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Baryonium masses in a quark model

K. Maharana,* D. Mishra, and C. V. Sastry
Regional Engineering College, Rourkela 8, Orissa, India
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We derive certain mass relations among the various isospin multiplets of the baryonium states in a quark model. The mass spectrum for these states is also predicted by identifying the interquark-interaction-potential parameters used for this calculation with those used previously for the calculation of the masses of mesons and baryons. We predict two spin-1 states at 1497 and 1799 MeV, which could be identified with the experimentally observed states at 1495 and 1820 MeV, respectively. Three other spin-1 states are calculated to have masses of 1692, 1898, and 1932 MeV. These are very close to the mass values of the experimentally found states 1684, 1897, and 1935 MeV.

I. INTRODUCTION

Recently narrow structures in $N\bar{N}$ scattering have been observed[1] which have a characteristic reluctance to decay into mesons. These objects were first predicted from duality considerations[2] as the multiquark bound states, being dual to the ordinary $q\bar{q}$ meson trajectories in baryon-antibaryon scattering. These states are generally known as baryoniums. The reluctance of the baryonium states to decay into mesons could possibly be explained by a new selection rule of the Okubo-Zweig-Iizuka (OZI) type in a color-gauge model.[3]

However, in this paper we address ourselves to the problem of calculation of masses of these baryonium states from the quark model. We derive certain mass relations among various isospin multiplets constructed out of the $q\bar{q}q\bar{q}$ system where the q's are the $\mathcal{P}$, $\mathcal{N}$, and λ quarks. Misra and Sastry[4] have obtained a number of mass relations among the baryon multiplets and also among the meson multiplets by parametrizing the different quark-quark and quark-antiquark interaction amplitudes with the usual additivity assumption among the amplitudes. The results obtained by them agree well with the experimental values. In this paper we apply their method to obtain mass relations among baryonium states of spin 2, 1, and 0. By identifying the parameters with those for mesons and baryons[4] we predict the masses of different multiplets. It is assumed that the quarks and antiquarks interact with each other directly or by exchanging spin or unitary spin or both. The noninteracting quarks and antiquarks remain passive or as spectators. The quark-quark and quark-antiquark interactions are parametrized as[5]

$$
\begin{aligned}
V|\mathcal{P}_+\mathcal{N}_+\rangle &= V_{dd}|\mathcal{P}_+\mathcal{N}_+\rangle + V_{de}|\mathcal{N}_+\mathcal{P}_+\rangle + V_{ed}|\mathcal{P}_+\mathcal{N}_+\rangle + V_{ee}|\mathcal{N}_+\mathcal{P}_+\rangle\,,\\
V|\mathcal{P}_+\mathcal{N}_-\rangle &= V_{dd}|\mathcal{P}_+\mathcal{N}_-\rangle + V_{de}|\mathcal{N}_+\mathcal{P}_-\rangle + V_{ed}|\mathcal{P}_-\mathcal{N}_+\rangle + V_{ee}|\mathcal{N}_-\mathcal{P}_+\rangle\,,\\
V|\mathcal{P}_+\lambda_-\rangle &= V_{dd}^{(1)}|\mathcal{P}_+\lambda_-\rangle + V_{de}^{(1)}|\lambda_+\mathcal{P}_-\rangle + V_{ed}^{(1)}|\mathcal{P}_-\lambda_+\rangle + V_{ee}^{(1)}|\lambda_-\mathcal{P}_+\rangle\,,\\
V|\lambda_+\lambda_-\rangle &= V_{dd}^{(2)}|\lambda_+\lambda_-\rangle + V_{de}^{(2)}|\lambda_+\lambda_-\rangle + V_{ed}^{(2)}|\lambda_-\lambda_+\rangle + V_{ee}^{(2)}|\lambda_-\lambda_+\rangle
\end{aligned}
\tag{1}
$$

