PHYSICAL REVIEW D VOLUME 20, NUMBER 12 15 DECEMBER 1979

Grand unified theory with heavy color

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(Received 6 August 1979)

An example of the unification of electroweak, color, and heavy-color forces in the unifying group SU(7) is presented. This simple toy model predicts a nontrivial mass spectrum for two families of quarks and leptons. The usual Higgs scalar sector is replaced by the strong interaction heavy-color sector at ~1 TeV.

I. INTRODUCTION

In this paper we demonstrate the possibility of combining the ideas of the Georgi-Glashow (GG) grand unified $SU(5)_{GG}$ model[1] with the heavy-color mechanism[2-5] which has been suggested as a replacement for the fundamental Higgs fields of the Weinberg-Salam model. There are a number of features of the $SU(5)_{GG}$ group which make it a particularly plausible candidate for a unifying group. The organization of particle multiplets is very elegant and naturally explains the observed quantum numbers of the quarks and leptons. The quantization of electric charge is a straightforward consequence. Furthermore the unrenormalized Weinberg angle satisfies $\sin^2\theta = \frac{3}{8}$. This is modified by renormalization[6] so that the real angle satisfies $\sin^2\theta \approx 0.2$. Another nontrivial consequence is that the lifetime of the proton is greater than the current lower bound.[6]

On the other hand, the theory also has bad features. Among them is the existence of two vastly different scales of Higgs expectation values.[7] The parameters of the Higgs sector must be tuned to ridiculous precision to maintain this "gauge hierarchy." Furthermore, the number of parameters involving the lower-mass Higgs sector is excessive. In addition to the Higgs self-coupling there are a large number of Yukawa couplings. No natural explanation for their extremely small magnitude has been given. Finally the simplest Higgs assignments lead to the unacceptable bare-mass relation

$$m_e = m_d .$$

Apparently the successes of the theory involve those features which are independent of the light Higgs sector. The failures suggest that the 100-GeV symmetry breaking is being incorrectly treated. It therefore seems reasonable to explore alternatives to the usual Higgs sector. In this paper a toy model is used to illustrate how the lowest-mass Higgs sector of $SU(5)_{GG}$ can be replaced by heavy color in a simple and elegant unification.

II. HEAVY COLOR

In this section we will briefly review the salient features of Refs. 2 and 3, in which heavy color (HC) was introduced. The reader is urged to consult the original papers for more complete explanations.

(1) HC is an unbroken gauge group with a running coupling which becomes strong at a scale ~1 TeV. It is essentially a scaled-up version of quantum chromodynamics (QCD) involving heavy-color fermions U and D which parallel the ordinary u, d quarks. These heavy-color quarks may or may not have color (C) but definitely have conventional electroweak (EW) interactions.

(2) The strong interactions at 1 TeV cause a spontaneous breaking of the flavor-chiral $SU(2)^{\text{left}} \times SU(2)^{\text{right}}$ of heavy-color quarks.[2] The result is massless Goldstone heavy-color pions. These objects replace the Higgs fields and induce a mass for the Z and $W^{\pm}$.

(3) Global isospin conservation of the heavy-color-quark sector is sufficient to guarantee the empirical relation

$$\frac{M_W}{M_Z} = \cos\theta_W . \tag{2.1}$$

(4) Heavy color should be part of a bigger group[3] with ordinary quarks and heavy-color quarks in the same multiplet. This is to allow ordinary quarks to gain mass through radiative corrections as shown in Fig. 1. The gauge bosons b which mediate transitions from ordinary quarks q to heavy-color quarks Q have mass[3] ~10–100 TeV.

One serious deficiency of Refs. 2 and 3 was that no consistent example was offered in which both leptons and quarks gain mass. Two obvious

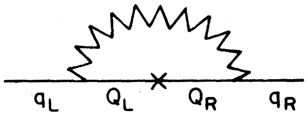

FIG. 1. Light quarks q gain mass by coupling to heavy-color quarks Q through emission of b bosons.

possibilities come to mind. In the first, additional heavy-color particles called heavy-color leptons are introduced. These feed mass to the ordinary leptons as shown in the graph of Fig. 2.

