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Quark mass differences and ρ-ω mixing

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The experimental data on ρ-ω mixing are reanalyzed in order to extract a new value for the ratio $r_- = (m_d - m_u)/2m_s$, where m_i is the current quark mass of quark i. After correcting for the purely electromagnetic contribution to the mixing, one obtains $r_- = 0.010 \pm 0.002$, which agrees well with the average $r_- = 0.014 \pm 0.003$ obtained previously from kaon and baryon mass differences and the $\eta \to 3\pi$ decay and disagrees substantially with the value $\pm(0.031 \pm 0.007)$ that would be required if m_u or m_d were zero. A new determination of the current quark mass ratios, including the effects of ρ-ω mixing, then yields $m_u/m_d = 0.47 \pm 0.11$ and $m_d/m_s = 0.042 \pm 0.007$.

There has been much discussion recently[1-7] concerning the ratios of the current masses of the light quarks. These are of interest because the current quark masses determine the pattern of chiral-symmetry breaking. If $m_d \neq m_u$ for example, the quark mass terms generate a nonelectromagnetic violation of isospin. This is just a concrete realization of the old Coleman-Glashow tadpole[8] idea. Also, certain[9] weak-interaction gauge theories and most[10] grand unified theories make predictions for the mass ratios. Finally, if any of the current quark masses were zero, the possible P and T violation associated with Θ vacuums would become unobservable.[11]

The ratio $r_+ = (m_d + m_u)/2m_s = 0.031 \pm 0.007$ can be reliably determined[12] from the pseudoscalar-meson masses and decay constants. $r_- = (m_d - m_u)/2m_s$, which has been determined[1-3] from the kaon masses, the baryon masses, and the $\eta \to 3\pi$ decay width, is more uncertain mainly because one must first estimate and subtract the purely electromagnetic contribution to the quantity.[13] It is therefore useful to have as many independent estimates of r_- as possible. In this comment, a new determination of r_- is obtained from the data on ρ-ω mixing.[14] The technical assumptions made are first-order SU(3) breaking, ideal ω-ϕ mixing, and the neglect (as is implied by the Zweig rule) of ρ-ϕ mixing. The ρ-ω system has the advantage that the electromagnetic contribution to the mixing is small and can be estimated rather reliably. Furthermore, the experimental data have improved considerably since the previous analyses[15] of some ten years ago. The result is $r_- = 0.010 \pm 0.002$, which is compatible with other determinations[1-3] and implies $m_d \neq m_u$, $m_u \neq 0$, and $m_d \neq 0$.

Evidence for ρ-ω mixing has been observed in many interactions,[16] but the cleanest reactions for disentangling the ρ-ω mixing strength $m_{\rho\omega}^2 = \langle \rho | H(0) | \omega \rangle$ from the ρ and ω production amplitudes are the production of $\pi^+\pi^-$ pairs in photoproduction[17] from nuclei and in e^+e^- annihilation.[18] The effect of ρ-ω mixing is to replace the ρ propagator by

$$\frac{1}{s - m_\rho^2 + i m_\rho \Gamma_\rho} + \frac{T_\omega}{T_\rho} \frac{m_{\rho\omega}^2}{(s - m_\rho^2 + i m_\rho \Gamma_\rho)(s - m_\omega^2 + i m_\omega \Gamma_\omega)}, \quad (1)$$

where T_ρ and T_ω are the ρ and ω production amplitudes and $s = (p_{\pi^+} + p_{\pi^-})^2$. The interpretation of Eq. (1) is that the produced ω couples to the ρ with strength $m_{\rho\omega}^2$. The ρ then decays into $\pi^+\pi^-$. For e^+e^- annihilation,

$$T_\omega / T_\rho = f_\rho / f_\omega, \quad (2)$$

where the photon–vector-meson coupling strength is $e m_V^2 / f_V$. For photoproduction, T_ω/T_ρ is usually approximated by the vector-meson-dominance (VMD) expression

$$T_\omega / T_\rho = (f_\rho / f_\omega)(A_\omega / A_\rho) \approx f_\rho / f_\omega, \quad (3)$$

where A_ω (A_ρ) is the amplitude for $\omega(\rho)$ to scatter elastically from the target. It is usually assumed that $A_\omega = A_\rho$. Equation (1) can be written in terms of the quantities $\xi(s)$ and $\alpha(s)$, defined by

$$\xi(s) e^{i\alpha(s)} = \frac{T_\omega(s)}{T_\rho(s)} \frac{m_{\rho\omega}^2}{s - m_\rho^2 + i m_\rho \Gamma_\rho}. \quad (4)$$

Experimental data are generally parametrized under the reasonable simplification of taking ξ and α to be constant. Then

$$\alpha(s) \approx \alpha(m_\omega^2 - i m_\omega \Gamma_\omega) = \tan^{-1}\left(\frac{m_\rho \Gamma_\rho - m_\omega \Gamma_\omega}{m_\rho^2 - m_\omega^2}\right) + \arg\left(\frac{T_\omega}{T_\rho}\right) + \arg(-m_{\rho\omega}^2). \quad (5)$$

The first term is 95.1° and the second term may reasonably be taken to be zero since SU(3) and

TABLE I. ρ-ω mixing parameters determined from various experiments. α is the relative phase and $(f_\omega/f_\rho)^2$ is the value used to extract $m_{\rho\omega}{}^2$ from the data.

