Horizontal symmetry and mass of the t quark

Tsutomu Yanagida
Department of Physics, Tohoku University, Sendai 980, Japan
(Received 31 May 1979)

We estimate masses of the b and t quarks as $m_b = 4$–6 GeV and $m_t = 16$–19 GeV, assuming the horizontal gauge symmetry, $SU_F(3)$.

The idea of unifying electronic and muonic matter by adding the horizontal symmetry $SU_F(2)$[1,2] to the weak and electromagnetic group SU(2) $\times$U(1)[3] has recently been suggested. As a consequence of gauging the symmetry, the conservation of muon number is violated. Exchange of extra gauge bosons S_μ^a ($a=1-3$) also induces a superweak type of CP nonconservation.[4,1] From the data on CP violation in $K_L^0 \to 2\pi$ decay, the effective coupling constant G_S of S_μ^a to leptons and quarks is determined[1] as $G_S \sim 10^{-15}$ GeV^{-2} unless an accidental cancellation occurs. The strength is weak enough to avoid unwanted flavor-changing transitions.

If six leptons and six quarks exist, it is natural to extend the horizontal symmetry $SU_F(2)$ to $SU_F(3)$. All fermions are assumed to transform as $SU_F(3)$ triplets. The triangle anomalies[5] in the lepton sector can be removed by introducing right-handed neutrinos.[6] Another possibility of assigning the fermions to $SU_F(2)$ triplets has been studied by Wilczek and Zee.[2] In this case the anomalies are not generated even without right-handed neutrinos. However, the former assignment seems more natural from the viewpoint of the composite model of leptons and quarks proposed by Terazawa[7] and Ne'eman.[8]

The purpose of this short note is to estimate masses of the b and t quarks on the basis of the $SU_F(3)\times SU(2)\times U(1)$ gauge model. In order to calculate the effects of the mass renormalization, we will also assume an asymptotically free Yang-Mills gauge theory of color SU(3) for strong interactions.[9]

The assignment for leptons and quarks is as follows

$$\begin{bmatrix}\nu\\ l\end{bmatrix}_L = \underset{(3,\ 2,\ -1)}{\begin{bmatrix}\nu_e & \nu_\mu & \nu_\tau\\ e & \mu & \tau\end{bmatrix}_L},\quad \underset{(3,\ 1,\ 0)}{\nu_R = [\nu_e\ \ \nu_\mu\ \ \nu_\tau]_R},\quad \underset{(3,\ 1,\ -2)}{l_R = [e\ \ \mu\ \ \tau]_R}, \tag{1}$$

$$\begin{bmatrix}u\\ d\end{bmatrix}_L = \underset{(3,\ 2,\ \frac{1}{3})}{\begin{bmatrix}\mathcal{P} & c & t\\ \mathcal{N} & \lambda & b\end{bmatrix}_L},\quad \underset{(3,\ 1,\ \frac{4}{3})}{u_R = [\mathcal{P}\ \ c\ \ t]_R},\quad \underset{(3,\ 1,\ -\frac{2}{3})}{d_R = [\mathcal{N}\ \ \lambda\ \ b]_R}. \tag{2}$$

Here the first two values in each pair of parentheses are the representation dimensions of $SU_F(3)\times SU(2)$ and the last one the U(1) hypercharge.

We introduce a Higgs scalar $\chi^{ab} = (6, 1, 0)$ to give large masses to the horizontal gauge bosons, S_μ^a ($a=1-8$). This scalar has a Yukawa coupling with right-handed neutrinos and their charge-conjugated fields.[6] Therefore, it may be attributed to large vacuum expectation values $\langle\chi^{ab}\rangle$ that the right-handed neutrinos have not been found at energies so far explored. We assume, furthermore, a Higgs scalar $\phi^a = (8, 2, 1)$ to break the symmetry $SU(2)\times U(1)$ down to the electromagnetic one.