Unfortunately this type of model has too much symmetry. In particular the weak hypercharges of leptons and of quarks would be separately conserved[8] leading to two Goldstone bosons. Only one of these could be absorbed by the Higgs effect, leaving a massless "axion." This is empirically unacceptable.

A second possibility would be to allow both leptons and quarks to couple to heavy-color quarks so that leptons would gain mass from the graph shown in Fig. 3. This would allow lepton-quark transitions as in Fig. 4, thus risking baryon violation by 100-TeV bosons.

One other potential danger implicit in Refs. 2 and 3 is the existence of stable heavy-color-quark bound states analogous to protons in QCD. While these are very heavy (1 TeV) they could lead to unpleasant astrophysical or cosmological consequences.

In this paper we shall see that all of the above difficulties are surmounted in our toy model, while the good features of $\mathrm{SU}(5)_{GG}$ are preserved.

III. THE MODEL

For the purpose of simplicity we will choose to build our toy model out of the simplest possible parts. In particular we shall choose the HC group to be the smallest possible non-Abelian group—$\mathrm{SU}(2)_{HC}$. We shall ultimately pay a price for the smallness of the HC group.

The minimal extension of Georgi-Glashow SU(5) to include HC is SU(7). This is our choice of unifying group. The components of a fundamental seven-dimensional representation are labeled

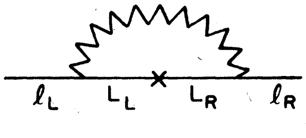

FIG. 2. Leptons l gain mass by coupling to heavy-color leptons L.

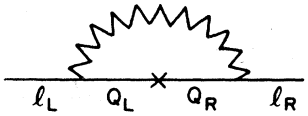

FIG. 3. Leptons l gain mass by coupling to heavy-color quarks.

$$\mathrm{SU}(5)_S\left\{\begin{array}{c}\\ \\ \\ \\ \\ \end{array}\right.\left(\begin{array}{c}\psi_{T_1}\\ \psi_{T_2}\\ \psi_{d_R}\\ \psi_{d_Y}\\ \psi_{d_B}\\ \psi_{\bar e}\\ \psi_{\bar\nu}\end{array}\right)\begin{array}{l}\left.\begin{array}{c}\\ \\ \end{array}\right\}\mathrm{HC}\\ \left.\begin{array}{c}\\ \\ \\ \end{array}\right\}\mathrm{C}\\ \left.\begin{array}{c}\\ \\ \end{array}\right\}\mathrm{EW}\end{array}\left.\begin{array}{c}\\ \\ \\ \\ \\ \end{array}\right\}\mathrm{SU}(5)_{GG} \tag{3.1}$$

(HC = heavy color, C = color, EW = electroweak, GG = Georgi-Glashow, S = strong, R, Y, B = red, yellow, blue).

Following arguments of Georgi,[9] we choose an anomaly-free set of representations formed from antisymmetric products of seven-dimensional representations. *All fermions are two-component (Weyl) left-handed fermions.* Our particular choice is $[2]+[4]+[6]$, where m means the antisymmetric products of m seven-dimensional representations. For example, the $[2]$ has representation vectors $\psi_{ij}=-\psi_{ji}$. The choice $[2]+[4]+[6]$ is not *ad hoc* but follows from the requirement of no anomalies.

Under the breakdown $\mathrm{SU}(2)_{HC}\times\mathrm{SU}(5)_{GG}$ these representations transform as follows:

$$\begin{aligned}[][2]&=(1,10)+(2,5)+(1,1),\\ [4]&=(1,10)+(2,\overline{10})+(1,\overline{5}),\\ [6]&=(1,\overline{5})+(2,1).\end{aligned} \tag{3.2}$$

For example, the $(2,\overline{10})$ in the $[4]$ consists of tensors with one HC index (called p) and three $\mathrm{SU}(5)_{GG}$ indices

$$(2,\overline{10})=\psi_{p\alpha\beta\gamma}. \tag{3.3}$$

According to Georgi,[9] the number of observable families [a family means a $(\overline{5}+10)$ of $\mathrm{SU}(5)_{GG}$] is given by the total number of $\overline{5}$'s minus the total number of 5's. For the example under consideration, this would mean zero. The point according

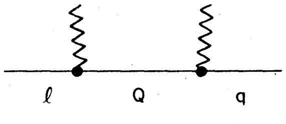

FIG. 4. Lepton goes to quark plus b mesons.

to Georgi is that a left-handed $\underline{\bar{5}}$ and $\underline{5}$ can be paired into a single Dirac fermion with a large mass term which does not violate $SU(5)_{GG}$. Such particles could well have superheavy masses. The same is true for the $\underline{10}$'s and $\underline{\overline{10}}$'s.