and

$$
\begin{aligned}
V|\mathcal{P}_+\bar{\mathcal{N}}_+\rangle &= \bar{V}_{dd}|\mathcal{P}_+\bar{\mathcal{N}}_+\rangle\,,\\
V|\mathcal{P}_+\bar{\mathcal{N}}_-\rangle &= \bar{V}_{dd}|\mathcal{P}_+\bar{\mathcal{N}}_-\rangle + \bar{V}_{ed}[|\mathcal{P}_+\bar{\mathcal{N}}_-\rangle - |\mathcal{P}_-\bar{\mathcal{N}}_+\rangle]\,,\\
V|\mathcal{P}_+\bar{\lambda}_-\rangle &= \bar{V}_{dd}^{(1)}|\mathcal{P}_+\bar{\lambda}_-\rangle + \bar{V}_{ed}^{(1)}[|\mathcal{P}_+\bar{\lambda}_-\rangle - |\mathcal{P}_-\bar{\lambda}_+\rangle]\,,\\
V|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle &= \bar{V}_{dd}|\mathcal{P}+\bar{\mathcal{P}}_-\rangle + \bar{V}_{de}[|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle + |\mathcal{N}_+\bar{\mathcal{N}}_-\rangle] + \bar{V}_{de}^{(1)}|\lambda_+\bar{\lambda}_-\rangle + \bar{V}_{ed}[|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle - |\mathcal{P}_-\bar{\mathcal{P}}_+\rangle]\\
&\quad + \bar{V}_{ee}[|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle - |\mathcal{P}_-\bar{\mathcal{P}}_+\rangle + |\mathcal{N}_+\bar{\mathcal{N}}_-\rangle - |\mathcal{N}_-\bar{\mathcal{N}}_+\rangle] + \bar{V}_{ee}^{(1)}[|\lambda_+\bar{\lambda}_-\rangle - |\lambda_-\bar{\lambda}_+\rangle]\,.
\end{aligned}
\tag{2}
$$

The usual additivity assumptions relate the amplitude as

$$
V_{ij}^{(2)} + V_{ij} = 2V_{ij}^{(1)}
$$

and

$$
\bar{V}_{ij}^{(2)} + \bar{V}_{ij} = 2\bar{V}_{ij}^{(2)}\,, \tag{3}
$$

where i and j stand for either d or e.

II. MASS RELATIONS

Now we shall proceed to calculate the masses of the two-quark–two-antiquark systems of the 405 representation which occurs in the direct product of $6\times\bar{6}\times6\times\bar{6}$. We shall mainly concentrate on the states contained in (27, 5), (8_1, 5), and (1, 5) of the 405 representation. The states in these representations are symmetric in the quark (antiquark) indices. If we denote by $s(q_1q_2q_3q_4)$ the symmetrized state of four quarks (antiquarks) with the minimum number of terms, we can write[6]

$$|B_m(27;2,0,2)\rangle = \frac{1}{\sqrt{6}} s(\mathcal{P}_+\mathcal{P}_+\overline{\mathcal{N}}_+\overline{\mathcal{N}}_+)\,, \tag{4}$$

where 2, 0, and 2 stand for the isospin, hypercharge, and the third components of isospin, respectively, and 27 is the SU(3) representation. Similarly we can write

$$|B_m(27;1,2,1)\rangle = \frac{1}{\sqrt{6}} s(\mathcal{P}_+\mathcal{P}_+\overline{\lambda}_+\overline{\lambda}_+)\,. \tag{5}$$

Now we have

$$\langle B_m(27;2,0,2)|V|B_m(27;2,0,2)\rangle = 2V_{dd} + 2V_{ed} + 2V_{ee} + 4\overline{V}_{dd} \equiv 2A \tag{6}$$

and

$$\langle B_m(27;1,2,1)|V|B_m(27;1,2,1)\rangle = 2V_{dd}^{(1)} + 2V_{de}^{(1)} + 2V_{ed}^{(1)} + 2V_{ee}^{(1)} + 4\overline{V}_{dd}^{(1)} = 2A^{(1)}\,. \tag{7}$$

