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Conditions for renormalizability of quantum flavor dynamics

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The presence of a new anomaly, in addition to the Adler-Bell-Jackiw (ABJ) anomaly, is hinted at for gauge theories with γ_5 couplings. The ABJ anomaly is discussed first by translation of variables of a linearly divergent integral, then by dimensional regularization. Using the second method, the general non-Abelian case is considered in the presence of an overlapping divergence. A new anomaly is suggested which is not, in general, canceled by the usual restrictions because fermion masses are involved. Assuming no cancellation between different Feynman diagrams and current-algebra quark masses then leads to the conclusion that the standard model of quantum flavor dynamics (i.e., that of Glashow, Salam, and Weinberg) might be nonrenormalizable. Imposition of renormalizability would then imply that new dynamical constraints be met. Hence, only that part of the quark mass corresponding to the lepton mass in the same quark-lepton generation is generated by the electroweak interactions. The remaining mass comes presumably from the strong interaction for which the dynamical theory must therefore have at least some flavor dependence.

I. INTRODUCTION

The demonstration[1] that Yang-Mills theory with spontaneous symmetry breaking and the Higgs mechanism is renormalizable marked a major turning point in weak-interaction theory. A unified theory[2] of electromagnetic and weak interactions now exists and agrees[3] remarkably well with low-energy data. Further experiments at high energy are needed to search for the intermediate vector bosons and Higgs scalar particles; if these are successfully found and have the predicted properties, the theory will become well established.

In proving the renormalizability[4,5] of gauge theory, it is crucial that the renormalized Lagrangian be itself locally gauge invariant under a group of transformations isomorphic to those leaving the bare Lagrangian invariant. This is necessary because gauge invariance must be preserved order by order in the renormalized perturbation series. This is a nontrivial requirement because renormalization involves an infinite reordering of the perturbation expansion. It is necessary because, otherwise, perturbative unitarity, and hence full unitarity, is violated.

To preserve gauge invariance, the most suitable method is to use dimensional regularization[6] which, in general, preserves the form of the relevant Ward identities while rendering divergent integrals finite in a generic space-time dimension, n. The counterterms are separated off as poles in $(4-n)^{-1}$.

Without fermions coupling to the gauge fields according to $\bar{\psi}\gamma_\mu\gamma_5\psi A_\mu$, the procedure is straightforward. For example, the perturbative renormalizability of quantum chromodynamics has no problem in this respect.

For weak interactions, however, such parity-violating couplings are inevitable and one must confront the triangle anomaly[7] of Adler, Bell, and Jackiw (ABJ). This presents no difficulty for open fermion lines which both enter and leave the diagram as external particles; for such a case, one simply defines[8] an entity γ_5 in arbitrary dimension which anticommutes with all γ_μ.

For closed fermion loops involving an odd number of γ_5 couplings, the handling of dimensional regularization is more problematic, and that is our present subject. Since a Dirac trace is involved, there is no really satisfactory generalization of γ_5 to arbitrary dimension, despite several attempts.[9] For example, we may observe that the lowest-dimensional representation of the Dirac algebra has dimension $2^{[d/2]}$ in d space-time dimensions. An analog $\Gamma^7, \Gamma^9, \ldots$ etc., exists for γ_5 in $6, 8, \ldots$ and all even dimensionalities, d. However, one then has the difficulty that e.g.

$$\mathrm{Tr}(\Gamma^7\Gamma_\alpha\Gamma_\beta\Gamma_\gamma\Gamma_\delta)=0 \text{ (for } d=6), \tag{1}$$

whereas

$$\mathrm{Tr}(\gamma_5\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta)=4i\epsilon_{\alpha\beta\gamma\delta} \text{ (for } d=4) \tag{2}$$

and this would lead to obvious inconsistencies.

The conventional wisdom for a general Feynman diagram containing, say a small triangular loop (Fig. 1) is to give up any attempt to generalize γ_5. Instead, the Bardeen prescription[10] is to regularize dimensionally all meson loops first, then to evaluate the triangle Dirac trace in four dimensions. This has been checked[11] for cer-

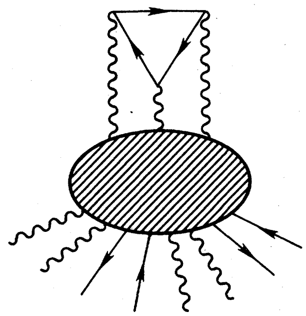

FIG. 1. General Feynman diagram containing the triangle anomaly.

tain two-loop diagrams and leads to no inconsistency. The type of problem that we discuss in this paper does not appear until at least three loops so it is technically difficult.