However, in our case all $\underline{5}$'s and $\underline{\overline{10}}$'s belong to HC doublets. This prevents them from combining with the heavy-color-free $\underline{\bar{5}}$'s and $\underline{10}$'s. Thus we expect two families of ordinary particles and a doublet of heavy-color families formed from $(2, \overline{10})$ and $(2, 5)$.

We now display the particle identification in terms of $SU(5)_{GG}$ representations. A subscript p means that each entry is a HC doublet. A capital letter indicates a heavy-colored particle.

The [2]:

$$(1,10)=\begin{pmatrix} 0 & \bar{c}_B & -\bar{c}_Y & u_R & d_R \\ -\bar{c}_B & 0 & \bar{c}_R & u_Y & d_Y \\ c_Y & -\bar{c}_R & 0 & u_B & d_B \\ -u_R & -u_Y & -u_B & 0 & \bar{e} \\ -d_R & -d_Y & -d_B & -\bar{e} & 0 \end{pmatrix}, \tag{3.4a}$$

$$(2,5)=\begin{pmatrix} D_R \\ D_Y \\ D_B \\ \bar{E} \\ \bar{N} \end{pmatrix}_p, \tag{3.4b}$$

$$(1,1)=(\bar{\nu}_\mu). \tag{3.4c}$$

The [4]:

$$(1,10)=\begin{pmatrix} 0 & \bar{u}_B & -\bar{u}_Y & c_R & s_R \\ -\bar{u}_B & 0 & \bar{u}_R & c_Y & s_Y \\ \bar{u}_Y & -\bar{u}_R & 0 & c_B & s_B \\ -c_R & -c_Y & -c_B & 0 & \bar{\mu} \\ -s_R & -s_Y & -s_B & -\bar{\mu} & 0 \end{pmatrix}, \tag{3.5a}$$

$$(2,\overline{10})=\begin{pmatrix} 0 & U_B & -U_Y & \bar{U}_R & \bar{D}_R \\ -U_B & 0 & U_R & \bar{U}_Y & \bar{D}_Y \\ U_Y & -U_R & 0 & \bar{U}_B & \bar{D}_B \\ -\bar{U}_R & -\bar{U}_Y & -\bar{U}_B & 0 & E \\ -\bar{D}_R & -\bar{D}_Y & -\bar{D}_B & -E & 0 \end{pmatrix}_p, \tag{3.5b}$$

$$(1,\bar{5})=\begin{pmatrix} \bar{s}_R \\ \bar{s}_Y \\ \bar{s}_B \\ \mu \\ \nu_\mu \end{pmatrix}. \tag{3.5c}$$

The [6]:

$$(1,\bar{5})=\begin{pmatrix} \bar{d}_R \\ \bar{d}_Y \\ \bar{d}_B \\ e \\ \nu_e \end{pmatrix}, \tag{3.6a}$$

$$(2,1)=(N)_p. \tag{3.6b}$$

The objects $D, U, \bar{D}, \bar{U}$ are colored, heavy-colored HC-quarks. $\bar{E}, E, \bar{N}$, and N are heavy-colored HC-leptons with the ordinary quantum numbers of $\bar{e}$, e, $\bar{\nu}$, and ν. One especially interesting feature of (3.4a) and (3.5a) is the interchange of the roles of $\bar{u}$ and $\bar{c}$ between the two multiplets. We will see that this identification keeps the mass matrix diagonal.

The 48 gauge bosons of SU(7) can be classified into several groups. First of all there are the usual 8 color gluons and 4 electroweak bosons. Twenty generators connect the 6 and 7 components $\psi_{\bar{e}}$, $\psi_{\bar{\nu}}$ to the remaining 5 components. These generators change an $SU(2)_{EW}$ index to a color or HC index. They will become superheavy ($\sim 10^{16}$ GeV) and will be ignored for the most part. There are 3 HC generators which remain unbroken. The corresponding heavy-color gluons mediate a confining force with a scale ~ 1 TeV. The 12 bosons which can connect the components ψ_T and ψ_d are called b bosons.

