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Electromagnetic interactions of hadrons in the relativistic harmonic-oscillator quark model

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Various electromagnetic properties of hadrons are investigated systematically by applying many possible types of "minimal currents" in the framework of the relativistic harmonic oscillator (H.O.) which are conserved in the symmetric limit. As a result it is shown that a general minimal current in the $SU(6)_M$ scheme [minimally boosted SU(6)] with definite-metric H.O. reproduces satisfactorily experimental behaviors of almost all hadron electromagnetic (EM) properties: EM form factors of nucleons and pions (including nonvanishing electric form factor of the neutron), magnetic and transition moments of ground-state mesons and baryons, and helicity couplings of photon and baryon resonances. Other types of currents, a vector-meson-dominant one and a general minimal one in the $\tilde{U}(12)$ scheme with shell-type definite-metric H.O., are also interesting in giving similar desirable behaviors with some exceptions.

I. INTRODUCTION

The success of the nonrelativistic harmonic-oscillator (H.O.) quark model is now widely accepted as the $SU(6)\otimes O(3)$ scheme.[1] It seems to us that, as a relativistic generalization of it, the $\tilde{U}(12)\otimes O(3,1)$ scheme or the $SU(6)_M\otimes O(3,1)$ scheme is most promising. The $\tilde{U}(12)\otimes O(3,1)$ scheme[2-4] was first used independently by Feynman, Kislinger, and Ravndal[2] (in their famous work) and by us[3] (in our attempt for a unified theory of hadrons), where the Pauli spinor for the SU(6) space is replaced by the Dirac spinor [$\tilde{U}(12)$], and the three-dimensional harmonic oscillator for the O(3) space is extended to the four-dimensional H.O. for the internal Lorentz space [$O(3,1)$]. Another scheme, the $SU(6)_M\otimes O(3,1)$ scheme [whose original intuitive form was proposed by Fujimura, Kobayashi, and Namiki[5] prior to the $\tilde{U}(12)\otimes O(3,1)$ scheme and later formulated rigorously by Matsuda, Namiki, and one (S.I.) of the present authors[6]], is also interesting; where the nonrelativistic SU(6) wave functions are extended to the covariant $SU(6)_M$ [minimally boosted SU(6)] ones with a minimal number of components, and there appear no such extra form factors of nucleons and pions as occurred due to boosting in the $\tilde{U}(12)$ case. In this line of approaches each of the two parts, SU(6) and O(3), of the $SU(6)\otimes O(3)$ scheme is separately boosted and it may be called the boosted $SU(6)\otimes O(3)$ scheme or the boosted L-S coupling scheme. This may be rather contrasting with conventional relativistic schemes[7] based on the bound-state picture of compositeness which naturally leads to a j-j coupling pattern for the hadron level scheme.

The generalization of the three-dimensional H.O. to the four-dimensional one has a rather long history,[8] and there are two typical kinds, indefinite- or definite-metric H.O., depending upon their treatment of relative time freedom. There is also some difference in treatment of quark coordinates between the conventional one and the one in the scheme of Ref. 3 (where is shown clearly a unified character of mesons and baryons), and they shall be called the B coordinate (bound-state coordinate) and the S coordinate (shell coordinate), respectively. Thus there are many variants of relativistic H.O. quark model in the framework of boosted $SU(6)\otimes O(3)$ scheme depending upon their choices on the above various points.

Many efforts have been done in applying these respective models to electromagnetic interactions of hadrons. In this paper we shall investigate *systematically* various electromagnetic properties of ground and excited hadrons in these models comparing their results. For completeness some essential results of previous works are also briefly recapitulated in appropriate places. In this paper there are two improved or new points: First, we apply the *most-general* form of minimal currents,[2] which is conserved[9-11] in the symmetrical limit so far as hadron masses are given by the H.O., while usually only a special form of it is considered. Second, we also consider a *conserved* (in the same limit) vector-meson-dominant (VMD) current, while our previous VMD current[12] lacked a proper consideration on conservation.