Concerning the ABJ anomaly, it is most easily obtained by considering the translation of variables in the linearly divergent integral. It is found in this way in Sec. II. It is known[12] that cancellation[13] of this (AVV) triangle anomaly cancels the anomaly at the one-loop level in related graphs (Fig. 2). Also, radiative corrections to the anomaly have been considered in the Adler-Bardeen theorem[14] which, however, will not be used here.

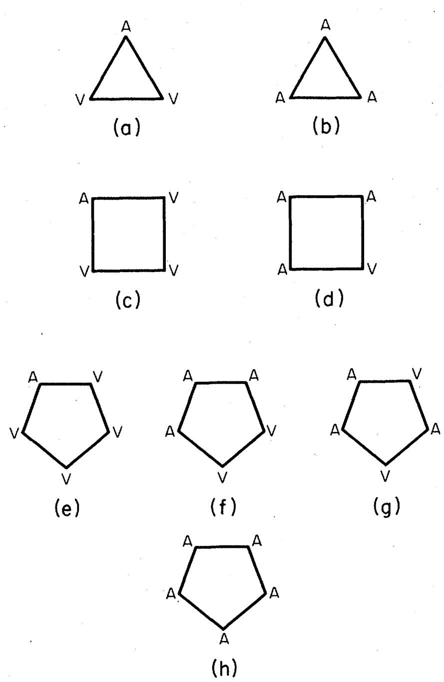

FIG. 2. One-loop diagrams affected by the anomaly (cf. Ref. 12).

The ABJ anomaly is obtained by the dimensional-regularization method in Sec. III, since we wish to expand in $(4-n)$ in Sec. IV which considers the general non-Abelian case and develops the new anomaly, not canceled by the usual restrictions on flavor. Finally, in Sec. V is a discussion of the status of renormalizability and of the dynamical constraints arising from the imposition of renormalizability.

II. SHIFTING INTEGRATION VARIABLE

The shortest route to obtain the correct ABJ anomaly is to consider the effect of translating the variable in the linearly divergent momentum integral occurring in the triangle diagram.

We take the (Abelian) interaction

$$\mathcal{L} = -e_A\bar{\psi}\gamma_\mu\gamma_5\psi Z_\mu^A - e_V\bar{\psi}\gamma_\mu\psi A_\mu\,, \tag{3}$$

and consider the case where the loop momentum k_μ is defined as indicated in Fig. 3(a). The Feynman rules then give the corresponding amplitude

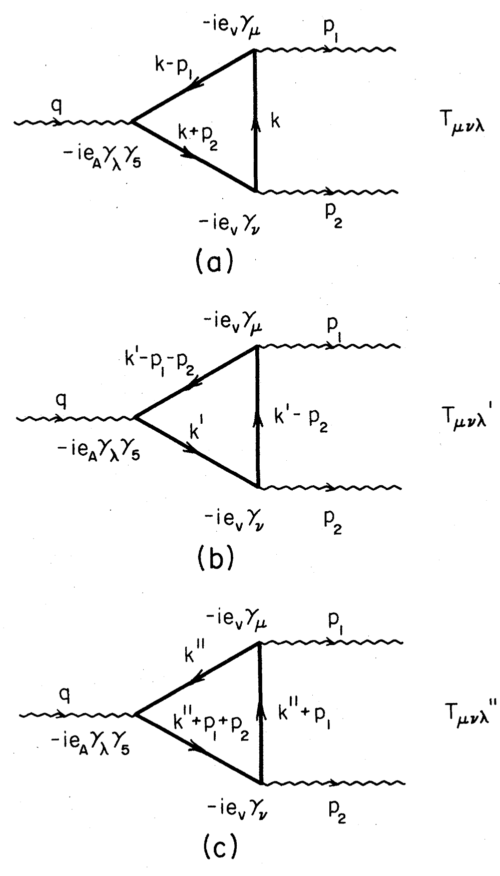

FIG. 3. Shifts of integration variable.