Finally a diagonal generator, orthogonal to hypercharge,

$$\frac{1}{\sqrt{140}}\begin{pmatrix} -5 & & & & & & \\ & -5 & & & & & \\ & & 2 & & & & \\ & & & 2 & & & \\ & & & & 2 & & \\ & & & & & 2 & \\ & & & & & & 2 \end{pmatrix}$$

is coupled to the gauge boson b'. The bosons b, the color and heavy-color gluons, and one linear combination of b' and hypercharge together correspond to a subgroup of $SU(5)_S$. The breaking of $SU(5)_S$ down to $SU(3)_G \times SU(2)_{HC}$ will cause b and b' to become massive.

IV. SYMMETRY BREAKING

At some very high energy ($\sim 10^{16}$ GeV) called the grand unification mass (GUM) SU(7) is a good symmetry. We shall require three separate stages of symmetry breakdown. The first occurs near the GUM and breaks SU(7) to $[SU(2)\times U(1)]_{EW} \times SU(5)_S$. [Note that we do *not* go through a stage where we break down to $SU(5)_{GG}$.] The group $SU(5)_S$ contains both color and heavy color. We shall not speculate further about this stage.

As energy comes down from the GUM the electroweak and $SU(5)_S$ coupling constants evolve independently according to the renormalization group.[6] This is depicted in Fig. 5. The second stage is also at a presently inaccessible energy of order[3] 100 TeV and breaks $SU(5)_S$ to $SU(3)_C \times SU(2)_{HC}$. (The unification of color and HC above 100 TeV has been suggested as a solution to the strong *CP* problem.[4,5]) We shall not speculate about the origin of this breakdown. However, it can be parametrized phenomenologically by a Higgs multiplet ϕ in the conjugate of the [2]. Indeed if

$$\langle\phi^{12}\rangle = -\langle\phi^{21}\rangle \neq 0\,, \tag{4.1}$$

then the required breakdown occurs. The 12 gauge bosons b receive equal masses and the boson b' also becomes massive.

During the first stage of symmetry breaking no fermion can gain mass. The invariance $[SU(2)\times U(1)]_{EW}\times SU(5)_S$ protects all fermions from mass generation. The second symmetry-breaking stage allows only one mass term to occur. For example, if a Higgs field ϕ^{ij} is used for the second stage, then the coupling (σ is a 2×2 spin matrix needed to make a scalar)

$$\psi_{ij}\,\sigma\,\psi_{kl}\,\phi^{ij}\phi^{kl} \tag{4.2}$$

between the Higgs field and the [2] is allowed by group theory. If $\langle\phi^{12}\rangle \neq 0$ then (4.2) generates the Majorana mass for $\bar{\nu}_\mu$. Thus we can assume that the $\bar{\nu}_\mu$ becomes a massive Majorana particle with $m \sim 100$ TeV.

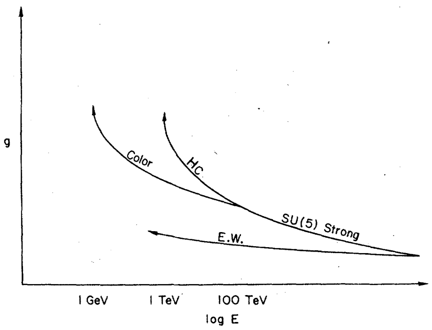

FIG. 5. Evolution of coupling constants with energy.

After this second symmetry breaking the coupling constants of $SU(2)_{HC}$ and $SU(3)_C$ evolve independently. We assume the HC coupling becomes large at ~1 TeV while the color coupling remains weak until ~1 GeV. [In fact, this is contrary to the expected renormalization behavior. This is of course due to the simplifying assumption that HC is SU(2).]

The strong HC interaction at 1 TeV precipitates the last stage of symmetry breakdown. At this energy only the HC interactions are large so it makes sense to study the HC world in isolation from the other interactions. Consider a closed world of HC gluons interacting with $U, \bar{U}, D, \bar{D}$ and $E, \bar{E}, N, \bar{N}$. As long as the other interactions are ignored the system has a global SU(16) symmetry which mixes the 16 left-handed fields among themselves. (Remember that heavy-color quarks come in three colors.)

