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Mass-splitting sum rules for the charmed baryons

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The relations between the masses of charmed baryons and uncharmed baryons are derived from the I-, U-, and L-spin properties of the mass-splitting interactions.

The discovery of resonances $\psi(3.1)$ and $\psi'(3.7)$ (Ref. 1) led to the study of theories with charmed quarks in the framework of SU(4).[2] The subsequent discovery of some new particles carrying charm[3] lends credence to these ideas. In looking for charmed baryons, it will be of much help to know any sum rules between the masses of charmed and uncharmed baryons.

In a recent paper,[4] we studied the mass splitting of charmed baryons using the SU(2) subgroups of SU(4), namely, I, U, V, R, P, and L spin.[5] Values of L spin for quarks and $\frac{3}{2}^+$ baryons are given in Ref. 6 and for $\frac{1}{2}^+$ in Ref. 4. We have written a parallelogram law both for $\frac{3}{2}^+$ baryons[6] and $\frac{1}{2}^+$ baryons.[4] Application of the parallelogram law to the parallelogram 7, 6, 5, and 12 (Fig. 2, Ref. 4) gives

$$C_1^0 - \Sigma^- + \Xi^- - S^0 + \sqrt{3}\,(S^0A^0) = 0\,, \tag{1}$$

and to the parallelogram 8, 9, 10, and 12 (Fig. 2, Ref. 4) yields

$$T^0 - X_s^+ + X_d^+ - S^0 + \sqrt{3}\,(S^0A^0) = 0\,, \tag{2}$$

where (S^0A^0) and $(S_u^0A_u^0)$ are the transition masses in I-spin and U-spin representations respectively, and

$$S_u^0 = \tfrac{1}{4}S^0 + \tfrac{3}{4}A^0 - \frac{\sqrt{3}}{2}(S^0A^0)\,. \tag{3}$$

Now if the medium-strong interaction is restricted to first order in H_{ms} (where H_{ms} is the Hamiltonian of "medium-strong interaction," then for any U-spin multiplet the mass operator may be expressed as

$$\alpha_1 + \alpha_2 U_3\,. \tag{4}$$

This will then imply the equality of first differences between the masses within any U-spin multiplet. For the $\frac{3}{2}^+$ baryon multiplet we will have

$$\Delta^- - \Sigma^{*-} = \Sigma^{*-} - \Xi^{*-} = \Xi^{*-} - \Omega^-\,, \tag{5}$$

$$\Delta^0 - \Sigma^{*0} = \Sigma^{*0} - \Xi^{*0}\,, \tag{6}$$

$$\Delta_c^0 - \Delta_c^{*0} = \Delta_c^{*0} - \Sigma_c^0\,. \tag{7}$$

For $\frac{1}{2}^+$ baryon 20-plet we will have

$$n + p + \Xi^0 + \Xi^- = \Sigma^+ + \Sigma^- - \Sigma^0 + 3\Lambda\,, \tag{8}$$

$$C_1^0 - S_u^0 = S_u^0 - T^0\,. \tag{9}$$

From (3) and (9) we have

$$2(C_1^0 + T^0) = S^0 + 3A^0 - 2\sqrt{3}(S^0A^0)\,,$$

which upon combining with (1) and (2) yields

$$C_1^0 + T^0 + \Sigma^- + X_s^+ = \Xi^- + X_d^+ - S^0 + 3A^0\,. \tag{10}$$

Similarly for any L-spin multiplet the mass operator may be expressed as

$$\beta_1 + \beta_2 L_3\,.$$

Then for $\frac{3}{2}^+$ baryons we will have

$$\Delta^{++} - \Delta_c^{++} = \Delta_c^{++} - \Sigma_{cc}^{++} = \Sigma_{cc}^{++} - \Xi_{ccc}^{++}\,, \tag{11}$$

$$\Delta^+ - \Delta_c^+ = \Delta_c^+ - \Sigma_{cc}^+\,, \tag{12}$$

$$\Sigma^{*+} - \Delta_c^{*+} = \Delta_c^{*+} - \Xi_{cc}^{++}\,. \tag{13}$$

And for $\frac{1}{2}^+$ baryons we have

$$p - C_{L^1}^+ = C_{L^1}^+ - X_d^+\,, \tag{14}$$

$$\Sigma^+ - S_L^+ = S_L^+ - X_s^+\,, \tag{15}$$

where again $(C_{L^1}^+C_{L^0}^+)$ and $(S_L^+A_L^+)$ are the transition masses in the L-spin representation.