$$T_{\mu\nu\lambda}=-e_A e_V{}^2\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}[\not k\gamma_\mu(\not k-\not p_1)\gamma_\lambda\gamma_5(\not k+\not p_2)\gamma_\nu]}{k^2(k-p_1)^2(k+p_2)^2}. \quad (4)$$

Here we have set the fermion mass equal to zero since the linear divergence is independent of this mass. Computation of $T_{\mu\nu\lambda}$ reveals that it is Bose symmetric under the interchange $\{p_1,\mu\}\leftrightarrow\{p_2,\nu\}$ so that addition of the crossed diagram gives merely a factor of 2. If we contract $T_{\mu\nu\lambda}$ with $q_\lambda=(p_1+p_2)_\lambda$ and rewrite

$$(\not p_1+\not p_2)\gamma_5=-(\not k-\not p_1)\gamma_5-\gamma_5(\not k+\not p_2)\,, \quad (5)$$

then $q_\lambda T_{\mu\nu\lambda}$ is seen to be the sum of two terms, each of which is a second-rank pseudotensor depending on only one four-momentum and hence vanishes.

To examine the contractions of $p_{1\mu}$ and $p_{2\nu}$ with $T_{\mu\nu\lambda}$ it is necessary to make shifts in the integration variable $k'_\mu=(k+p_2)_\mu$ and $k''_\mu=(k-p_1)_\mu$, respectively, whereupon the result vanishes by an argument similar to that of the previous paragraph. These two integration shifts correspond to the momentum labelings of Figs. 3(b) and 3(c), respectively.

Shifting the integration variable in $T_{\mu\nu\lambda}$ by an amount $k'_\mu=(k+a)_\mu$ results in the change (defining $T_{\mu\nu\lambda}=-e_A e_V{}^2 t_{\mu\nu\lambda}$)

$$t'_{\mu\nu\lambda}=t_{\mu\nu\lambda}+c_{\mu\nu\lambda\alpha}a_\alpha\,, \quad (6)$$

where, after evaluating the Dirac trace,

$$c_{\mu\nu\lambda\alpha}=-\frac{4i}{(2\pi)^4}\int d^4k\,\frac{\partial}{\partial k_\alpha}\left(\frac{\epsilon_{\mu\nu\lambda\beta}k_\beta}{k^4}\right) \quad (7)$$

$$=-\frac{1}{8\pi^2}\epsilon_{\mu\nu\lambda\alpha}\,. \quad (8)$$

To ensure that the vector Ward identities are satisfied we choose the contact term as

$$t'_{\mu\nu\lambda}=t_{\mu\nu\lambda}-\frac{1}{8\pi^2}\epsilon_{\mu\nu\lambda\alpha}(p_1-p_2)_\alpha\,, \quad (9)$$

which satisfies

$$p_{1\mu}t'_{\mu\nu\lambda}=p_{2\nu}t'_{\mu\nu\lambda}=0\,, \quad (10)$$

and then gives for the axial-vector anomaly

$$q_\lambda t'_{\mu\nu\lambda}=+\frac{1}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\,. \quad (11)$$

This establishes notation and evaluates uniquely in lowest order the well-known ABJ anomaly.[7]

III. DIMENSIONAL REGULARIZATION: MASSLESS CASE

Let us reexamine the quantity $t_{\mu\nu\lambda}$ of the previous section, now using the technique of dimensional regularization. This enables us to rederive the ABJ anomaly again, but the main motivation is to set the stage for the general non-Abelian case in the next section. Unlike the ABJ anomaly, we are now able to derive the new results given there only by the dimensional method. Nevertheless, this choice of regularization method is only one of mathematics not of physics, and the results do not depend on it. For the non-Abelian case, the dimensional method is the only one available that is otherwise consistent, and it is also the one involved in the Bardeen prescription.[10]

With this motivation, we therefore rewrite

$$t_{\mu\nu\lambda}=\frac{8i}{(2\pi)^4}\int_0^1 dx\int_0^{1-x}dy\int\frac{d^4k}{(k^2+2k\cdot Q-M^2)^3}\times T_{\delta\epsilon\xi,\,\mu\nu\lambda}(k+p_2)_\delta k_\epsilon(k-p_1)_\xi\,, \quad (12)$$

where

$$Q_\mu=xp_{2\mu}-yp_{1\mu}\,, \quad (13)$$

$$M^2=-p_2{}^2x-p_1{}^2y\,, \quad (14)$$

$$T_{\delta\epsilon\xi,\,\mu\nu\lambda}=\epsilon_{\rho\delta\nu\epsilon}(g_{\rho\mu}g_{\xi\lambda}-g_{\rho\xi}g_{\lambda\mu}+g_{\rho\lambda}g_{\mu\xi})-\epsilon_{\rho\mu\xi\lambda}(g_{\rho\delta}g_{\nu\epsilon}-g_{\rho\nu}g_{\delta\epsilon}+g_{\rho\epsilon}g_{\delta\nu})\,. \quad (15)$$