A subgroup of this symmetry is ordinary chiral $SU(2)_{\text{left}}\times SU(2)_{\text{right}}$. To see this it is convenient to replace all the barred fields by their right-handed charge conjugates U_R, D_R, E_R, N_R. Because of the reality of the representations of $SU(2)_{HC}$ the right-handed particles transform equivalently to the left-handed ones under HC. The $SU(2)_{\text{left}}\times SU(2)_{\text{right}}$ subgroup is defined to act on the doublets $(U,D)_{\text{left}}$, $(U,D)_{\text{right}}$ and $(N,E)_{\text{left}}$, $(N,E)_{\text{right}}$.

As in QCD, we expect the HC interactions to cause a spontaneous breakdown of chiral symmetry by precipitating vacuum condensates which up to an SU(16) rotation have the form[2]

$$\langle\bar{U}U\rangle = \langle\bar{D}D\rangle = S$$

(no color sum) (4.3)

$$\langle\bar{E}E\rangle = \langle\bar{N}N\rangle = S\,.$$

When we turn on the color and electroweak interactions the condensates becomes determined up to an $SU(2)_{\text{left}}\times U(1)$ rotation. Requiring color invariance and electric charge conservation fixes the condensates to have the form (4.3).

In fact when ordinary color interactions as well as b' exchange are accounted for the HC-lepton and HC-quark condensates need not be equal. Thus we write

$$\begin{aligned}\langle\bar{U}U\rangle &= \langle\bar{D}D\rangle = S_Q\,,\\ \langle\bar{E}E\rangle &= \langle\bar{N}N\rangle = S_L\,.\end{aligned} \tag{4.4}$$

These condensates violate $[SU(2)\times U(1)]_{EW}$ according to the standard pattern and give mass to

Z and W bosons leaving the photon massless. The isospin $[\mathrm{SU}(2)_{\mathrm{left}}+\mathrm{SU}(2)_{\mathrm{right}}]$ invariance of the HC world insures[2] the empirically successful relation

$$\frac{M_W}{M_Z}=\cos\theta_W\,. \tag{4.5}$$

The nonvanishing expectation values of $\bar{U}U$, $\bar{D}D$, $\bar{N}N$, $\bar{E}E$ spontaneously violate 119 of the 255 generators of SU(16). Among these, 3 have the quantum numbers of the π^+, π^-, π^0 and these are absorbed by the $W^{\pm}$, Z. The remaining 116 are pseudo-Goldstone bosons which receive mass when the color and/or $[\mathrm{SU}(2)\times\mathrm{U}(1)]_{\mathrm{EW}}$ interactions are turned on.

V. INTERACTIONS MEDIATED BY *b* BOSONS

New interactions connecting heavy-colored states to heavy-color singlets are mediated by the heavy b bosons. The new interaction vertices are always between particles in the same SU(7) representation. The transitions can occur between two states if they are related by changing a color index ($i=3,4,5=R,Y,B$) to a heavy-color index ($i=1,2$). Thus, for example, we identify [see Eqs. (3.4a) and (3.4b)]

$$D_{Rp}=\psi_{3p}\quad(p=1,2)\,.$$

Changing the p index to a color index, say 4, gives the transition

$$\psi_{3p}\to\psi_{34}+\bar{b} \tag{5.1}$$

or

$$D_{Rp}\to\bar{c}_B+\bar{b}\,. \tag{5.2}$$

The new vertices are listed in Fig. 6 (by convention we label the negatively charged heavy boson b and its antiparticle $\bar{b}$).

Labeling HC-quarks, HC-leptons, quarks, and leptons by Q, L, q, l we see that three types of processes occur:

$$\begin{aligned}&Q\to\bar{q}+\bar{b}\,,\\&\bar{L}\to q+\bar{b}\,,\\&\bar{Q}\to l+\bar{b}\,.\end{aligned} \tag{5.3}$$