Experiment	α	$B_{\omega\to\pi^+\pi^-}{}^{1/2}$	$m_{\rho\omega}{}^2\,(10^{-3}\ \mathrm{GeV}^2)$	$(f_\omega/f_\rho)^2$	$\frac{m_d-m_u}{2m_s}$
Biggs *et al.*, Ref. 17 $\gamma N\to\pi^+\pi^-N$	$104.0°\pm 5.1°$	0.09 ± 0.014	-2.6 ± 0.4	$7.0^{+2.1}_{-1.5}$	0.0081 ± 0.001
Alvensleben *et al.*, Ref. 17. $\gamma N\to\pi^+\pi^-N$	$96°\ \pm 15°$	0.11 ± 0.014	-3.2 ± 0.4	$9.4^{+2.6}_{-1.6}$	0.0096 ± 0.001
Benaksas, *et al.*, Ref. 18 $e^+e^-\to\pi^+\pi^-$	$85.7°\pm15.3°$	0.19 ± 0.05	-5.5 ± 1.4	8.1 ± 1.2	0.015 ± 0.004
Quenzer *et al.*, Ref. 18 $e^+e^-\to\pi^+\pi^-$	$98°\ \pm13.6°$	0.13 ± 0.03	-3.7 ± 0.9	8.1 ± 1.2	0.011 ± 0.002

ideal ω-ϕ mixing predict $f_\omega=3f_\rho$. From the experimental phases given in Table I, we see that $m_{\rho\omega}{}^2=2m_\rho\Delta_{\rho\omega}$ may be taken to be real and negative. The experimental results on the magnitude ξ are usually presented in terms of the branching ratio for $\omega\to\pi^+\pi^-$, which is related to $m_{\rho\omega}{}^2$ by

$$B_{\omega\to\pi^+\pi^-}=\frac{(m_{\rho\omega}{}^2)^2}{(m_\omega{}^2-m_\rho{}^2)^2+(m_\rho\Gamma_\rho-m_\omega\Gamma_\omega)^2}\times\frac{\Gamma_{\rho\to\pi^+\pi^-}}{\Gamma_\omega}\,. \tag{6}$$

The experimental values for $B_{\omega\to\pi^+\pi^-}{}^{1/2}$ and the corresponding values for $m_{\rho\omega}{}^2$ are given in Table I. The various groups have used different values of $(f_\omega/f_\rho)^2$ in their extraction of $B_{\omega\to\pi^+\pi^-}$ from ξ, but the differences are within the quoted errors and have therefore not been corrected for. It should be noted that the photoproduction experiments have smaller statistical errors, but their interpretation requires the assumption that $A_\omega=A_\rho$, the accuracy of which is hard to evaluate. Keeping this uncertainty in mind, the agreement between the photoproduction and annihilation experiments is satisfactory.

There are two contributions to $m_{\rho\omega}{}^2$, a tadpole part $(m_{\rho\omega}{}^2)^{\mathrm{tad}}$ and an electromagnetic part $(m_{\rho\omega}{}^2)^{\mathrm{em}}$. The electromagnetic part of the mixing receives contributions from $\rho\to\gamma\to\omega$, $\rho\to\pi^0\gamma\to\omega$, $\rho\to\eta\gamma\to\omega$, etc. The one-photon contribution is

$$m_{\rho\omega}{}^2(1\gamma)=\frac{4\pi\alpha m_\rho{}^2}{f_\rho f_\omega}=+(0.69\pm0.05)\times10^{-3}\ \mathrm{GeV}^2\,, \tag{7}$$

where the values of f_ω and f_ρ, determined by Benaksas *et al.*,[18] have been used. The other electromagnetic contributions are negligible. For example, the absorptive part of the $\pi^0\gamma$ term is[19]

$$\mathrm{abs}\,m_{\rho\omega}{}^2(\pi^0\gamma)=-m_\rho\Gamma_{\rho\to\pi\gamma}{}^{1/2}\Gamma_{\omega\to\pi\gamma}{}^{1/2}=-1.4\times10^{-4}\ \mathrm{GeV}^2\,. \tag{8}$$