After dimensional regularization (setting $d^4k\to d^nk$) one evaluates

$$t_{\mu\nu\lambda}=-\frac{1}{4\pi^2}\int\frac{dxdy}{(-Q^2-M^2)^{3-n/2}}[\Gamma(3-\tfrac{1}{2}n)A_{\mu\nu\lambda}+\Gamma(2-\tfrac{1}{2}n)(-Q^2-M^2)B_{\mu\nu\lambda}]\,, \quad (16)$$

$$A_{\mu\nu\lambda}=(A_1p_{1\alpha}+A_2p_{2\alpha})\epsilon_{\mu\nu\lambda\alpha}+(_3p_{1\lambda}+A_4p_{2\lambda})\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}+(A_5p_{1\nu}+A_6p_{2\nu})\epsilon_{\mu\lambda\alpha\beta}p_{1\alpha}p_{2\beta}+(A_7p_{1\mu}+A_8p_{2\mu})\epsilon_{\nu\lambda\alpha\beta}p_{1\alpha}p_{2\beta}\,, \quad (17)$$

$$B_{\mu\nu\lambda}=(B_1p_{1\alpha}+B_2p_{2\alpha})\epsilon_{\mu\nu\lambda\alpha}, \quad (18)$$

with

$$A_1=-p_1\cdot p_2y(1-y)(1-2x)+p_2{}^2x(1-x)(1-y)-p_1{}^2y^2(1-y)\,, \quad (19)$$

$$A_2=-p_1\cdot p_2xy(1-2x)+x^2(1-x)p_2{}^2-xy^2p_1{}^2\,, \quad (20)$$

$$A_3=-A_7=-y(1-y)\,, \quad (21)$$

$$A_4=-A_8=-xy\,, \quad (22)$$

$$A_5=y(1-2x-y)\,, \quad (23)$$

$$A_6 = -x(2 - 2x - y)\ , \tag{24}$$

$$B_1 = 3y - 1 + \tfrac{1}{2}(4-n)(1-y)\ , \tag{25}$$

$$B_2 = 1 - 3x + \tfrac{1}{2}(4-n)x\ . \tag{26}$$

With these algebraic results, we may then calculate the contractions of $p_{1\mu}$, $p_{2\nu}$, and q_λ with $t_{\mu\nu\lambda}$. The results are that the vector Ward identities are violated by $t_{\mu\nu\lambda}$, as expected, and thus a contact term must be added of the form

$$t'_{\mu\nu\lambda} = t_{\mu\nu\lambda} + \epsilon_{\mu\nu\lambda\alpha}(\gamma^1 p_{1\alpha} + \gamma^2 p_{2\alpha})\ . \tag{27}$$

The coefficients γ^1, γ^2 are determined by the requirement of Eq. (10) and hence the axial-vector anomaly is found with the unique answer (in n dimensions)

$$q_\lambda t'_{\mu\nu\lambda} = \frac{1}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\int \frac{dx\,dy}{(-Q^2-M^2)^{2-n/2}}\Gamma(3-\tfrac{1}{2}n)\ ,$$

$$= \frac{1}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta} \tag{28}$$

$$\times \int_0^1 dx \int_0^{1-x} dy[1 - \tfrac{1}{2}(4-n)\ln(-Q^2-M^2) + \cdots]$$

$$\times [1 + \tfrac{1}{2}\Gamma'(1)(4-n) + \cdots]\ . \tag{29}$$

For $n = 4$, we obtain the same result as in the previous section, namely, Eq. (11).

If we now insert a nonzero fermion mass m, there are two changes: (i) In the propagator denominators, the value of M^2 changes from its value given in Eq. (14) by an amount

$$\Delta M^2 = m^2\ . \tag{30}$$

(ii) In the Dirac trace, there are terms proportional to m^2; however, these contributions are the expected ones from the pseudoscalar source current $(\bar\psi\gamma_5\psi)$ in the Ward identity.