The mass of all the remaining states in the 27 and the masses of the states in 8_1 and 1 can similarly be calculated. They are shown in Table I. The masses in the 8_1 representation satisfy the Gell-Mann–Okubo mass formula

$$B_m(8;\tfrac{1}{2},1,\tfrac{1}{2}) = \tfrac{3}{4}B_m(8;0,0,0) + \tfrac{1}{4}B_m(8;1,0,1) \tag{8}$$

as expected. The other relations that follow from the table are

$$\begin{aligned}
&B_m(27;2,0,2) + B_m(27;1,2,1) = 2B_m(27;\tfrac{3}{2},1,\tfrac{3}{2})\,,\\
&5B_m(27;1,0,1) = 4B_m(27;1,2,1) + B_m(27;2,0,2)\,,\\
&10B_m(27;\tfrac{1}{2},1,\tfrac{1}{2}) = 11B_m(27;1,2,1) - B_m(27;2,0,2)\,,\\
&5B_m(27;0,0,0) = 6B_m(27;1,2,1) - B_m(27;2,0,2)\,,
\end{aligned} \tag{9}$$

and

$$15B_m(1;0,0,0) + 3B_m(27;2,0,2) + 6B_m(27;1,2,1) = 8B_m(8;1,0,1) + 16B_m(8;\tfrac{1}{2},1\tfrac{1}{2})\,,$$

where the particle index stands for the (mass)^2.

In a like manner states with spins 1 and 0 can be constructed and mass eigenvalues of the multiplets obtained.

It is to be noted that not all the above states in the mass relations are physical states. States with the same quantum numbers in the different SU(3) representations will get mixed. Knowledge of some of the masses of these states would be necessary to determine the mixing angles.

III. MASSES

Now we make use of the values obtained for the different quark-quark and quark-antiquark amplitudes in Refs. 4 and 5. We neglect $\overline{V}_{de}$ and $\overline{V}_{de}^{(1)}$ to ensure ideal mixing of ω-ϕ (Ref. 5) system. Then using squared masses for both mesons and baryons, we obtain the masses of the baryoniums listed in Table II. For example,

$$m^2(2,0,2;5) = 2A = \tfrac{2}{3}N^{*2} + 4\rho^2\,, \tag{10}$$

where the particle symbols on the right-hand side stand for the masses of the corresponding particles. This gives the mass of the baryonium state with quantum numbers $T=2$, $T_3=2$, and $Y=0$ belonging to the (27, 5) representation as 1.841 GeV. Similarly, the masses of the baryonium contained in (27, 3) and (27, 1) representations can be expressed in terms of the known masses of the decuplet baryons and vector and pseudoscalar mesons. For example, the wave functions of the $T=2$, $T_3=2$ and $Y=0$ states contained in (27, 3) and (27, 1) representations are

$$|B_m(2,0,2;3)\rangle = \frac{1}{2\sqrt{6}}[s(\mathcal{P}_+\mathcal{P}_+\overline{\mathcal{N}}_+\overline{\mathcal{N}}_-) - s(\mathcal{P}_+\mathcal{P}_-\overline{\mathcal{N}}_+\overline{\mathcal{N}}_+)]$$

and

$$|B_m(2,0,2;1)\rangle = \frac{1}{4\sqrt{3}}[2s(\mathcal{P}_+\mathcal{P}_+\overline{\mathcal{N}}_-\overline{\mathcal{N}}_-) + 2s(\mathcal{P}_-\mathcal{P}_-\overline{\mathcal{N}}_+\overline{\mathcal{N}}_+) - s(\mathcal{P}_+\mathcal{P}_-\overline{\mathcal{N}}_+\overline{\mathcal{N}}_-)] \tag{11}$$

and their mass values are $2(A+2\overline{V}_{ed})$ and $2(A+3\overline{V}_{ed})$, respectively. Therefore

$$m^2(2,0,2;3) = \tfrac{2}{3}N^{*2} + 2\rho^2 + 2\pi^2$$

and

$$m^2(2,0,2;1) = \tfrac{2}{3}N^{*2} + \rho^2 + 3\pi^2\,. \tag{12}$$

It may be mentioned here that in calculating the masses of the baryonium belonging to the octet and singlet representations, the mixing angle for η and η' mesons is taken as $\theta = -10°$ as usual.[7]