II. CONSERVED CURRENTS IN THE RELATIVISTIC QUARK MODEL

A. Relativistic quark model

First we recapitulate briefly the framework of our relativistic quark model (RQM). In our scheme hadrons are described[3] by appropriate multilocal fields in the boosted $SU(6)\otimes O(3)$ space, which play the role of wave functions in the usual approach and are, at the same time, second-quantized as Q-number fields. In this paper we shall call them simply wave functions. They may be (in the case of B coordinates) also regarded as the Bethe-Salpeter amplitudes in the conventional color-quark model.

In the B-coordinate RQM, which is natural from the usual bound-state picture of compositeness, the wave functions are described for (nonexotic) hadrons as

$$\Phi^{B}_{(p)A}(x_1, x_2) \text{ mesons},$$
$$\Phi_{(p)A_1A_2A_3}(x_1, x_2, x_3) \text{ baryons}, \tag{1a}$$

where x_i's represent Lorentz four-vectors representing space-time coordinates of "constituents," $A\equiv(\alpha, a)$ [$a=(1,2,3)$ for flavors, $\alpha=(1,2,3,4)$ and $(1,2)$, respectively, for Dirac spinor in the $\tilde{U}(12)$ scheme and Pauli spinor in the $SU(6)_M$ (Ref. 6) scheme], and the suffix $p=(1,2)$ is for the ρ-matrix space needed only in the $SU(6)_M$ scheme.

In the S-coordinate RQM, which appeared in the scheme of Ref. 3, the wave functions are given as

$$\Phi^{B}_{(p)A}(x;\xi\eta) \text{ mesons},$$
$$\Phi_{(p)A_1A_2A_3}(x;\xi_1\xi_2\xi_3) \text{ baryons}, \tag{1b}$$

where x_μ is a center-of-mass coordinate of hadrons and ξ, η, and ξ_i's are Lorentz vectors representing internal coordinates of the "constituent" measured from the center-of-mass point. Here the *external* coordinate x_μ is, from the beginning, set *independently* of the *internal* coordinates, and its meaning as center-of-mass coordinate is achieved only in the form of expectation value in the internal space as

$$\langle\xi+\eta\rangle=\langle\partial/\partial\xi+\partial/\partial\eta\rangle=0 \quad \text{mesons},$$
$$\left\langle\sum_i \xi_i\right\rangle=\left\langle\sum_i \partial/\partial\xi_i\right\rangle=0 \quad \text{baryons} \tag{2}$$

by imposing some appropriate subsidiary conditions.[3] This kinematical independence of ξ, etc., from x greatly facilitates calculations in this scheme and makes clear a unified character of hadrons.

B. Minimal currents

A prescription to get conserved "minimal" electromagnetic currents in the relativistic H.O. quark model was first given by Feynman *et al.*,[2] and now it has been developed in various respects.[9-11] Here, following our prescription[10] given previously, we derive, as a simple example, the most general form of minimal baryon currents in the case[13] of S-coordinate H.O. in the $\tilde{U}(12)\otimes O(3,1)$ scheme. The most general Lagrangian density $\mathcal{L}$ in this case, which leads to a Klein-Gordon equation with the H.O. mass term, is given by

$$-\mathcal{L}=\frac{\partial\bar{\Phi}}{\partial x_\mu}\frac{\partial\Phi}{\partial x_\mu}+\frac{\lambda}{2}\left[f_E\sum_i\left(\frac{\partial\bar{\Phi}}{\partial\xi_{i\mu}}\frac{\partial\Phi}{\partial\xi_{i\mu}}+\kappa^2\xi_{i\mu}{}^2\bar{\Phi}\Phi\right)+f_M\sum_i\left(\frac{\partial\bar{\Phi}}{\partial\xi_{i\mu}}\gamma_\mu^{(i)}\gamma_\lambda^{(i)}\frac{\partial\Phi}{\partial\xi_{i\lambda}}+\kappa^2\xi_{i\mu}{}^2\bar{\Phi}\Phi\right)\right]+S_0\bar{\Phi}\Phi, \tag{3a}$$

where $\gamma^{(i)}$'s are Dirac matrices for the ith "constituent." Here λ, κ, f_E, f_M, and S_0 are constant parameters which satisfy (without loss of generality) a restriction

$$f_E+f_M=1. \tag{3b}$$

From the Lagrangian density (3) we derive the Euler equation

$$\left[-\frac{\partial^2}{\partial x_\mu{}^2}+\sum_i\frac{\lambda}{2}\left(-\frac{\partial^2}{\partial\xi_{i\mu}{}^2}+\kappa^2\xi_{i\mu}{}^2\right)+S_0\right]\times\Phi(x;\xi_1\xi_2\xi_3)=0. \tag{4}$$

