Reconnection of strings and quark matter

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(Received 18 June 1979)

Quarks are bound by strings attached to them. When two strings come close, a rearrangement of connections or "flip-flop" takes place. This effect plays an important role in a dense ensemble of quarks, quark matter. A phase transition from nuclear matter to quark matter occurs at the nucleon density $\rho < 5\rho_0$ where ρ_0 is the density of the ordinary nuclear matter. The string flip-flop causes a new type of attractive interaction between hadrons at short distances or among quarks at high density. This interaction can produce multiquark bound states such as dibaryon or dimeson resonant states.

I. INTRODUCTION

Since 1932 when the nucleons were discovered, a nucleus has been regarded as made of nucleons and possibly a few number of pions. Actually, however, nucleons and pions are composed of quarks and probably strings that bind them, and a nucleus must be looked at as an ensemble of quarks and strings. In ordinary nuclei or normal nuclear matter, nucleons are well separated, and it is a fair approximation to regard them as a set of nucleons.

When nuclear matter is compressed, the quark wave functions of nucleons overlap and rearrangement of string connections takes place. In this case it is no longer possible to think in terms of nucleons. The system must be described by quarks and strings joining them. In the present paper this state, the quark-matter state,[1] will be investigated. In turns out that phase transition from nuclear matter to quark matter takes place at a density less than five times the density of normal nuclear matter.

The Yukawa potential, the one-pion-exchange potential, is no longer applicable in quark matter. Instead, the reconnection of strings, the string "flip-flop," acts as an agent to bind the multiquark system. Thus there will be (metastable) quark matter: A dibaryon resonance is an example. This mechanism is also responsible for exotic states such as dimeson resonances.

In the following section, Sec. II, we summarize the interaction of quarks and strings. In Sec. IV we investigate the quark-matter state, and the string flip-flop interaction will be discussed in Sec. V.

II. QUARKS AND STRINGS

From the linearity of Regge trajectories we conclude that the potential energy acting on a quark is linear in the distance r. The origin of this confining potential is not yet clear. In the present paper we take a phenomenological standpoint and assume a model of quarks and strings[2] as explained below. An example of such string is provided by a flux of magnetic lines of force in a type II superconductor, which forms a string of constant line density. We assume a similar situation to hold in our case also. In quantum chromodynamics (QCD), color is a source of lines of force and lines emitted from a color end on its complementary color. Red, blue, and green form a colorless system, so lines of force close in this three-quark system.

The vacuum in QCD is expected to be a superstate and to squeeze the lines to form strings. From a colored system a string or strings must extend to some point in space. Only a colorless group of quarks can form a closed system disconnected from others. [Strictly speaking, for a quark system to form a closed system, it must be not only colorless (i.e., $I_3^{\text{color}}=0$, $Y^{\text{color}}=0$), but also color singlet. In the following, however, the colorless condition only is considered in quark matter.] Strings are formed so that the length of strings is as short as possible. Accordingly the string of a R-B-G system is Y shaped, with two strings making an angle of 120° each, if the three quarks are placed in an acute-angles triangle. Here we are fixing the position of quarks, i.e., we are considering adiabatic potential. Although this is not a good approximation for ordinary quarks which have a small rest mass, we shall see later that quarks have an effective mass and the velocity is not extremely relativistic.

Regarding the applicability of the string picture in hadrons, the following facts are noted: In a nucleon the string potential dominates over the one-gluon-exchange potential (see the end of this section), so the strings are certainly there. On the other hand, in the pion which is much smaller than the nucleon, the potential based only on the string no longer holds. The very small mass of the pion implies that here the attractive gluon potential is the dominant one. We conclude that for interquark distance of the order of or less than the pion radius

$$r_\pi \approx 0.4 \text{ fm},$$

the string-only picture fails and some other potential must be considered.

So far strings were treated classically. There are two quantum-mechanical effects that are important to us:

(1) Tearing of string. A stretched string is torn, i.e., it makes a transition to a state of meson and string. This is an emission of a meson and is described by the Yukawa interaction (see Fig. 1).

(2) Flip-flop of strings. Strings can make a transition to another configuration provided the latter is a possible one consistent with the color condition. This is a kind of tunneling effect: The strings stretch themselves, violating energy conservation, and when they touch each other, they switch to the other configuration (Fig. 2). As we shall see in Sec. V, the transition amplitude is

$$T \propto e^{-a' A},$$

where A is the area the strings have to sweep in making this transition.