Masses of fermions are generated by the Yukawa interactions of ϕ^a as follows (neglecting the neutrino sector):

$$\mathcal{L}_{\rm mass} = -G^l \bar{l}_R\langle\hat{\Phi}\rangle^\dagger l_L - G^d \bar{d}_R\langle\hat{\Phi}\rangle^\dagger d_L - G^u \bar{u}_R\langle\hat{\Phi}\rangle u_L + \text{H.c.}, \tag{3}$$

where $\hat{\Phi} = \sum\lambda^a\phi^a$. If all vacuum expectation values $\langle\phi^a\rangle$ are not in phase, we have nonvanishing weak mixing angles. The weak mixing matrix $U_{\rm weak}$ is defined in terms of the quark-rotation matrices as $U_{\rm weak} = U_L^{\rm up\dagger} U_L^{\rm down}$. The fermion-rotation matrices have the following relations to each other:

$$U_L^{\rm lepton} = U_L^{\rm down} = U_R^{\rm up}, \tag{4}$$

$$U_R^{\text{lepton}} = U_R^{\text{down}} = U_L^{\text{up}} . \tag{5}$$

The lepton-mixing angles are of interest in discussing neutrino oscillations, since the left-handed neutrinos have small masses in general through Yukawa couplings to heavy Majorana neutrinos (right-handed neutrinos).[6]

The mass matrices, Eq. (3), lead to relations among the diagonalized masses,

$$m_e^0/m_\mu^0 = m_{\mathfrak{N}}^0/m_\lambda^0 = m_{\mathcal{P}}^0/m_c^0 , \tag{6}$$

$$m_\mu^0/m_\tau^0 = m_\lambda^0/m_b^0 = m_c^0/m_t^0 . \tag{7}$$

Equation (7) has recently been derived in a different context by using a permutation symmetry for Higgs couplings.[10] The first equalities in Eqs. (6) and (7) also hold in the SU(5) model.[11] The $\mathfrak{N}$-quark mass may be underestimated as in the SU(5) model,[12] since we usually take $m_n/m_\lambda \gg m_e/m_\mu, m_{\mathcal{P}}/m_c$ at short distances.[13] A detailed discussion on such a problem will be given elsewhere.[14]

In this note we pay attention to the heavy-quark masses. The most significant part of the mass renormalization comes from the strong interactions. Taking into account only the color SU(3) term without mass corrections for simplicity, we find[12]

$$\ln\left(\frac{m_\lambda}{m_b}\right) = \ln\left(\frac{m_\mu}{m_\tau}\right) + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_s(2m_\lambda)}{\alpha_s(2m_b)}\right) , \tag{8}$$

$$\ln\left(\frac{m_c}{m_t}\right) = \ln\left(\frac{m_\mu}{m_\tau}\right) + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_s(2m_c)}{\alpha_s(2m_t)}\right) . \tag{9}$$

Here we have defined the quark masses, m_λ, m_c, m_b, and m_t, to be half of the threshold energies of producing the pairs of quarks.[15] The gluon coupling constant $\alpha_s(Q^2)$ is assumed to be in a form motivated by the renormalization group,

$$\alpha_s(Q^2) = \frac{12\pi}{25\ln(Q^2/\Lambda^2)} . \tag{10}$$

The results of Eqs. (8) and (9) for the b and t quark masses are shown in Table I for the six-quark scheme ($n_f = 6$). We have used $m_\mu = 0.106$ GeV, $m_\tau = 1.8$ GeV, $m_\lambda = 0.5$ GeV, $m_c = 1.5$ GeV, and values of $\alpha_s(Q^2 = 10\ \text{GeV}^2)$, 0.2–0.4, which are suggested by quantum chromodynamic analyses on charmonium[16] and deep-inelastic electroproduction.[17]

TABLE I. Masses of the quarks. Λ is defined by $\alpha_s(Q^2) = 12\pi/[25\ln(Q^2/\Lambda^2)]$. Inputs in the calculations are $m_\mu = 0.106$ GeV, $m_\tau = 1.8$ GeV, $m_\lambda = 0.5$ GeV, $m_c = 1.5$ GeV, and $\alpha_s(Q^2)$ at $Q^2 = 10$ GeV^2.