Thus the slope parameter of orbital Regge trajectory α' is given by

$$\alpha'=\omega^{-1}, \quad \omega\equiv\lambda\kappa. \tag{5}$$

Now, substituting "covariant derivatives" for usual derivatives in the free $\mathcal{L}$ (3), we get a minimal interaction with electromagnetic field $A_\mu(x)$ as

$$L_I=-\int dx\prod_j d\xi_j\sum_i j_{i\mu}(x;\xi_1\xi_2\xi_3)A_\mu(x+\xi_i), \tag{6a}$$

where explicit expressions of "multilocal current densities" $j_{i\mu}$'s are given (up to the first order coupling) in Table I. In the conventional case of B coordinates the interaction takes the form

$$L_I=-\int\prod_j dx_j\sum_i j_{i\mu}(x_1,x_2,x_3)A_\mu(x_i). \tag{6b}$$

In Table I all explicit forms of these multilocal current densities for mesons and baryons in the various RQM's, which are obtained similarly as above, are collected. In the following, to discriminate these various types of minimal currents, we shall use the notation as B (S) D (I) $\tilde{U}$ (M), which means bound-state- (shell-) type coordinates, definite- (indefinite-) metric H.O. and $\tilde{U}(12)$ ($SU(6)_M$) boosting, respectively; for example, $BD\tilde{U}$ current means a minimal current in the case of B-type coordinate definite-metric H.O. in the $\tilde{U}(12)\otimes O(3,1)$ scheme.

C. Vector-meson-dominant currents

The electromagnetic (EM) interactions given above are, so the speak, the ones for "bare com-

TABLE I. Minimal current densities.

