigma''}{\sigma},\\ R^2{}_2=R^3{}_3&=-\frac{\ddot a}{a}-2\frac{\dot a^2}{a^2}+\frac{1}{a^2}\left(\frac{\sigma''}{\sigma}+\frac{\sigma'^2-1}{\sigma^2}\right), \end{aligned} \tag{30}$$

in which an overdot means time derivative.

The covariant derivatives of Ω are given by

$$\begin{aligned} \Omega^{|0}{}_{||0}&=\ddot\Omega,\\ \Omega^{|1}{}_{||1}=\Omega^{|2}{}_{||2}=\Omega^{|3}{}_{||3}&=-\frac{\dot a}{a}\dot\Omega. \end{aligned} \tag{31}$$

From this, we obtain the result that the 3-curvature ${}^{(3)}R$ must be a constant.

Let us define $\epsilon \equiv -\frac{1}{6}{}^{(3)}R$. Then ϵ may assume the values 0, +1, −1. Correspondingly, the function $\sigma(\chi)$ may be χ, $\sin\chi$, or $\sinh\chi$. The solution is easily obtained:

$$a(t)=(-\epsilon t^2+bt+c)^{1/2}, \tag{32}$$

$$\Omega=\frac{\Omega_0}{a}(-2\epsilon t+b). \tag{33}$$

Let us make some comments on these solutions. We remark first of all that, as we have said, a simple explicit form for the function $a(t)$ is available when the cosmos is filled with such nonlinear photons.

Constants b, c, and Ω_0 are not completely arbitrary. They have to satisfy a constraint which is linked to the definition of Ω. As in the isotropic world there is a privileged direction, the vector W^μ must be of the form $W^\mu=(\dot\phi, 0, 0, 0)$. We have set a derivative on ϕ just to recall that W^μ is a gradient. Thus, we have

$$1+\lambda\dot\phi^2=\Omega_0(-2\epsilon t+b)(-\epsilon t^2+bt+c)^{-1/2}. \tag{34}$$

Let us examine this relation for the three possible values of ϵ separately. In the case of $\epsilon=0$, then $\lambda\dot\phi^2=\Omega_0 b/a-1$. If λ is negative, then Ω_0 must be negative too, once b is a positive constant.

In the closed universe, $\lambda\dot\phi^2=(\Omega_0/a)(-2t+b)-1$. In the case of a negative λ, then Ω_0 must be positive and b negative. Finally, for the open model, if λ is negative, b must be positive and Ω_0 negative. Now let us turn to the function $a(t)$. The possibility of a real solution is dominated by the sign

$$\Delta \equiv b^2+4\epsilon c.$$

In the case of a closed model, a positive value of Δ implies the existence of two real roots t_1 and t_2. The universe starts to exist at $t=t_1$ and ends at $t=t_2$. The value t_2-t_1 measures the total life of such a universe. The maximum point of expansion is given by the value $t=\frac{1}{2}b$. A remarkable consequence of the above set of equations is the impossibility of a steady-state regime. Indeed, for the Euclidean section, the equations of motion are given by

$$-3\frac{\ddot a}{a}=\frac{\ddot\Omega}{a}, \tag{35}$$

$$\frac{\ddot a}{a}+2\frac{\dot a^2}{a^2}=-\frac{\dot a}{a}\frac{\dot\Omega}{\Omega}, \tag{36}$$

$$\dot\Omega a^3=\text{constant}. \tag{37}$$

It is easy to recognize that this set does not admit the de Sitter function e^{Ht}, where H is a constant, as a possible value for the solution. This could well be guessed by Eq. (25) which is nothing, in the Lyttleton-Bondi formulation, but the annihilation of the photon mass. Thus, although there is a point of contact between our present model and LB's suggestion, their main result—besides other properties—does not occur in our theory.

The above cosmological solution is stable against a small perturbation generated by the introduction of a small quantity of matter. Actually, this property does not depend on our specific model but is a consequence of the absence of density of matter in the expanding background.

As a consequence of the energy-balance equation, and owing to the absence of electric and magnetic fields, the stress-energy tensor of the matter must be conserved. Let us consider a fluid (dust) with an energy-momentum tensor given by $T_{\mu\nu}=(\delta\rho)V_\mu V_\nu$, where $\delta\rho$ is a small density.