Evidently some exotic kinds of interactions can be mediated by exchange of two b's or $\bar{b}$'s. For example,

$$\bar{l}\to\bar{q}+2\bar{b}\,. \tag{5.4}$$

One might worry that baryon number might be violated by such processes. However, it is easy to see that there are two conserved quantities

$$\begin{aligned}&N_1=N_q-\tfrac{3}{2}N_L-\tfrac{1}{2}N_Q-\tfrac{1}{2}N_b\,,\\&N_2=N_l+\tfrac{1}{2}N_L-\tfrac{1}{2}N_Q+\tfrac{1}{2}N_b\,,\end{aligned} \tag{5.5}$$

where N_i is the number of particles minus antiparticles of type i. Since no Q's, $\bar{Q}$'s, L's, $\bar{L}$'s, or b's occur in final states of low-energy reactions, the conservation of N_1 and N_2 guarantee baryon and lepton conservation. Of course the 20 superheavy bosons mediate baryon violation as in $\mathrm{SU}(5)_{\mathrm{GG}}$.

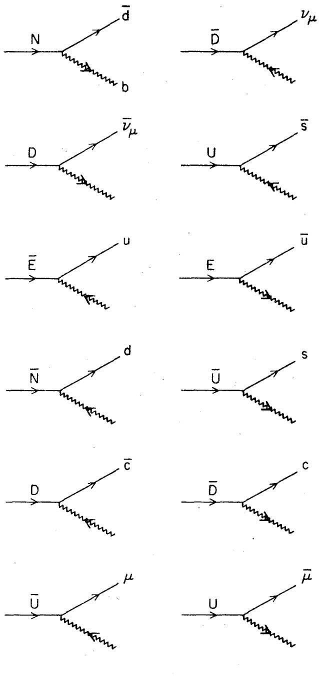

FIG. 6. Vertices involving the b meson.

VI. IMPLICATIONS

The main important effect mediated by b exchange is the mass generation of the ordinary fermions. This mass generation is a kind of radiative feeddown of the HC-quark and HC-lepton masses. The relevant graphs are shown in Fig. 7. The crosses on the HC-fermion lines in Fig. 7 indicate the absorption by vacuum condensates of a pair.[3] The induced masses will approximately be of the form

$$m_e=m_{\nu_e}=0\,, \tag{6.1a}$$

$$m_\mu\simeq\frac{3S_Q}{m_b^{\,2}}g_5^{\,2}\,, \tag{6.1b}$$

$$m_c=m_s\simeq\frac{2S_Q}{m_b^{\,2}}g_5^{\,2}\,, \tag{6.1c}$$

$$m_u=m_d\simeq 1\frac{S_L}{m_b^{\,2}}g_5^{\,2}\,. \tag{6.1d}$$

$g_5^{\,2}$ is the $\mathrm{SU}(5)_{\mathrm{S}}$ coupling constant at 100 TeV. It is common to all graphs and has no reason to be far from unity. The factors 3, 2, 1 are group

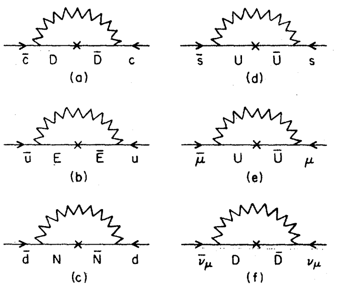

FIG. 7. Ordinary fermion mass generations. Note that an incoming left-handed particle is equivalent to an outgoing right-handed charge conjugate.

theoretic in origin. Taking $m_b \sim 100$ TeV would give ordinary masses in the hundreds of MeV.

At this point the reader can see why we made the unusual identification of particles in the $\underline{10}$'s. If we interchanged the $\bar{u}$ and $\bar{c}$ quarks the up-quark mass matrix would have been off diagonal.

The mass matrix is deficient in two ways. First of all the Cabibbo angle is zero. Secondly there are disappointing u-d and c-s degeneracies.

However, the mass matrix has two interesting features. The standard $\mathrm{SU}(5)_{\mathrm{GG}}$ mass relations $m_e = m_d$ and $m_\mu = m_s$ following from the simplest choice of Higgs fields do not follow. Also the radiative character of the mass mechanism naturally accounts for the smallness of fermion masses relative to electroweak bosons.

The muon neutrino ν_μ is coupled through the graph of Fig. 7(f) to a particle we have called $\bar{\nu}_\mu$. As discussed previously the $\bar{\nu}_\mu$ gets a Majorana mass ~100 TeV. The mixing of Fig. 7(f) results in a small Majorana mass order $m_\mu{}^2/m_{\bar{\nu}}$ for the ν_μ.