IV. DISCUSSION AND CONCLUDING REMARKS

Two states with spin 1 have been experimentally observed at 1495 and 1820 MeV.[8] These could be identified with spin-1 states with quantum numbers $T=2$, $Y=0$ and $T=1$, $Y=0$ with masses 1497 and 1799 MeV that occur in the 27 representation (Table II). However, the parity assigned to these states in Ref. 8 does not agree with ours. The much talked about state at 1935 (Refs. 3, 9, and 10) could be associated with either the spin-1, $T=0$, $Y=0$ state of the 27 representation with mass

TABLE I. Wave functions of (1, 5), (8_1, 5), and (27, 5) in the 405 irreducible representation that occurs in $6\times\bar{6}\times 6\times\bar{6}$ of SU(6) and their mass values (the spin index is omitted).

T	Y	T_3	States	Mass values
			1	
0	0	0	$\frac{1}{12}[2s(\mathcal{P}\mathcal{P}\bar{\mathcal{P}}\bar{\mathcal{P}})+2s(\mathcal{N}\mathcal{N}\bar{\mathcal{N}}\bar{\mathcal{N}}) - s(\mathcal{P}\mathcal{N}\bar{\mathcal{P}}\bar{\mathcal{N}})+s(\mathcal{P}\lambda\bar{\mathcal{P}}\bar{\lambda}) - s(\mathcal{N}\lambda\bar{\mathcal{N}}\bar{\lambda})+2s(\lambda\lambda\bar{\lambda}\bar{\lambda})]$	$\frac{2}{3}A+\frac{4}{3}A^{(1)}+\frac{8}{3}\bar{V}_{de}+\frac{16}{3}\bar{V}_{de}^{(1)}$
			8_1	
1	0	1	$\frac{1}{2\sqrt{30}}[2s(\mathcal{P}\mathcal{P}\bar{\mathcal{P}}\bar{\mathcal{N}})-2s(\mathcal{P}\mathcal{N}\bar{\mathcal{N}}\bar{\mathcal{N}}) + s(\mathcal{P}\lambda\bar{\mathcal{N}}\bar{\lambda})]$	$\frac{8}{5}A+\frac{2}{5}A^{(1)}+3\bar{V}_{de}+2\bar{V}_{de}^{(1)}$
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2\sqrt{30}}[2s(\mathcal{P}\mathcal{P}\bar{\mathcal{P}}\bar{\lambda})-s(\mathcal{P}\mathcal{N}\bar{\mathcal{N}}\bar{\lambda}) +2s(\mathcal{P}\lambda\bar{\lambda}\bar{\lambda})]$	$\frac{1}{5}A+\frac{9}{5}A^{(1)}+\bar{V}_{de}+4\bar{V}_{de}^{(1)}$
0	0	0	$\frac{1}{12\sqrt{5}}[4s(\mathcal{P}\mathcal{P}\bar{\mathcal{P}}\bar{\mathcal{P}})+4s(\mathcal{N}\mathcal{N}\bar{\mathcal{N}}\bar{\mathcal{N}}) -2s(\mathcal{P}\mathcal{N}\bar{\mathcal{P}}\bar{\mathcal{N}})-s(\mathcal{P}\lambda\bar{\mathcal{P}}\bar{\lambda}) +s(\mathcal{N}\lambda\bar{\mathcal{N}}\bar{\lambda})-8s(\lambda\lambda\bar{\lambda}\bar{\lambda})]$	$-\frac{4}{15}A+\frac{34}{15}A^{(1)}+\frac{1}{3}\bar{V}_{de}+\frac{14}{3}\bar{V}_{de}^{(1)}$
			27	
2	0	2	$\frac{1}{\sqrt{6}}s(\mathcal{P}\mathcal{P}\bar{\mathcal{N}}\bar{\mathcal{N}})$	$2A$
$\frac{3}{2}$	1	$\frac{3}{2}$	$\frac{1}{2\sqrt{3}}s(\mathcal{P}\mathcal{P}\bar{\mathcal{N}}\bar{\lambda})$	$A+A^{(1)}$
1	2	1	$\frac{1}{\sqrt{6}}s(\mathcal{P}\mathcal{P}\bar{\lambda}\bar{\lambda})$	$2A^{(1)}$
1	0	1	$\frac{1}{2\sqrt{30}}[s(\mathcal{P}\mathcal{P}\bar{\mathcal{P}}\bar{\mathcal{N}})-s(\mathcal{P}\mathcal{N}\bar{\mathcal{N}}\bar{\mathcal{N}}) -2s(\mathcal{P}\lambda\bar{\mathcal{N}}\bar{\lambda})]$	$\frac{2}{5}A+\frac{8}{5}A^{(1)}$
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{6\sqrt{5}}[2s(\mathcal{P}\mathcal{P}\bar{\mathcal{P}}\bar{\lambda})-s(\mathcal{P}\mathcal{N}\bar{\mathcal{N}}\bar{\lambda}) -3s(\mathcal{P}\lambda\bar{\lambda}\bar{\lambda})]$	$-\frac{1}{5}A+\frac{11}{5}A^{(1)}$
0	0	0	$\frac{1}{24\sqrt{5}}[4s(\mathcal{P}\mathcal{P}\bar{\mathcal{P}}\bar{\mathcal{P}})+4s(\mathcal{N}\mathcal{N}\bar{\mathcal{N}}\bar{\mathcal{N}}) -2s(\mathcal{P}\mathcal{N}\bar{\mathcal{P}}\bar{\mathcal{N}})-6s(\mathcal{P}\lambda\bar{\mathcal{P}}\bar{\lambda}) -6s(\mathcal{N}\lambda\bar{\mathcal{N}}\bar{\lambda})+12s(\lambda\lambda\bar{\lambda}\bar{\lambda})]$	$-\frac{2}{5}A+\frac{12}{5}A^{(1)}$