Baryon	
S coordinate	$j_{i\mu}=\bar{\Phi}_B Q^{(i)}\left\{-i\frac{\overleftrightarrow{\partial}}{\partial x_\mu}-\frac{\lambda f_E}{2}i\frac{\overleftrightarrow{\partial}}{\partial \xi_{i\mu}}-\frac{\lambda f_M}{2}i\left[\frac{\overleftrightarrow{\partial}}{\partial \xi_{i\mu}}+i\sigma_{\mu\nu}^{(i)}\left(\frac{\overrightarrow{\partial}}{\partial \xi_{i\nu}}+\frac{\overleftarrow{\partial}}{\partial \xi_{i\nu}}\right)\right]\right\}\Phi_B$
B coordinate	$j_{i\mu}=\bar{\Phi}_B Q^{(i)}\left\{-ci\frac{\overleftrightarrow{\partial}}{\partial x_\mu}-3\nu g_E i\frac{\overleftrightarrow{\partial}}{\partial x_{i\mu}}-3\nu g_M i\left[\frac{\overleftrightarrow{\partial}}{\partial x_{i\mu}}+i\sigma_{\mu\nu}^{(i)}\left(\frac{\overrightarrow{\partial}}{\partial x_{i\nu}}+\frac{\overleftarrow{\partial}}{\partial x_{i\nu}}\right)\right]\right\}\Phi_B$
Meson	
S coordinate	$j_{1\mu}=\bar{\Phi}_M Q^{(1)}\left\{-i\frac{\overleftrightarrow{\partial}}{\partial x_\mu}-\frac{\lambda f_E}{2}i\frac{\overleftrightarrow{\partial}}{\partial \xi_{\mu}}-\frac{\lambda f_M}{2}i\left[\frac{\overleftrightarrow{\partial}}{\partial \xi_{\mu}}+i\sigma_{\mu\nu}^{(1)}\left(\frac{\overrightarrow{\partial}}{\partial \xi_{\nu}}+\frac{\overleftarrow{\partial}}{\partial \xi_{\nu}}\right)\right]\right\}\Phi_M$
	$j_{2\mu}=\bar{\Phi}_M (-Q^{(2)T})\left\{-i\frac{\overleftrightarrow{\partial}}{\partial x_\mu}-\frac{\lambda f_E}{2}i\frac{\overleftrightarrow{\partial}}{\partial \eta_{\mu}}-\frac{\lambda f_M}{2}i\left[\frac{\overleftrightarrow{\partial}}{\partial \eta_{\mu}}-i\sigma_{\mu\nu}^{(2)T}\left(\frac{\overrightarrow{\partial}}{\partial \eta_{\nu}}+\frac{\overleftarrow{\partial}}{\partial \eta_{\nu}}\right)\right]\right\}\Phi_M$
B coordinate	$j_{1\mu}=\bar{\Phi}_M Q^{(1)}\left\{-ci\frac{\overleftrightarrow{\partial}}{\partial x_\mu}-2\nu g_E i\frac{\overleftrightarrow{\partial}}{\partial x_{1\mu}}-2\nu g_M i\left[\frac{\overleftrightarrow{\partial}}{\partial x_{i\mu}}+i\sigma_{\mu\nu}^{(1)}\left(\frac{\overrightarrow{\partial}}{\partial x_{1\nu}}+\frac{\overleftarrow{\partial}}{\partial x_{1\nu}}\right)\right]\right\}\Phi_M$
	$j_{2\mu}=\bar{\Phi}_M (-Q^{(2)T})\left\{-ci\frac{\overleftrightarrow{\partial}}{\partial x_\mu}-2\nu g_E i\frac{\overleftrightarrow{\partial}}{\partial x_{2\mu}}-2\nu g_M i\left[\frac{\overleftrightarrow{\partial}}{\partial x_{2\mu}}-i\sigma_{\mu\nu}^{(2)T}\left(\frac{\overrightarrow{\partial}}{\partial x_{2\nu}}+\frac{\overleftarrow{\partial}}{\partial x_{2\nu}}\right)\right]\right\}\Phi_M$
Restrictions on parameters	
S coordinate	$f_E+f_M=1$, $\lambda\kappa=\omega$; $\omega=(\alpha')^{-1}=1.10\ \mathrm{GeV}^2$
B coordinate	$g_E+g_M=1$, $\nu\Omega=\omega$, $c+\nu=1$

posite systems" and contain no effects due to strong interactions. So they are not applicable, for example, to problems concerned with a timelike photon where the effects of vector mesons play an essential role. For many years, on the other hand, we have stressed[14] the importance of vector-meson effects on the EM properties of composite hadrons. Especially, we have shown[12] that a VMD current in the $SD\tilde{U}$ scheme [where a coupling between nucleons and vector mesons is derived by applying the SU(6) symmetry heuristically] reproduces the experimental EM properties of nucleons quite well. However, this current has an unpleasant feature that it gives[15] no contribution to electric radiative decay amplitudes of excited hadrons.

Now we shall propose a semiphenomenological VMD current which has no such difficulty and preserves essentially the interesting results of nucleon properties. We suppose that interactions of baryons with an external vector meson V_μ in the shell-type $\tilde{\mathrm{U}}(12)\otimes\mathrm{O}(3,1)$ scheme are given by

$$L^V=-\int dx\prod_j d\xi_j\sum_{i=1}^{3}j^V_{i\mu}(x;\xi_1\xi_2\xi_3)V_\mu(x+\xi_i)\,,\quad (7)$$

with

$$j^V_{i\mu}\equiv j^{(E)}_{i\mu}+j^{(M)}_{i\mu}\,,\qquad (8a)$$

$$j^{(E)}_{i\mu}(p'p;\xi_1\cdots)=C_E\bar{\Phi}(p';\xi_1\cdots)\times\left[(p'+p)_\mu-i\frac{\lambda}{2}\frac{\overleftrightarrow{\partial}}{\partial\xi_{i\mu}}\right]\Phi(p;\xi_1\cdots)\,,\qquad (8b)$$