As mentioned previously, there are various pseudo-Goldstone bosons in the spectrum connected with the SU(16) chiral group of the HC sector. In particular, one of these bosons with $\overline{L}\overline{L}$ quantum numbers only receives mass from b's and $\mathrm{U}(1)_{\mathrm{EW}}$. This spinless boson has the quantum numbers of $\overline{E}\overline{N}$. The vertices in Fig. 6 permit it to decay to a muon antineutrino and a charmed baryon composed of u, d, c. The leading process is shown in Fig. 8.

The U(1) contribution to the mass of this $\overline{L}\overline{L}$ pseudo-Goldstone boson is analogous to the electromagnetic shift of the pion mass. Roughly speaking we should expect the mass of the $\overline{L}\overline{L}$ state to be

$$m_{\bar{L}\bar{L}}{}^2 \sim (m_{\pi^+}{}^2 - m_{\pi^0}{}^2)\frac{F_\pi{}^2}{f_\pi{}^2}\,, \tag{6.2}$$

where F_π and f_π are the axial-vector couplings (pion decay constants) for HC-pions and pions.

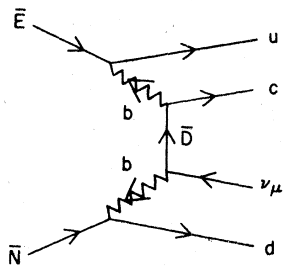

FIG. 8. The decay of the $\overline{E}\overline{N}$ state.

From the relation

$$M_W = g_{\mathrm{EW}}\frac{F_\pi}{2}$$

we know[2] $F_\pi/f_\pi \sim 2000$. This gives

$$m_{\bar{L}\bar{L}} \sim 70 \text{ GeV}.$$

The contribution arising from b exchange may be comparable. Accordingly $m_{\bar{L}\bar{L}}$ ought to be comparable to the Z and W masses.

An additional amusing feature of the toy model is the absence of nucleon decay. The usual $\mathrm{SU}(5)_{\mathrm{GG}}$ proton decay process is shown in Fig. 9. Because of the peculiar interchange of $\bar{c}$ and $\bar{u}$ in the $\underline{10}$'s the top vertex of Fig. 9 has $\bar{u}$ replaced by $\bar{c}$, thus eliminating proton decay. This corresponds to a 90° right-handed "Cabibbo-type rotation" as discussed by Jarlskog.[10]

We do not suggest that proton decay is readily forbidden. In a more realistic model with nonvanishing Cabibbo angle, the natural interpretation would be that proton decay is Cabibbo suppressed.

Similarly a nonvanishing θ_c might also allow the c quark in Fig. 8 to be replaced by a u quark. This would lead to the interesting prediction of an $\overline{E}\overline{N}$ narrow resonance coupled to the proton-muon-antineutrino channel.

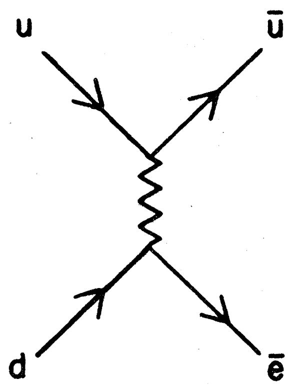

FIG. 9. Proton decay in ordinary $\mathrm{SU}(5)_{\mathrm{GG}}$.

VII. COMMENTS

We have exhibited a toy model of a grand unified theory including the known forces and heavy color. The representation content of the model yields two ordinary fermion families and a heavy-color family. The HC sector generates masses both for electroweak bosons and the ordinary quarks and leptons.

One of the most significant features of our model is the way in which the relation

$$\cos\theta_W = M_W/M_Z$$

arises. This relation is not a general consequence of the $SU(2)\times U(1)_{EW}$ structure. It follows from the higher $SU(2)_{\text{left}}\times SU(2)_{\text{right}}$ symmetry of the heavy color world. This symmetry requires the somewhat surprising occurrence of right-handed heavy-color neutrinos N_R. In general, a different group structure or choice of fermion representation would not reproduce this result.