Application of the parallelogram law to the parallelogram 1, 13, 14, and 6 (Fig. 2, Ref. 4) yields

$$\begin{aligned}&\Sigma^- - n + C_1^{++} - S^+ + \gamma(S^+A^+)\\ &\quad\equiv \Sigma^- - n + C_1^{++} - S_L^+ + \gamma'(S_L^+A_L^+)\\ &\quad= 0\,,\end{aligned} \tag{16}$$

where

$$S_L^+ = \tfrac{1}{4}S^+ + \tfrac{3}{4}A^+ - \frac{\sqrt{3}}{2}(S^+A^+)\,. \tag{17}$$

Similarly for parallelogram 4, 5, 15, and 13 (Fig. 2, Ref. 4), we have

$$\begin{aligned}&C_1^{++} - \Xi^0 + \Xi^- - C_1^+ + \delta(C_1^+C_0^+)\\ &\equiv C_1^{++} - \Xi^0 + \Xi^- - C_{L^1}^+ + \delta'(C_{L^1}^+C_{L^0}^+)\\ &= 0\,,\end{aligned} \tag{18}$$

where

$$C^+_{L^1} = \tfrac{1}{4}C^+_1 + \tfrac{3}{4}C^+_0 - \frac{\sqrt{3}}{2}(C^+_1 C^+_0) . \qquad (19)$$

The identities are satisfied for

$$\gamma = \gamma' \text{ and } \delta = \delta' .$$

A comparison of (14), (18), and (19) yields

$$2p + 2X^+_d + 2\Xi^- - 2\Xi^0 = 2C^{++}_1 - C^+_1 + 3C^+_0 , \qquad (20)$$

and a comparison of (15), (16), and (17) yields

$$2n + 2X^+_s + 2\Sigma^+ - 2\Sigma^- = 2C^{++}_1 - S^+ + 3A^+ . \qquad (21)$$

If electromagnetic splitting is neglected then from (20) we will have

$$2p + 2X_d = 3C_0 + C_1 . \qquad (22)$$

C_1 has a mass of 2430 MeV and C_0 has a mass of 2260 MeV.[3] Taking the proton mass to be ~938 MeV, the mass of X_d will be ~3667 MeV.

Applying the parallelogram law to the parallelogram 2, 3, 9, and 8 (Fig. 2, Ref. 4) we have

$$p + X^+_s = \Sigma^+ + X^+_d ,$$

which yields $X^+_s = 3927$ MeV.

Combining (10) and (21) and neglecting the electromagnetic splitting we have

$$T^0 = 2n + X_s + X_d + \Xi - 3C_1 - \Sigma$$

or

$$T^0 \approx 2301 \text{ MeV} .$$

Finally if the electromagnetic interaction is also restricted to first order in H_{em} (where H_{em} is the Hamiltonian of electromagnetic interaction), then within an I-spin multiplet the masses of any order in H_{ms} can be written as

$$\gamma_1 + \gamma_2 I_3 .$$

Then for $\frac{3}{2}^+$ baryons we have

$$\Sigma^{*+} - \Sigma^{*0} = \Sigma^{*0} - \Sigma^{*-} , \qquad (23)$$

$$\Delta^{++}_c - \Delta^+_c = \Delta^+_c - \Delta^0_c , \qquad (24)$$

and for $\frac{1}{2}^+$ baryons we have

$$\Sigma^+ - \Sigma^0 = \Sigma^0 - \Sigma^- , \qquad (25)$$

$$C^{++}_1 - C^+_1 = C^+_1 - C^0_1 . \qquad (26)$$

Thus using an restrictive assumption that H_{ms} and H_{em} can be taken to first order only, we have been able to derive sum rules concerning the mass splitting of charmed baryons. In the above relations, those concerning only the uncharmed baryons are satisfied to a reasonable extent, thus lending credence to other relations as well.

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[1] J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, *ibid.* 33, 1406 (1974); C. Bacci *et al.*, *ibid.* 33, 1408 (1974).

[2] M. K. Gaillard, B. W. Lee, and J. Rosner, Rev. Mod. Phys. 47, 277 (1975); T. Das, P. Divakaran, L. Pandit, and V. Singh, Pramana 4, 105 (1975); J. W. Moffat, Phys. Rev. D 12, 288 (1975); M. Ahmad and Tej. K. Zadoo, Nuovo Cimento 36A, 86 (1976); Phys. Rev. D 15, 2483 (1977).

[3] Particle Data Group, Phys. Lett. 75B, 1 (1978).

[4] Tej K. Zadoo and M. Ahmad, Phys. Rev. D 18, 4206 (1978).

[5] M. Ahmad and Tej K. Zadoo, Czech. J. Phys. B27, 1337 (1977).

[6] Tej K. Zadoo and M. Ahmad, Can. J. Phys. 55, 783 (1977). See also G. Feldman and P. Matthews, Ann. Phys. (N.Y.) 31, 469 (1965); ICTP Report No. IC/77/99, 1977 (unpublished).