The real part can be estimated if one assumes a VMD-type form factor $F(K^2)=m_V{}^2/(m_V{}^2-K^2)$ at each $V\pi^0\gamma(K)$ vertex. Then neglecting the pion mass, the real part of $m_{\rho\omega}{}^2(\pi^0\gamma)$ is just $3/\pi$ times the absorptive part, so that $m_{\rho\omega}{}^2(\pi^0\gamma)$ can be neglected. $m_{\rho\omega}{}^2(\eta\gamma)$ is an order of magnitude smaller still. Hence it appears safe to approximate $(m_{\rho\omega}{}^2)^{\mathrm{em}}$ by the one-photon contribution, which is small compared to the experimental value but not entirely negligible.

The remainder of $m_{\rho\omega}{}^2$ is assumed to be due to the tadpole term, which is related to quark masses by

$$\frac{m_d-m_u}{2m_s-(m_d+m_u)}=\frac{(m_{\rho\omega}{}^2)^{\mathrm{tad}}}{-2(m_{K^*}{}^2-m_\rho{}^2)}\,. \tag{9}$$

Equation (9) is derived from the fact that the tadpole operator $\bar dd-\bar uu$ and the SU(3) breaking operator $\bar dd+\bar uu-2\bar ss$ belong to the same octet. Ideal ω-ϕ mixing and the Zweig-rule assumption $(m_{\rho\phi}{}^2)^{\mathrm{tad}}=0$ are also required. (The results are insensitive to small deviations from these assumptions.) Using the values for $m_{\rho\omega}{}^2$ in Table I, as well as Eq. (7) and the value for $r_+\simeq0.031$, one can then compute r_- for each experiment. The results are given in Table I. Finally, if one increases the uncertainties in the photoproduction experiments to allow for uncertainties in the $A_\omega=A_\rho$ assumption, one arrives at an average value of $r_-=0.010\pm0.002$ from ρ-ω mixing. This is in excellent agreement with the average value $r_-=0.014\pm0.003$ obtained[1-3] from the kaon and baryon mass differences and the $\eta\to3\pi$ decay and therefore supports the consistency of the tadpole picture. This value for r_- is also very far from the value

TABLE II. Values of $r_- = (m_d - m_u)/2m_s$ obtained from various sources (Refs. 1–3). $r_+ = (0.031 \pm 0.007)$ is from the pseudoscalar masses (Refs. 1–3). The errors include estimates of theoretical uncertainties.

Kaon mass difference	0.0175 ± 0.003
Baryon mass difference	0.0105 ± 0.003
$\eta \to 3\pi$	0.0145 ± 0.008
ρ-ω mixing	0.010 ± 0.002

of ± 0.031 which would correspond to $m_u(m_d) = 0$.

The values of r_- obtained from different sources are given in Table II. They are all in reasonable agreement with each other, although the value obtained from the kaon masses is rather high compared with the other values.[3]

A new weighted mean, including the value from ρ-ω mixing, is $r_- = 0.011 \pm 0.002$, from which the new ratios $m_u/m_d = 0.47 \pm 0.11$ and $m_d/m_s = 0.042 \pm 0.007$ are obtained.

As a final consistency check, note that the ρ mass difference $m_{\rho^0} - m_{\rho^+}$ receives no tadpole contribution, while

$$(m_{K^{*+}}{}^2 - m_{K^{*0}}{}^2)^{\mathrm{tad}} = [m_{\rho\omega}{}^2]^{\mathrm{tad}}$$

But in the SU(3) limit with ideal mixing one has

$$m_{\rho^0}{}^2 - m_{\rho^+}{}^2 = (\sqrt{2}\, m_{\rho\phi}{}^2 + m_{\rho\omega}{}^2)^{\mathrm{em}} - (m_{K^{*+}}{}^2 - m_{K^{*0}}{}^2)^{\mathrm{em}} \tag{10}$$

Saturating $(m_{\rho\phi}{}^2)^{\mathrm{em}}$ and $(m_{\rho\omega}{}^2)^{\mathrm{em}}$ by the one-γ contribution and using $r_- = 0.010 \pm 0.002$, one has $m_{\rho^0} - m_{\rho^+} = 5.4 \pm 1.9$ MeV, in agreement with the value 2.4 ± 2.1 MeV found by Pisut and Roos.[20]

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[13] Deshpande and Soper (Ref. 7) have also questioned the value of f_{K^+}/f_{K^0}. See Ref. 1, however. For $\eta \to 3\pi$, the principal difficulty is in estimating the tadpole matrix element.

[14] ρ-ω mixing is also important because of its implications for isospin violation in the *NN* interaction. See P. Langacker and D. Sparrow, University of Pennsylvania Report No. UPR-0126T (unpublished), and references therein.

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