The shortcomings of the model are serious. No mechanism was offered to explain the 100-TeV symmetry breaking of the strong SU(5) down to color and HC. Furthermore, the subsequent evolution of the $SU(3)_C$ and $SU(2)_{HC}$ couplings is required to be perverse in that the HC coupling must become strong before color. Both of these features and the lack of another family (t, b, τ, ν_τ) point toward a larger group structure.

A candidate model can easily be constructed to include three families of fermions. Consider the group SU(9) and the anomaly-free representation $[2]+[4]+[6]+[8]$. At the grand unification mass it breaks down to $SU(2)\times U(1)_{EW}\times SU(7)_S$. At 100 TeV $SU(7)_S$ breaks down to $SU(3)_C\times SP(4)_{HC}$ where SP(4) is the subgroup of SU(4) matrices which have the property

$$U\eta U^T = \eta \quad (T = \text{transpose}).$$

Here η is the symplectic metric

$$\eta = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} .$$

The breakdown can be induced by a Higgs field in the [2] with expectation value

$$\begin{array}{cc} & \begin{array}{ccc} \text{HC} & \text{C} & \text{EW} \end{array} \\ \phi_{ij} = & \begin{bmatrix} \eta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} .$$

The representations of SP(4) are all real like those of SU(2). The $\underline{4}$ and $\underline{\bar{4}}$ of SU(4) both transform as $\underline{4}$'s of SP(4). The antisymmetric tensor which is a $\underline{6}$ in SU(4) becomes a $\underline{5}+\underline{1}$ in SP(4). The breakdown of the fermion content into $SP(4)_{HC}\times SU(5)_{GG}$ is as follows:

$$[2] = (1, 10) + (4, 5) + (1, 1) + (5, 1),$$

$$[4] = (1, \bar{5}) + (4, \overline{10}) + (5, 10) + (1, 10) + (4, 5) + (1, 1),$$

$$[6] = (4, 1) + (5, \bar{5}) + (1, \bar{5}) + (4, \overline{10}) + (1, 10),$$

$$[8] = (4, 1) + (1, \bar{5}).$$

This material is just sufficient to assemble two families of heavy-color families in the $\underline{4}$ and one in the $\underline{5}$, $\underline{3}$ ordinary families and 2 heavy Majorana neutrinos.

This model and our toy model belong to a sequence in which all of the good features of our toy model are preserved. This sequence, discovered by Georgi,[9] is defined by the group $SU(2n+1)$ with fermions in the representations $[2], [4], \ldots, [2n]$. The strong group $SU(2n-1)$ is broken to SU(3) $\times SP(2n-4)$. These models contain $n-1$ ordinary families. A paper discussing these points in detail is in preparation.

Notes added:

(1) Closely related models based on O(18) have been considered by H. Georgi and E. Witten and discussed by H. Georgi at the Lepton-Quark meeting in Hamburg in 1978.

(2) Ken Lane has discovered a pair of true Goldstone bosons in our model. To see them let $\chi[m]$ be the field for fermions in the $[m]$ of SU(7). Then the current densities

$$j_0[m] = \chi^\dagger[m]\chi[m]$$

are conserved up to anomalies. Since $j[m]$ is SU(7) invariant, its anomaly is determined up to a numerical factor

$$\partial_\mu j_\mu[m] = c(m)\tilde{F}F$$

Evidently two anomaly-free linear combinations fail to commute with the condensates and therefore are realized in the Goldstone-Nambu mode. The problem is obviously due to the reducibility of the fermion representation. Fortunately an elegant escape is available. Georgi, and independently, Bjorken, have pointed out to us that our fermion representation is actually a single spinor representation of O(14) in which SU(7) is embedded. There are 43 new generators and a subset of these can mediate transitions between the different SU(7) multiplets, thus eliminating the unwanted conserved currents.

(3) P. Ramond has emphasized the possible occurrence of Majorana neutrino masses in models based on orthogonal groups. See his lecture at the

Sanibel Symposium in Florida in 1979, Caltech Report No. CALT-68-709.

ACKNOWLEDGMENTS

We wish to thank S. Raby, P. Sikivie, and J. Bjorken for helpful discussions. We would like to thank Ken Lane for his continuing effort to root out all the difficulties with our model. This work was supported in part by the Department of Energy under Contract No. DE-AC03-76SF00515. The work of L.S. was supported in part by the National Science Foundation.

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