Binding energy of quarkonium systems and the mass spectra

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We show that if a quark (or antiquark) with mass m_a is replaced by a heavier quark (or antiquark) with mass m_b in a quarkonium state, the increase of the mass of the quarkonium system is substantially less than $m_b - m_a$ due to the increase of the binding energy (in some cases it is only 1/6 of $m_b - m_a$). It is shown that for the heavy quarkonia the F-family mesons get close to the D-family mesons for a large class of potentials. The spectrum of all known mesons is explained if the change of the binding energy is taken into account.

In our previous paper[1] (we call it model I) the mass of S-wave hadrons was calculated by using the Fermi-Breit interaction which comes from one-gluon exchange in asymptotically free quantum chromodynamics (QCD).[2] We take $m_u = m_d = 0.336$ GeV which is obtained from the magnetic moment of baryons. This value gives a reasonable explanation for photoproduction amplitudes.[3]

We found that the mass of the charmonium becomes too large if the mass of the charmed quark is determined by using the mass of charmed meson and charmed baryons (see column th1 in Table I of Ref. 1). The purpose of this paper is to solve this puzzle.

In Ref. 1 we neglected the change of the binding energy of quarkonium. Recently Kraseman and the present author[4] studied the quarkonium spectrum by using several potentials. One of the potentials studied in Ref. 4 is (this potential is motivated from the asymptotically free QCD and we call it model II)

$$V_{\rm II}(R) = V_{\rm AF}(R) + V_{\rm INT}(R) + aR \;, \qquad (1)$$

where

$$V_{\rm AF}(R) = -\frac{4}{3}\,\frac{\alpha_s(R)}{R}\,, \quad \alpha_s(R) = \frac{12\pi}{25}\,\frac{1}{2\ln(\mu/R)}\,,$$

$$\mu = (\Lambda e^{\gamma}), \quad \Lambda = 0.5 \text{ GeV}, \quad \gamma = 0.5772, \qquad (2)$$

$$a \approx 0.787 \text{ GeV/fm} \;.$$

The intermediate potential assumed ($V_{\rm INT}$) is

$$V_{\rm INT} = -b\exp(-R/c) \;, \qquad (3)$$

which is similar to the potential used by Celmaster, Georgi, and Machacek.[5] The form of the potential is plotted in Fig. 1. We have tried many other intermediate potentials besides Eq. (3) and have found that this gives the nearest values for the energy level of the P-wave chamonium state and for the ratio $\Gamma_{\psi' \to l\bar{l}}/\Gamma_{\psi \to l\bar{l}}$.

In order to show how our results depend on the form of potential let us use another potential (model III, the form of this potential is shown in Fig. 1) which does not use the running coupling constant (i.e., α_s is a constant).

To calculate the fine and hyperfine structures of S and P states we use the Fermi-Breit Hamiltonian (for the detailed calculations and assumptions see Ref. 4).

In our model there are two free parameters b and c which are fitted to the experimental data $\psi' - \psi = 0.59$ GeV and $\Upsilon' - \Upsilon = 0.56$ GeV. It should be noted that these experimental values do not correspond to the mass splitting between 2S and 1S because the correction which comes from the hyperfine structure in 2S and 1S levels cannot be neglected. The splitting between $^3S_1$ and $^1S_0$ for the $c\bar{c}$ system is around 100 MeV in our model. Using the potential II and III for the Schrödinger equation one obtains the center of gravity (c.o.g.) of 1S and c.o.g. of 2S state. Using the Fermi-Breit Hamiltonian perturbatively one gets the hyperfine structure and the mass of ψ, ψ', Υ, and Υ' is determined.

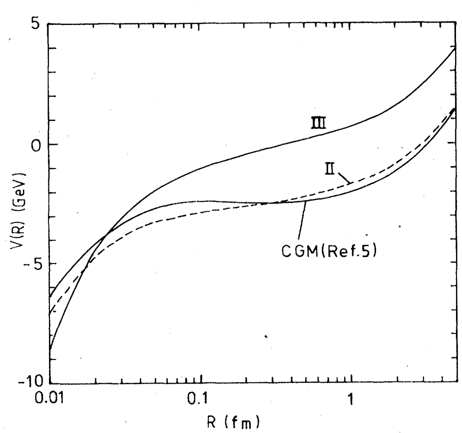

FIG. 1. The potentials II and III.

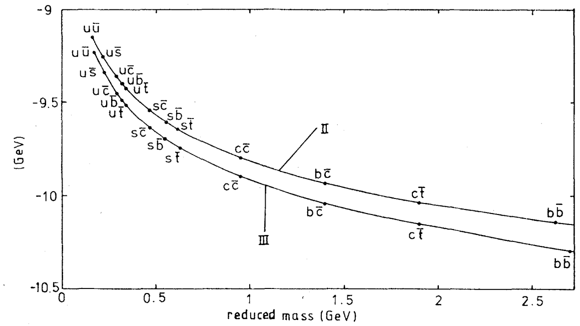

FIG. 2. The energy levels of the S-wave quarkonium states for the models II and III. The mass of the t quark is assumed to be infinity.