$$j^{(M)}_{i\mu}(p'p;\xi_1\cdots)=C_M\bar{\Phi}(p';\xi_1\cdots)\times\left[\epsilon_{\mu\nu\kappa\lambda}\frac{q_\nu}{m_V}(p'+p)_\kappa i\gamma_5^{(i)}\gamma_\lambda^{(i)}\right]\times\Phi(p;\xi_1\cdots)\,,\qquad (8c)$$

where the expressions for $j_{i\mu}$ is given, for convenience, in the momentum space, m_V is a mass of relevant vector meson, and we assume the Okubo-Zweig-Iizuka rule for the flavor degree of freedom. To get a whole EM current we introduce a factor $m_V^2/(m_V^2+q^2)$ (q^2 = four-momentum square of photon) into (8). The two parameters C_E and C_M in (8) are fixed by charge and magnetic moment of proton, respectively. This current (8) is identical to our previous one except for an addition of the second term in (8b), which is concerned essentially with excited hadrons, and we expect that they preserve the good properties concerning ground hadrons. It is notable that our electric current (8b) is conserved in the symmetric limit (since it is essentially a minimal current), while the magnetic current is strictly conserved as is easily seen from (8c). This type of current is denoted as the VD1 type in the following. We can also obtain the other various conserved (in the symmetric limit) VMD currents by adding a factor $m_V^2/(m_V^2+q^2)$ to the currents in Table I. These EM currents may be interpreted to include effects of both direct photon coupling and indirect photon coupling through vector mesons. In this work we shall treat this type of "vector-meson-dominant minimal" current only in the case of *BIM* type with the normal ordering (see Sec. III) and denote it as the VD2 type.

Finally, in this section we add a general remark. Strictly our currents may be significant only in the symmetric limit, where their conservation is guaranteed. However, in actual application to physical world in case of a broken symmetry kinematical effects should be properly taken into account, of which procedure is somewhat ambiguous. We have made this generally as follows: Our effective interactions (after the integrals on internal variables being carried out) in the symmetric limit can be reduced, aside from some factors, to corresponding simple local interactions among usual particle fields. In these "local parts" of interactions all fields are regarded as physical with real masses. In the following applications this case is denoted as the symmetric mass case. For comparison we have also examined the case of real masses, where all parts of our interactions are treated as physical with real masses.

III. APPLICATIONS TO GROUND-STATE HADRONS

In this section we shall investigate the EM properties of ground-state hadrons by applying the interactions derived in Sec. II, comparing our various models. From the multilocal current densities given in Table I, following the usual techniques in our scheme (which is essentially equivalent to the ordinary composite-model calculation) with aids of tables[3,6] in our previous works, we can derive unifiedly the effective currents $J_\mu(q)$ for respective process, which are directly related with our relevant physical quantities.

A. Nucleon form factors and various models

The expressions of nucleon electromagnetic form factors (EMFF) thus obtained are collected in Table II. First we explain characteristic features and relations among respective models taking this example of the FF.

$SU(6)_M$ or $\tilde{U}(12)$. The boosted spin wave functions contribute the extra factors to the FF, respectively, as

$$S_{\tilde{U}} \equiv \frac{[\bar{u}(p')u(p)]^3}{\bar{u}(p')u(p)} = \left(1+\frac{q^2}{4m^2}\right) \text{ for } \tilde{U}(12)\,, \tag{9a}$$

$$S_M \equiv \frac{\bar{u}(p')u(p)}{\bar{u}(p')u(p)} = 1 \text{ for } SU(6)_M\,, \tag{9b}$$

where $u(p)$ $[\bar{u}(p')]$ represents an initial [final] Dirac spinor with the four-momentum p_μ $[p'_\mu]$. In (9) the factor in the denominator comes from the Dirac spinor for nucleon as a whole, while the power 3 [1] of $\bar{u}u$ in the numerator reflects a fact that the space of ρ matrix is introduced to each constituent (only a whole nucleon) in the $\tilde{U}(12)$ $[SU(6)_M]$ scheme.