We choose the comoving frame in order to set the fluid velocity V^μ to have the value $V^\mu=\delta^\mu_0$. Conservation of $T^{\mu\nu}$, projected in the V^μ direction, gives

$$(\delta\rho)^{\cdot}+(\delta\rho)\,\theta=0.$$

In the case of the Euclidean section, using the results obtained above, the expansion θ equals $\frac{3}{2}b(bt+c)^{-1}$. A direct integration yields

$$\delta\rho=(\delta\rho)(bt+c)^{-3/2}.$$

Thus, as time goes on the total perturbation decreases showing the stability of the model under a small injection of matter in our nonlinear-photon cosmos.

Actually, one can show a result stronger than this, e.g., that our model universe cannot share the bending of space-time with a finite density of matter. This can be seen by a direct inspection on equations $R=0$ and $\Box\Omega=0$. These two equations specify the functions $a(t)$ and $\Omega(t)$, giving no possibility of inserting another function $\rho(t)$ in our equations.

VII. THE ANISOTROPIC UNIVERSE

Although there is no possibility of having a non-null electric and/or magnetic field as a source of an isotropic world, this is not the case in an anisotropic cosmos. Indeed, cosmological solutions of Maxwell's equations with a privileged direction have been analyzed by many authors. In the present situation we will show the possibility of having nonlinear photons as the source of an anisotropic universe.

Remarkably enough, our solution will have a null electric and magnetic field but a non-null potential vector pointing in a privileged direction. We start by considering the general equations (2) and (4), under such conditions, for a Bianchi type-I cosmological model. The fundamental length is given by

$$ds^2=dt^2-a^2(t)dx^2-b^2(t)dy^2-c^2(t)dz^2. \tag{38}$$

Then a straightforward calculation gives

$$\frac{\ddot a}{a}+\frac{\ddot b}{b}+\frac{\ddot c}{c}=-\frac{\ddot\Omega}{\Omega}, \tag{39}$$

$$\frac{\ddot a}{a}+\frac{\dot a}{a}\frac{\dot b}{b}+\frac{\dot a}{a}\frac{\dot c}{c}=\frac{\dot a}{a}\frac{\dot\Omega}{\Omega}, \tag{40}$$

$$\frac{\ddot b}{b}+\frac{\dot a}{a}\frac{\dot b}{b}+\frac{\dot b}{b}\frac{\dot c}{c}=\frac{\dot b}{b}\frac{\dot\Omega}{\Omega}, \tag{41}$$

$$\frac{\ddot c}{c}\frac{\dot a}{a}\frac{\dot c}{c}\frac{\dot b}{b}\frac{\dot c}{c}=\frac{\dot c}{c}\frac{\dot\Omega}{\Omega}, \tag{42}$$

$$(abc\dot\Omega)^{\cdot}=0, \tag{43}$$

in which a dot, as usual, means time derivative. We limit our discussion here to the case in which $b=c=0$. Then, there remain only two equations,

$$\frac{\ddot a}{a}=\frac{\dot a}{a^2}\frac{m}{\Omega}, \tag{44}$$

$$\dot\Omega=\frac{m}{a}, \tag{45}$$

where m is a constant. Then we obtain

$$a\ddot a=\text{constant}, \tag{46}$$

$$\Omega=m\int\frac{dt}{a}. \tag{47}$$

We will extend an analysis of this model elsewhere.

VIII. FINAL REMARKS

In this paper we have considered the nonminimal coupling of electrodynamics and gravidynamics. As a consequence of such coupling the behavior of electrodynamics is governed by nonlinear equations. The nonlinear terms can be interpreted as due to the presence of massive photons, in which the space-time-dependent photon mass is proportional to the scalar of curvature.

The above model gives an effective modification of electrodynamics only in those regions in which the gravitational field is strong, that is, for high values of the curvature.

The effects of such modification in homogeneous isotropic/anisotropic cosmological models are examined in Secs. VI and VII. We show that a Friedmann-type solution can be found and we give explicitly the dependence of the radius of curvature with the cosmic time.

Finally, let us point out that in the general case a non-null scalar of curvature may induce some new effects—as, for instance, the decay of the photon into other particles. The consequences of this should be a matter for future investigation.

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