T is not large so the mixture of ψ_2 in ψ_1, which is given by

$$\frac{T}{E_1 - E_2},$$

is in general small. However, when E_2 becomes equal to E_1, a resonance takes place and the state is a mixture of equal amounts of ψ_1 and ψ_2, irrespective of the magnitude of T. After passing the resonance, E_2 becomes less than E_1 and the state becomes ψ_2. As a result, for a given quark configuration, strings are formed so that the total length of the strings is the minimum,

$$V_{\text{string}} = a \min \sum r_i, \tag{1}$$

subject to the color condition that any colored system must be attached to a string and must not be left disconnected. The potential energy is not a simple sum of two-body or three-body potentials: It depends on the configuration of all quarks.

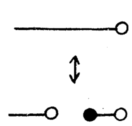

FIG. 1. Emission of a meson from a string. Open and solid circles represent quarks and antiquarks, respectively.

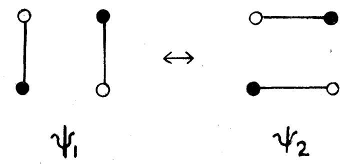

FIG. 2. Flip-flop of strings.

The quantum fluctuation of the strings has not been considered. As an example of the quantum theory of strings we take the lattice gauge theory in the strong coupling limit. In this theory the lowest-order correction to the energy is proportional to the string length, and it merely results in the renormalization of the coupling strength.

Up to now quarks were held fixed. If they are allowed to move, Eq. (1) is not a correct expression of the potential energy. The confining potential must be a Lorentz scalar rather than the fourth component of a vector. To see this let us consider a simple case, the individual-particle approximation. The Y-shaped string is very suitable for this, since the center of the Y is almost equal to the center of gravity. The Hartree potential is (almost) a linear one proportional to the distance from the center to the quark. The total energy is the sum of individual energy:

$$H^{(1)} = \alpha p + V, \tag{2}$$

$$E_{\text{tot}} = \sum E_i^{(1)}. \tag{3}$$

In Eq. (2) the rest mass of the ordinary quark is neglected. The equation (3) does not hold if the one-gluon-exchange force between quarks is not negligible (since the two-body potential energy is double-counted).

If the potential is of vector type, the single-body equation is

$$(\alpha p + ar)\psi = E\psi,$$

which gives

$$p^2\psi = (E - ar)^2\psi \approx a^2 r^2 \psi, \quad r \to \infty.$$

The wave is traveling with real momentum at $r \to \infty$, so no bound state exists in this case.[3] In contrast, if the potential is a scalar,

$$(\alpha p + ar\beta)\psi = E\psi \tag{4}$$

gives a second-order equation

$$(p^2 + a^2 r^2 + \{\alpha p, ar\beta\})\psi = E^2\psi.$$

This is of harmonic-oscillator type and certainly gives bound states. The rising potential must be a scalar in order to produce bound states.

Another nice feature of the scalar-type potential is the magnetic moment. From Eq. (4), we have for the electromagnetic interaction

$$\psi^*(\alpha A)\psi = \psi^*\left(\frac{p}{E}A + \frac{\sigma B}{2E}\right)\psi ,$$

where A and B are the vector potential and magnetic field, respectively. This shows that a quark with charge $Q_q e$ has a magnetic moment $Q_q e/2E$.[4] For the proton, Eq. (3) gives $E = M_p/3$, and the quark magnetic moment is

$$\mu_q = Q_q 3 \frac{e}{2M_p} ,$$

which is the amount needed for the nucleon magnetic moments.

The conclusion is that a model of the nucleon with a dominant string potential which is Lorentz scalar seems to work well. Further discussion of the nucleon structure will be given in Sec. VI. The potential energy of a multiquark system in a Hartree approximation is given by

$$V = a \min \sum r_i \beta^{(i)} , \tag{5}$$

where r_i is the distance from the center. The value of the constant a determined from quarkonia (spin-parallel configurations) is 0.15–0.2.[5] We shall see that in a single-particle approximation we are taking, $a = 0.1$ is the correct choice.

III. MULTINUCLEON SYSTEM

Consider two nucleons, each having an extension of the order $r_p = 0.8$ fm.