Definite- or indefinite-metric, and S- or B-coordinate H. O. The internal overlapping integrals between initial and final H.O. wave functions contribute the factors to the FF, respectively, as

$$F_{SD} = \left(1+\frac{q^2}{2m^2}\right)^{-3} \exp\left[-\frac{1}{4\kappa}\left(\frac{q^2}{1+q^2/2m^2}\right)\right] \text{ for } S \text{ coordinates, definite metric}\,, \tag{10a}$$

$$F_{SI} = \exp\left(-\frac{1}{4\kappa}q^2\right) \text{ for } S \text{ coordinates, indefinite metric} \tag{10b}$$

and

$$F_{BD} = \left(1+\frac{q^2}{2m^2}\right)^{-2} \exp\left[-\frac{1}{\Omega}\left(\frac{q^2}{1+q^2/2m^2}\right)\right] \text{ for } B \text{ coordinates, definite metric}\,, \tag{11a}$$

$$F_{BI} = \exp\left(-\frac{1}{\Omega}q^2\right) \text{ for } B \text{ coordinates, indefinite metric}\,. \tag{11b}$$

In the indefinite-metric case the exponentially damping [with regards to q^2; $q_\mu = (p'-p)_\mu$] factors lead, similarly as in the nonrelativistic case, to an obvious contradiction to experiments; while in the definite metric case these factors decrease slowly in comformity with experiments, and this is known as representing a "Lorentz-contraction effect"[16] of the internal extension. The numbers of negative powers 3 and 2 [(10a) and (11a)] reflect the numbers of independent (four-dimensional) H.O. in the respective cases.

Indefinite-metric H.O. with normal ordering. The exponential factors in (10) and (11) come from "expectation values" of the term, for example in the S-coordinate H.O.

$$e^{-iq\xi} = \exp\left(-\frac{1}{4\kappa}q^2\right)\exp\left(-\frac{1}{\sqrt{2\kappa}}q_\mu a_\mu^\dagger\right) \times \exp\left(\frac{1}{\sqrt{2\kappa}}q_\mu a_\mu\right)\,; \tag{12a}$$

TABLE II. Nucleon form factors G.

	$G \propto (q^2)^{-n}$ for $q^2 \to \infty$		
	n		
	Ũ(12)		SU(6)$_M$
S coordinate			
$G_E^p = \left[1 + \frac{\lambda f_E}{2} \frac{q^2}{q^2+2m^2} + \frac{\lambda f_M}{2}\left(\frac{q^2}{q^2+2m^2} - \frac{q^2}{q^2+4m^2}\right)\right]\left(1+\frac{q^2}{4m^2}\right) F_{SD} \times \begin{cases} 1+q^2/4m^2, & \tilde{\mathrm{U}}(12) \\ 1, & \mathrm{SU}(6)_M \end{cases}$	1		2
$G_M^p = \frac{\lambda f_M}{2} F_{SD} \times \begin{cases} 1+q^2/4m^2, & \tilde{\mathrm{U}}(12) \\ 1, & \mathrm{SU}(6)_M \end{cases} \quad G_M^n = -\frac{2}{3} G_M^p$	2		3
$G_E^n = F_{SD} \times \begin{cases} 0, & \tilde{\mathrm{U}}(12) \\ \frac{\lambda F_M}{2} \frac{q^2}{6m^2}, & \mathrm{SU}(6)_M \end{cases}$	...		2
B coordinate			
$G_E^p = \left[1 + 2\nu g_E \frac{q^2}{q^2+2m^2} + 2\nu g_M\left(\frac{q^2}{q^2+2m^2} - \frac{3}{2} \frac{q^2}{q^2+4m^2}\right)\right]\left(1+\frac{q^2}{4m^2}\right) F_{BD} \times \begin{cases} 1+q^2/4m^2, & \tilde{\mathrm{U}}(12) \\ 1, & \mathrm{SU}(6)_M \end{cases}$	0 ↓ (1	FKR	1 ↓ 2)
$G_M^p = 3\nu g_M F_{BD} \times \begin{cases} 1+q^2/4m^2, & \tilde{\mathrm{U}}(12) \\ 1, & \mathrm{SU}(6)_M \end{cases} \quad G_M^n = -\frac{2}{3} G_M^p$	1		2
$G_E^n = F_{BD} \times \begin{cases} 0, & \tilde{\mathrm{U}}(12) \\ 3\nu g_M q^2/6m^2, & \mathrm{SU}(6)_M \end{cases}$	...		1
VD1			
$G_E^p = \left(1 + \frac{\lambda}{2} \frac{q^2}{q^2+2m}\right)\left(1+\frac{q^2}{4m^2}\right)^2 \frac{m_V{}^2}{q^2+m_V{}^2} F_{SD}$	2		
$G_M^p = \mu_p \left(1+\frac{q^2}{4m^2}\right)^2 \frac{m_V{}^2}{q^2+m_V{}^2} F_{SD}, \quad G_M^n = -\frac{2}{3} G_M^p$	2		
$G_E^n = 0$	...		
VD2			
$G_E^p = \left(1 - 3\nu g_M \frac{q^2}{q^2+4m^2}\right)\left(1+\frac{q^2}{4m^2}\right) \frac{m_V{}^2}{q^2+m_V{}^2}$			0
$G_M^p = 3\nu g_M \frac{m_V{}^2}{q^2+m_V{}^2}, \quad G_M^n = -\frac{2}{3} G_M^p$			1
$G_E^n = 3\nu g_M \frac{q^2}{6m^2} \frac{m_V{}^2}{q^2+m_V{}^2}$			0
$F_{SD} = \left(1+\frac{q^2}{2m^2}\right)^{-3} \exp\left(-\frac{1}{4\kappa} \frac{q^2}{1+(q^2/2m^2)}\right) \xrightarrow[q^2 \to \infty]{} (q^2)^{-3},$			
$F_{BD} = \left(1+\frac{q^2}{2m^2}\right)^{-2} \exp\left(-\frac{1}{\Omega} \frac{q^2}{1+(q^2/2m^2)}\right) \xrightarrow[q^2 \to \infty]{} (q^2)^{-2}$			