The change represented by Eq. (30) has significance only when there is an overlapping divergence. Then it is possible for a $(4-n)$ factor to be canceled by a pole in $(4-n)^{-1}$ and the nonleading term of Eq. (29) becomes relevant. This term depends on the fermion mass. Thus, although the ABJ anomaly is mass independent, as it depends only on the leading linear divergence, the term on the right-hand side of Eq. (29) proportional to $(4-n)$ does involve m. Unlike the ABJ anomaly, therefore, this new anomaly cannot be canceled by the simple device of adding an arbitrarily massive additional fermion to the theory. We do not pursue this here, but study the only slightly more complicated non-Abelian case which is directly relevant to the more realistic quantum flavor dynamics of the electroweak interactions.

IV. GENERAL NON-ABELIAN CASE

In a non-Abelian gauge theory, the situation concerning the fermion masses in the triangle diagram is different from that discussed above. Because the group matrices have nondiagonal elements, the masses occurring on the three sides may be equal or unequal. The kinematics is illustrated in Fig. 4. There, the flavor matrices $T^a_{\alpha\beta}$ are understood to include the relevant coupling constant as well as the sign corresponding to right- or left-handed helicities. The amplitude appropriate to Fig. 4 is given by

$$T^{abc}_{\mu\nu\lambda} = T^a_{\alpha\beta}T^b_{\beta\gamma}T^c_{\gamma\alpha}t_{\mu\nu\lambda}(p_1,p_2;m_\alpha,m_\beta,m_\gamma;n)\ , \tag{31}$$

with

$$t_{\mu\nu\lambda} = \int \frac{d^4k}{(2\pi)^4}\frac{1}{[(k+p_2)^2-m_\gamma{}^2][(k^2-m_\alpha{}^2)][(k-p_1)^2-m_\beta{}^2]}$$

$$\times \mathrm{Tr}[\gamma_5(\not k + \not p_2 + m_\gamma)\gamma_\nu(\not k + m_\alpha)\gamma_\mu(\not k - \not p_1 + m_\beta)\gamma_\lambda]\ . \tag{32}$$

To include also the crossed graph, we form the combination

$$I^{abc}_{\mu\nu\lambda}(p_1,p_2;m_\alpha,m_\beta,m_\gamma;n) = T^{abc}_{\mu\nu\lambda}(p_1,p_2;m_\alpha,m_\beta,m_\gamma;n) + T^{bac}_{\nu\mu\lambda}(p_2,p_1;m_\alpha,m_\beta,m_\gamma;n)\ . \tag{33}$$

Finally, we sum over all flavors of quarks and lepton occurring in the theory to obtain the full triangular vertex

$$\Gamma^{abc}_{\mu\nu\lambda} = \sum_{\alpha,\beta\gamma} I^{abc}_{\mu\nu\lambda}(p_1,p_2;m_\alpha,m_\beta,m_\gamma;n)\ . \tag{34}$$

A necessary condition for renormalizability is then that

$$q_\lambda\Gamma^{abc}_{\mu\nu\lambda} = 0\ . \tag{35}$$

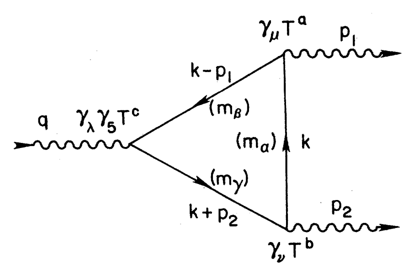

FIG. 4. Kinematics in the non-Abelian case.

We can compute the left-hand side of Eq. (35) by considering first $t'_{\mu\nu\lambda}$ obtained from $t_{\mu\nu\lambda}$ of Eqs. (31) and (32) by adding a contact term which ensures the vector Ward identities. By steps parallel to those of the previous section, we arrive at

$$q_\lambda t'_{\mu\nu\lambda} = \frac{1}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\Gamma(3-\tfrac{1}{2}n) \times \int dx \int dy \; \frac{1}{(-Q^2-M^2)^{2-n/2}}. \tag{36}$$