I. $r_{NN} > 1.6$ fm. When the nucleon-nucleon distance r_{NN} is larger than $2r_p$, there is no overlap of wave functions. The system can be described by two nucleons and the one-pion-exchange potential (OPEP):

$$\text{System} = \sum N + V_{\text{Yukawa}} . \tag{I}$$

II. $r_{NN} < 1.6$ fm. The wave functions overlap. The result is that a quark of the other nucleon is nearer to the center and string flip-flop occurs (Fig. 3). In a more symmetrical way, transitions as given in Fig. 4 will take place. The system must be described by six quarks and the string potentials:

$$\text{System} = \sum q + V_{\text{string}} . \tag{II}$$

III. $r_{NN} < 0.6$ fm or $r_{qq} < 0.4$ fm. Here the string picture loses its meaning and the one-gluon-exchange potential prevails. We may say that all quarks are in one bag[6] of normal phase surrounded by the superstate vacuum:

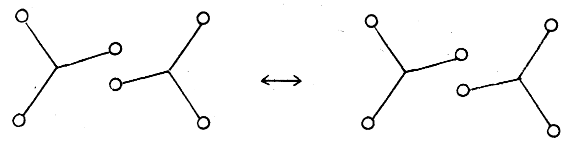

FIG. 3. Flip-flop of strings of two nucleons.

$$\text{System} = \sum q + V_{\text{normal}} \tag{III}$$

In the present paper we are concerned with the intermediate region II.

IV. QUARK MATTER

In ordinary nuclei the nuclear density is such that $\rho_0^{-1/3} = 1.9$ fm and nucleons can be said to be mostly in region I. However, as it is compressed, the string flip-flop begins to be important. In a highly compressed state a quark is no longer confined to a nucleon. It can travel within the whole volume by reconnecting strings. It feels an averaged constant potential and can be treated as a free particle. The system can be described by a Fermi gas of quarks. This is called quark matter.

We considere ordinary quarks only. There are six kinds, u and d each in three hues. The Fermi gas to be considered contains an equal number of each kind.

$$n(u_R) = n(u_B) = n(u_G) = n(d_R) = n(d_B) = n(d_G)$$
$$= \frac{A}{2}\,[= n(p) = n(n)] , \tag{6}$$

where $n(p)$ and $n(n)$ are the numbers of protons and neutrons of the system when it is regarded as nuclear matter. The Fermi momentum is given by

$$P_F = (3\pi^2\rho)^{1/3} = \left(\frac{3\pi^2}{2}\frac{A}{V}\right)^{1/3}, \quad \rho = \frac{n}{V} , \tag{7}$$

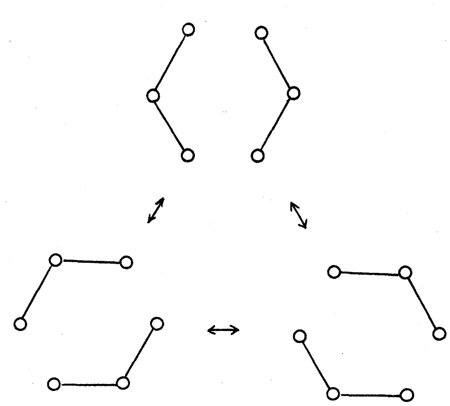

FIG. 4. Resonating group structure in six-quark system.

where V is the volume of the Fermi gas. The equality $n(q)=n(N)$, Eq. (6), tells us that the Fermi momentum calculated as nuclear matter is the same as that calculated as quark matter.

First we calculate the potential energy in the individual-particle approximation. From a center we look for the nearest red quark, nearest blue quark, and nearest green quark and draw strings. Stated in another way, a red quark searches for the nearest center which can accept it. Since centers are uniformly distributed, we take the average. When particles (centers) are uniformly distributed with a density $\rho(=A/V)$, the average of the distance to the nearest particle is given by

$$\bar{r}=\Gamma\left(\frac{4}{3}\right)\left(\rho\frac{4\pi}{3}\right)^{-1/3},$$

where Γ is the gamma function. In our case, the average of the string length is

$$\bar{r}=\Gamma\left(\frac{4}{3}\right)\left(\frac{A}{V}\ \frac{4\pi}{3}\right)^{-1/3}=0.89\left(\frac{8}{9\pi}\right)^{-1/3}P_F^{-1}$$

$$=1.36/P_F \quad (\text{in units GeV}=1)\,. \tag{8}$$

The potential energy per quark is

$$\langle V\rangle_{\text{per } q}=a\bar{r}\beta=\frac{1.36a}{P_F}\,\beta\equiv M(P_F)\beta\,.$$

The quantity multiplying β can be called the effective mass. The total energy of the quark matter is

$$E_{\text{tot}}=\sum_{|p|<P_F}[p^2+M^2(P_F)]^{1/2}\,,$$

and the energy per quark is

$$E_{\text{per } q}=\int [p^2+M^2(P_F)]^{1/2}d^3p\Big/\int d^3p'\equiv I(P_F)\,.$$