$\alpha_s(Q^2 = 10\ \text{GeV}^2)$	Λ (GeV)	m_b (GeV)	m_t (GeV)
0.4	0.48	4.0	16
0.3	0.26	4.8	17
0.2	0.07	5.8	19

Our model can be extended to more flavors by considering the horizontal $\mathrm{SU}_F(n)$ symmetry ($n \geq 4$). The weak SU(2)-doublet and -singlet fermions are assigned to the n-dimensional representation and the Higgs scalars χ^{ab} and ϕ^a to $n(n+1)/2$ and (n^2-1) under $\mathrm{SU}_F(n)$, respectively. As a straightforward extension of Eqs. (6) and (7), we obtain

$$m_e^0 : m_\mu^0 : m_\tau^0 : m_{\tau'}^0 : \cdots = m_{\mathfrak{N}}^0 : m_\lambda^0 : m_b^0 : m_{b'}^0 : \cdots = m_{\mathcal{P}}^0 : m_c^0 : m_t^0 : m_{t'}^0 : \cdots . \tag{11}$$

If $m_{\tau'}^0$ is known, we can predict the masses of quarks in the next generation.

Finally we note that the present model is a possible candidate for the spontaneous mass generation by dynamical symmetry breaking.[18] The Higgs scalars χ^{ab} and ϕ^a may be regarded as bound states of two right-handed neutrinos and of antifermion and fermion ($\bar{d}_R\lambda^a q_L + \cdots$), respectively. The large masses of the right-handed neutrinos (in the sense of Majorana particles) are therefore correlated with a large breaking of the horizontal symmetry.

Note added. In the $\mathrm{SU}_F(3)$ model we predict large weak-mixing angles among (tb) and ($c\lambda$) or ($\mathcal{P}\mathfrak{N}$). If the mixing angles turn out to be small, the horizontal group $\mathrm{SU}_F(3)$ would have to be enalrged to $\mathrm{SU}_F(4)$. In this case the large mixings may be caused among the heavy quarks, ($t'b'$) and (tb). We have restricted ourselves to the case $\mathrm{SU}_F(3)$ in this note, since the result on the b and t quark masses is not sensitive to the number of generations.

We would like to thank M. Yoshimura for a discussion.

[1] T. Maehara and T. Yanagida, Prog. Theor. Phys. 60, 822 (1978); 61, 1434 (1979).

[2] F. Wilczek and A. Zee, Phys. Rev. Lett. 42, 421 (1979).

[3] S. Weinberg, Phys. Rev. Lett. 19, 1204 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (*Nobel Symposium No.* 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[4] L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964).

[5] C. Bouchiat, J. Iliopoulos, and Ph. Meyer, Phys. Lett. 38B, 519 (1972); D. Gross and R. Jackiw, Phys. Rev. D 6, 477 (1972).

[6] T. Yanagida, in *Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories*, edited by S. Sawada and A. Sugamoto (National Laboratory for High Energy Physics, Tsukuba, Ibaraki, Japan).

[7] H. Terazawa, Y. Chikashige, and K. Akama, Phys. Rev. D 15, 480 (1977); K. Akama, Y. Chikashige, and T. Matsuki, Report No. INS-288, 1977 (unpublished).

[8] Y. Ne'eman, Phys. Lett. 82B, 69 (1979).

[9] D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973). H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).

[10] S. Pakvasa and H. Sugawara, Phys. Lett. 82B, 105 (1979).

[11] H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[12] A. J. Buras, J. Ellis, M. K. Gailard, and D.V. Nanopoulos, Nucl. Phys. B135, 66 (1978).

[13] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[14] In order to solve this problem, we, furthermore, introduce another Higgs scalar, $\phi^0 = (1,2,1)$, whose vacuum expectation value is small $[G_0^l \langle\phi^0\rangle \sim O(m_e^0, m_{\mathfrak{N}}^0 \text{ or } m_{\mathfrak{O}}^0)]$. In this case the fermion masses are shifted slightly and even if the additional mass terms are very small, Eq. (6) changes drastically. For example, $G_0^l \langle\phi^0\rangle \simeq 0$ MeV and $G_0^d \langle\phi^0\rangle \simeq 5$ MeV, which may give the correct m_n/m_λ ratio $\simeq \frac{1}{20}$. However, with respect to the masses of the heavy fermions, μ, λ, c, . . . , the small mass shifts are neglected and Eq. (7) holds approximately.

[15] H. Georgi and H. D. Politzer, Phys. Rev. D 14, 1829 (1976).

[16] T. Appelquist, R. M. Barnett, and K. Lane, Annu. Rev. Nucl. Sci. 28, 387 (1978).

[17] R. D. Field, in *Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Toyko, 1978), and references therein.

[18] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); 124, 246 (1961).