The only change for the present case lies in the formula for M^2 which is given by Eq. (14) with the addition of

$$\Delta M^2 = m_\alpha{}^2(1-x-y) + m_\beta{}^2 y + m_\gamma{}^2 x, \tag{37}$$

which is a generalization of Eq. (30). Expanding Eq. (36) in terms of $(4-n)$ now gives

$$q_\lambda t'_{\mu\nu\lambda} = \frac{1}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta} \times \left\{1 + \left[\tfrac{1}{2}\Gamma'(1) - \int dx\, dy \ln(-Q^2-M^2)\right] \times (4-n) + \cdots \right\}. \tag{38}$$

The 1 in the curly brackets of Eq. (38) is just the ABJ anomaly again, and it is canceled in Eq. (35) provided that

$$\mathrm{Tr}(T^c\{T^a,T^b\}_+) = 0, \tag{39}$$

which is the usual cancellation condition.[13]

When there is an overlapping divergence, however, we must consider the higher-order terms in $(4-n)$, and the crucial point is that these do depend on the flavors $\{\alpha,\beta,\gamma\}$ through the fermion masses in Eq. (37). Thus the flavor sum in Eq. (34) does not simplify to a trace [as in Eq. (39)], in general, unless further restrictions are met. Since these extra terms depend on the energies $p_1{}^2, p_2{}^2, q^2$ and since the cancellation must hold for all such energies, the necessary and sufficient condition is that the masses $m_\alpha, m_\beta, m_\gamma$ are degenerate within each generation of fermions for which Eq. (39) holds. For example, in the standard model[2] of quantum flavor dynamics, the quarks cancel against leptons within the sequential generations $(u,d,e),(c,s,\mu),(t,b,\tau)$. Thus these generations should be mass degenerate[15] to avoid the new anomaly.

To conclude this section, we add two remarks.

(1) We have chosen to employ dimensional regularization methods, but the new anomaly arising from the overlapping divergence is expected to be independent of this choice.

(2) The anomaly occurs in one particular Feynman diagram (or the sum of two if we include the crossed diagram), and it is still possible that there is a cancellation between different Feynman diagrams at the same order of perturbation theory. This logical possibility is the first of two considered in the next section.

V. STATUS OF RENORMALIZABILITY: DYNAMICAL CONSTRAINTS

Let us consider the standard model of quantum flavor dynamics.[2] This is the $SU(2)\otimes U(1)$ model of leptons and quarks with six flavors in sequential left-handed doublets. The ABJ anomaly is canceled between quarks and leptons within each quark-lepton generation g with $g_1=(u,d,e)$, $g_2=(c,s,\mu)$, $g_3=(t,b,\tau)$.

Concerning the new anomaly, there are two distinct logical possibilities as follows:

(1) There may be cancellation of the new anomaly between different Feynman diagrams at a fixed order of perturbation theory. In this case, there is no new dynamical constraint on the fermion masses. That such cancellation might take place is perhaps suggested (though not, of course, demonstrated) by the following considerations. If we examine the $SU(2)\otimes U(1)$ theory at finite temperature,[16] using Green's functions defined by

$$G_T(x_1\cdots x_n) = \frac{\sum_\alpha e^{-E_\alpha/kT}\langle\alpha|T(\phi(x_1)\cdots\phi(x_n))|\alpha\rangle}{\sum_\alpha e^{-E_\alpha/kT}\langle\alpha|\alpha\rangle}, \tag{40}$$

then above a critical temperature T_c the symmetry is expected to be restored, the stable vacuum has ${}_0\langle\phi(x)\rangle_0=0$, and the fermion masses become degenerate and equal to zero. The new anomaly is then absent. When the theory is cooled through the phase transition at T_c and down to $T=0$, there might persist sufficient memory of the original symmetry so that the new anomaly remains absent by interdiagrammatic cancellations. This appears technically difficult to check directly, since three-loop diagrams are involved. However, in this case the quark masses can achieve their current-algebra values through the Higgs mechanism.[17]

(2) If the cancellation described in case (1) does not take place, then renormalizability imposes further dynamical constraints in addition to the usual ones.[13] The constraints are that the electromagnetic and weak contributions to the quark masses are such that $m_u=m_d=m_e$, $m_c=m_s=m_\mu$, and so on. The quark masses can

now separate from the lepton mass in each generation through the color effect of quantum chromodynamics (QCD). However, it would then appear that the strong interactions should not be completely flavor independent as in QCD in order that the (large) mass difference $m_u \neq m_d$, $m_c \neq m_s$ be explained.

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