In the two extreme cases,

$$I(P_F)=M(P_F)=\frac{1.36a}{P_F}\,,\quad P_F\ll M(P_F)$$

$$=\tfrac{3}{4}P_F\,,\quad P_F\gg M(P_F)\,.$$

$I(P_F)=E_{\text{per } q}$ is plotted as a function of P_F in Fig. 5. (a is taken as 0.1.) It has a minimum at

$$P_F=0.42\ \text{GeV}\equiv P_F^c\,, \tag{9}$$

and

$$E_{\text{per } q}=0.46\ \text{GeV}\,,\qquad \rho=4.9\rho_0\,, \tag{10}$$

where ρ_0 is the ordinary nuclear matter density (corresponding to $P_F=0.25$ GeV).

At this point stable quark matter is formed. However, this matter has zero surface tension. Indeed, the average distance $\bar{r}$, Eq. (8), which

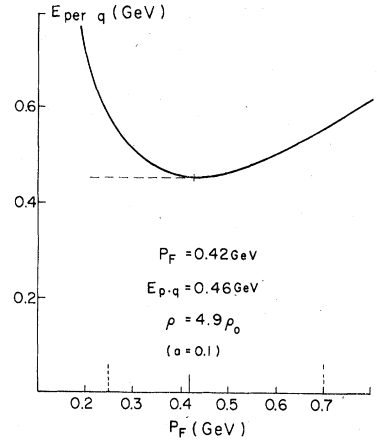

FIG. 5. Energy per quark as a function of Fermi momentum P_F. The minimum is given in the figure. $P_F > 0.7$ GeV correspond to region III, and the present calculation is inapplicable. $P_F = 0.25$ GeV is that of ordinary nuclear matter. Below the critical point P_F^c the system is an ensemble of baryons.

gives the potential energy of a quark, is unchanged even if the quark is near the surface or deeply embedded in the quark matter. Below $P_F < P_F^c$, i.e., if the volume is larger than the critical volume, the quark matter splits into pieces and finally reaches to $3q$ objects which cannot be broken further. Below the critical point, the system is an ensemble of baryons, and the minimum (9), (10) is the critical point of the phase transition

$$\sum B\ (\text{baryon matter}) \rightarrow \sum q\ (\text{quark matter})\,.$$

Here we used the words baryon matter rather than nuclear matter since in a slightly compressed nuclear matter, there may be some Δ resonances beside nucleons. $P_F=0.7$ GeV corresponds to $r_{qq}=0.4$ fm where the string picture loses its meaning.

We have been neglecting strange quarks in the quark matter. Comparing the top energy in the Fermi gas and the energy of a strange quark at rest, the condition for the stability of the ordinary quark matter against weak decay is

$$M_s+M(P_F)>[P_F^{\,2}+M^2(P_F)]^{1/2}\,,$$

where M_s is the rest mass of the strange quark. The crossing occurs at $P_F=0.46$ GeV for $M_s=0.25$ GeV, and $P_F=0.5$ GeV for $M_s=0.3$ GeV. Above this Fermi momentum the ordinary quark matter decays weakly into strange-quark matter.

V. STRING FLIP-FLOP

We shall now give a slightly more detailed discussion of these matters. The string flip-flop causes an attractive interaction among quark and string systems. If a state ψ_1 is connected to another state ψ_2 by a transition matrix element T, the energy is lowered by

$$\Delta E = \frac{E_2 - E_1}{2} - \left[\left(\frac{E_2 - E_1}{2}\right)^2 + T^2\right]^{1/2}, \qquad (11)$$

where E_i is the (unperturbed) energy of the state ψ_i. In the two extreme cases,

$$\Delta E = -|T|, \quad E_2 - E_1 \ll 2T,$$

$$= -\frac{T^2}{E_2 - E_1}, \quad E_2 - E_1 \gg 2T.$$

The second expression is the second-order perturbation-expansion term.

Two strings acquire an attractive interaction by flip-flop. This is the interaction among quarks in region II to play a role of the Yukawa interaction in region I. The case $E_2 = E_1$ is most favorable since the energy is linear rather than quadratic in T. The state is said to form a resonating group structure.

Lacking the correct theory to describe quarks and strings, the flip-flop interaction should be determined phenomenologically, in a similar way as the tearing of strings is described by the Yukawa coupling. Here we take the lattice gauge theory and estimate the order of magnitude of T.