Our parameters b and c

$$b = 1.378 \text{ GeV}, \quad c = 1.20 \text{ GeV}^{-1} \quad \text{for model II} \tag{4}$$

are chosen to give correct energy splittings $\psi' - \psi$ and $\Upsilon' - \Upsilon$ after the corrections due to the hyperfine structure. Our models II and III predict roughly the correct energy levels of the P-wave charmonium state.

We show the energy level of the S state as a function of the reduced mass of the quark-antiquark system in Fig. 2. The state falls deep into the potential well with increasing m_Q (or the reduced mass of quarkonium), i.e., the binding energy increases. This can be proved by semiclassical methods independent of the particular potential.[6]

It can easily be understood from Fig. 1 that if we neglect this increase of the binding energy which comes from the large reduced mass of charmonium states, we obtain too high a mass for charmonium.[1]

TABLE I. Quarkonium mass spectrum in MeV.

	c.o.g. exp (Ref. 9)	Model I	Model II[a]	Model III[b]
π, ρ	615	614	618	617
K, K^*	793	789	797	800
ψ, η_c	ψ = 3086	3303	3096	3081
	η_c = ?	(3089)[c]	(3075)[d]	(3063)[e]
D, D^*	1970	1959	1973	1964
F, F^*	2113 ± 60	2133	2072	2068
Υ, η_b	Υ = 9460, η_b = ?		9448	9458
D_b, D_b^*			5284	5316
F_b, F_b^*			5359	5394
$b\bar{c} + \bar{b}c$			6313	6328

[a] The assumed quark masses are $m_u = m_d = 0.336$ GeV, $m_s = 0.620$ GeV, $m_c = 1.90$ GeV, and $m_b = 5.25$ GeV.

[b] The assumed quark masses are $m_u = m_d = 0.336$ GeV, $m_s = 0.630$ GeV, $m_c = 1.90$ GeV, and $m_b = 5.29$ GeV.

[c] In model I, $\Delta M = {}^3S_1 - {}^1S_0 = 25$ MeV. Therefore, ${}^1S_0 = 3097 - 25 = 3072$ MeV and the center of gravity should be 3089 MeV.

[d] In model II, $\Delta M = 88$ MeV.

[e] In model III, $\Delta M = 136$ MeV.

Taking into account the change of the binding energy we find an excellent agreement with the experimental data, including charmonium states. In Table I we show our results, which are not very sensitive to the choice of the potential. This improvement gives further support to our potential picture.

We come to the following conclusion. If in a quarkonium system a quark (or antiquark) with m_a is replaced by a heavier quark (or antiquark) with mass m_b the increase of the mass of the quarkonium system is substantially less than $m_b - m_a$ due to the increase of the binding energy.

TABLE II. Mass differences between F-family and D-family mesons in MeV.

	exp (Ref. 9)	Model I	Model II	Model III
$m_s - m_u$		174	284	294
$\frac{K^* + 3K}{4} - \frac{\rho + 3\pi}{4}$	177	174	183	179
$\frac{F^* + 3F}{4} - \frac{D^* + 3D}{4}$	143 ± 60	174	104	99
$\frac{F_b^* + 3F_b}{4} - \frac{D_b^* + 3D_b}{4}$		174	78	75
$\frac{F_t^* + 3F_t}{4} - \frac{D_t^* + 3D_t}{4}$ $(m_t = \infty)$		174	62	58

This is true for any system composed of any number of quarks.

The mass difference between the c.o.g. of F-family mesons and that of D-family mesons is listed for various values of the heavier quark mass (m_Q) in Table II. In model I this mass difference is equal to $m_s - m_u$ and does not depend on m_Q. On the contrary, in models II and III as m_Q increases the mass difference decreases drastically and approaches a constant (~ 60 MeV) which is only $\frac{1}{6}$ of $m_s - m_u$. We have not made any attempt to see how the relativistic corrections affect the results.

Finally, it is worthwhile to note that similar results are obtained for the noncharmed baryon system by using the harmonic-oscillator potential.[7,8]

The $c\bar{q}$ mesons are probably very relativistic. This makes it difficult to trust quantitative results. Finally, it would be appropriate to mention that a semiclassical analysis showing that binding energy corrections modify naive expectation about quark mass differences was done by Celmaster[10] in the context of electromagnetic mass differences.

Note added. After the completion of this work we learned the experimental evidence of $U(2976) = \eta_c$ and $B(5.3) = D_b$. These data are very encouraging for our model.

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