TABLE II. Masses of the baryonium states in MeV.

T	Y	T_3	Spin 2	Spin 1	Spin 0
			27		
2	0	2	1841	1497	1291
$\frac{3}{2}$	1	$\frac{3}{2}$	1997	1692	1516
1	2	1	2148	1867	1713
1	0	1	2086	1799	1637
$\frac{1}{2}$	1	$\frac{1}{2}$	2170	1898	1749
0	0	0	2198	1932	1785
			8_1		
1	0	1	1905	1798	1743
$\frac{1}{2}$	1	$\frac{1}{2}$	2114	1975	1902
0	0	0	2180	2031	1953
			1		
0	0	0	2047	2011	1994

1932 MeV or the spin-2, $T=1$, $Y=0$ state of the 8 representation with mass 1905 MeV. We also predict that the experimentally observed states at 1684 (Ref. 11) and 1897 (Ref. 12) should have spin 1 and $T=\frac{3}{2}$, $Y=1$ and $T=\frac{1}{2}$, $Y=0$ by comparing with the states we obtain at 1692 and 1898 MeV, respectively, in the 27 representation. Also, the experimentally observed state at 1646 MeV could be identified with the spin-0 state at 1637 MeV and should have $T=1$, $Y=0$. Similarly the experimentally observed state at 2020 MeV[13] could be the spin-1 state of the 8 representation or singlet representation with $T=Y=0$, and the state at 1794 (Ref. 14) may be associated with the spin-1 state with $T=1$, $Y=0$ having the mass 1978 of the 8 representation. Besides these, the masses of a host of as yet undiscovered states and their quantum numbers are predicted in Table II. Many of these mass values lie close to the masses of the experimentally observed states. But we cannot accomodate in our scheme the observed baryonium state at 1395 MeV. However, in predicting the above masses we have neglected the mixing of the different multiplets with the same quantum numbers. If the physical states are mixtures of different states with the same quantum numbers, the masses predicted could have slight variation. Owing to the lack of proper identification, we have not taken this into account.

The observation that the baryoniums may be prohibited to decay into mesons due to the color quantum number does not alter the above mass relations.

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*Present address: Physics Department, Utkal University, Bhubaneswar 751004, India.

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