In the lattice gauge theory the Hamiltonian consists of an unperturbed term which is the string energy and an interaction term of order $1/g^4$ that creates (or annihilates) a tiny closed loop of string on a plaquette of lattice space. If one side of the plaquette coincides with the original string, the string is deformed through an area d^2, d being the lattice constant. The whole transition is an A/d^2-step process, where A is the area the strings sweep in making the flip-flop. The transition amplitude is therefore

$$T \propto \left(\frac{1}{g^4}\right)^{A/d^2} = e^{-a'A}, \quad a' = \frac{1}{d^2} \ln g^4. \qquad (12)$$

This is multiplied by a factor which is independent of the coupling constant g. The exact evaluation of this factor is difficult and is not attempted.[7] The transition amplitude T can be written as

$$T = acle^{-a'A},$$

where l is the sum of the lengths of strings making the flip-flop and c is some dimensionless quantity which in general depends on the lengths of the strings.

Once T is given we can calculate the string flip-flop energy of quark matter. The exact evaluation is difficult and not worth pursuing, since the original interaction was not defined with any precision. We shall make a rough estimate of an interaction energy between two centers separated by R. This energy corresponds to a nuclear potential between nucleons. It has a range of order $\bar{r}$, since for $R > 2\bar{r}$ there is no overlap of quark wave functions. For $R \approx \bar{r}$, a flip-flop takes place when the configuration of quarks and strings are such that $E_2 - E_1$ is not large compared with T and the area A is not large compared with $1/a'$. This gives

$$V_{cc}(R \approx \bar{r}) \approx -c^2 \frac{a}{a'} \frac{1}{\bar{r}}.$$

The interaction energy per center or per nucleon is

$$E_N = \tfrac{1}{2}\rho_N \int V_{cc}(R) d^3R \approx -c^2 \frac{a}{a'} P_F. \qquad (13)$$

The interaction V_{cc} is of the Wigner type which shows no saturation character and gives a positive surface tension of the quark matter. So with this interaction the quark matter is stable against separation into pieces. However, no such stable object is observed. Perhaps the binding energy (13) is not sufficient to hold a nucleon from evaporation. In the quark matter, if regarded as a Fermi gas of nucleons, the top nucleon has a kinetic energy

$$E_{\text{kin},N} = \frac{P_F^2}{2M_p} = -0.1 \text{ GeV} \quad (P_F = P_F^c).$$

The instability condition puts an upper limit of the constant c.

Quark matter will be formed when two nuclei collide and local high density is attained. The translational degrees of freedom are tripled by the phase transition and result in a change of some statistical properties, such as the compressibility or the sound velocity.

A very simple yet nontrivial quark matter is the nucleon-nucleon resonant state formed by six quarks. Some such resonances have been observed.[8] They can be either bound states of two baryons or states of six quarks (Table I). At present it is not clear which assignment is more appropriate.

VI. BARYONS

A baryon made of three quarks is the simplest quark matter. From Eq. (10) a baryon has an energy

TABLE I. Possibilities for nucleon-nucleon resonances.

	Bound by	Binding energy	Size
Dibaryon	OPEP	tens MeV	$\sim 1/m_\pi = 1.4$ fm
Hexaquark	String flip-flop	~100 MeV	$\sim r_p = 0.8$ fm

$$E_B = 1.38 \text{ GeV} \quad (\text{quark matter}),$$

although treating the three-body state as a Fermi gas is not reasonable.

By solving the Hartree equation (4), we have an energy of a baryon

$$E_B = 3\sqrt{2.6a} = 1.53 \text{ GeV} \quad (\text{Hartree}). \tag{14}$$

The Hartree approximation for the three-body problem is not a good one either, but we accept this approximation to be consistent with the previous arguments.

There are several corrections to the baryon energy:

(1) One-gluon-exchange potential. This vanishes for heavy-quark matter due to the saturation character. For a baryon it gives an energy

$$E_g = -2\alpha_s\left\langle \frac{1}{r} \right\rangle \approx -0.5\alpha_s \approx -0.2 \text{ GeV}, \tag{15}$$

The strong fine-structure constant α_s has not been obtained precisely. It should be slightly larger than the value used in charmonium binding, $\alpha_s = 0.2-0.4$, where the size is about half the baryon size. The value $\alpha_s = 0.4$ is used in Eq. (15).

(2) Pion emission and reabsorption. While the string flip-flop as discussed in Sec. V does not occur in a single hadron, the other type of quantum effect, the tearing of strings or the pion emission, is possible. In quark matter such a transition is suppressed or greatly modified due to the exclusion principle, but for a single baryon it can be describable by the Yukawa interaction with (pseudovector) coupling constant f. The self-energy of a nucleon due to $N \rightleftarrows N+\pi$ is

$$\Delta E_\pi = -\frac{f^2}{4\pi}\,\frac{3}{\pi}\,\frac{1}{m_\pi^{\,2}}\int \frac{k^4}{k^2+m_\pi^{\,2}}\,G^2(k)d^1k,$$

where $G(k)$ is the form factor of a nucleon emitting a pion. If $G(k)$ is taken equal to the proton electric form factor,

$$\Delta E_\pi \approx -0.2 \text{ GeV}. \tag{16}$$

Incidentally, the magnetic moment of the proton due to the pion cloud is[9]

$$\mu_{p,\pi} = \frac{f^2}{4\pi}\,\frac{4}{3\pi}\,\frac{M_p}{m_\pi} \times\left(\frac{4k^4}{(k^2+m_\pi^{\,2})^2} - \frac{3k^2}{k^2+m_\pi^{\,2}}\right)G^2(k)d^1k$$

$$= 0.05 \text{ nuclear magnetons},$$

and is very small. The nucleon magnetic moments are mostly due to the quark configuration as was discussed in Sec. II.

Using Eqs. (14), (15), and (16), the energy of an ordinary baryon is

$$M_B = 1.5 - 0.2 - 0.2 = 1.1 \text{ GeV}. \tag{17}$$

We have been neglecting the spin dependence of the interaction. Equation (17) should be compared with some average of ordinary baryon masses, M_N and M_Δ, which is about 1.1 GeV. This justifies the choice $a = 0.1 \text{ GeV}^2$.

VII. CONCLUSION AND DISCUSSIONS

Introducing the notion of the string flip-flop which takes place when two nucleons come close together, the energy of the quark matter was calculated. The phase transition from baryon (nuclear) matter to quark matter will occur at a density less than $5\rho_0$. The attractive string flip-flop interaction is expected to lower this critical point.

The string flip-flop interaction acts as a major agent to hold a dense multiquark system. It will produce metastable quark matter, the simplest example being the dibaryon resonant state. In a similar way we can think of quarkonium matter that consists of high density quarks and antiquarks bound by the string flip-flop. The simplest nontrivial one is made of $qq\bar{q}\bar{q}$ making a resonating group structure as given in Fig. 2. It decays into two mesons with a wide width. If the flip-flop attraction is sufficiently strong to form a stable multiquark-antiquark system, it correspond to the "pion condensation." For quantitative arguments, the precise form of the flip-flop interaction must be specified, either phenomenologically or from a more fundamental interaction.

The form (12) suggested by the lattice gauge theory cannot be taken as it stands. When two strings are aligned on a line, the area A is zero no matter how the two are separated. This produces a long-range potential of the form $1/R^n$ between hadrons in contradiction to observation. Actually the expression (12) no longer holds when the width of the area A is less than the lattice constant d. A linear term in the string length is needed in the exponent besides the term proportional to the area.

In Sec. IV, the Fermi gas consisting of ordinary quarks with saturated isospin was considered. This is sufficient for the medium-sized quark matter in laboratory experiments. Quark matter may also be found inside stars. In such a large-scaled case, neutral quark matter must be considered and strange quarks must be included as has been discussed at the end of that section.

In the present paper the Hartree approximation was introduced, and the problem was reduced to a single-quark problem. This treatment tends to overestimate the energy for a few-body problem. This may be a reason for the necessity of the small constant, $a = 0.1 \text{ GeV}^2$. It is possible to evaluate the potential energy among quarks in a more exact way. However, the form of the interaction energy for two- or more-particle systems in a relativistic theory is somewhat ambiguous.

ACKNOWLEDGMENTS

It is the author's pleasure to thank Professor T. Yamanouchi for his continual guidance during the author's study of high-energy physics. The author is grateful to Professor S. Gasiorowicz for hospitality extended and for very helpful comments on the subject of this paper. Thanks are also due to Professor D. Geffen, Professor J. Rosner, and other staff, of the School of Physics and Astronomy at the University of Minnesota for their warm hospitality. This work was supported in part by the Numatron Project, Institute for Nuclear Study, University of Tokyo and in part by the United States Department of Energy under Contract No. E(11-1)-1764.

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