a_μ and $a_\mu^\dagger$ are oscillator variables,[17] which reflects the physical situation that EM fields interact with the nucleon through the constituents being at some distance from its center of mass [see (6)]. This factor, as it is, led to the exponential decrease in the indefinite-metric case as in (10b) and (11b). However, if we take its normal product[13] concerning the internal oscillator variables, again in the example of S-coordinate H.O.

$$:e^{-iq\xi}: = \exp\left(-\frac{1}{\sqrt{2\kappa}} q_\mu a_\mu^\dagger\right) \exp\left(\frac{1}{\sqrt{2\kappa}} q_\mu a_\mu\right), \tag{12b}$$

the factors (10b) and (11b) become

$$F_{SI} = F_{BI} = 1, \tag{13}$$

and there is no exponentially damping factor. In this paper we shall adopt this normal ordering always with the indefinite-metric H.O. Here we note that this prescription preserves the conserving character of our currents and it gives no effects on real-photon processes.

B. EM form factors of nucleons and pions

In Table II (III) we collect the results for nucleon (pion) form factors in the respective models. Experimentally,[18] it is well known that nucleon form factors ($G^p_{M,E}, G^n_M$) obey the scaling law and show the dipole-like asymptotic behavior [$\approx 1/(1+q^2/0.71)^2$], while pion form factor does the simple pole-like one ($\approx 1/[1+q^2/(0.671)^2]$). In this respect it is interesting that the *BDM* case with a *special Feynman-Kislinger-Ravndal (FKR) choice* of parameters shows[6] all of these asymptotic behaviors, while with a general choice the behaviors for G^p_E and for F^π (in the second formulation[19]) are somewhat different from them. So we can naturally expect good results in the general *BDM* case. The $SD\tilde{U}$ case [VD1 case] shows also the desirable asymptotic behaviors except for G^p_E [F^π] with the behavior of $(q^2)^{-1}$ [$(q^2)^{-2}$]. The status of comparison[20] of our theory with experiments is shown in the following four figures.

First, in Fig. 1 the status for magnetic FF's of the nucleon is shown for the cases of (1) *BDM*, (2) $SD\tilde{U}$, (3) VD1, (4) $BD\tilde{U}$, (5) VD2, of which all satisfy the scaling law. The cases of (1), (2), and (3) [having the desirable asymptotic behavior $\propto (q^2)^{-2}$] and of (4) [$\propto (q^2)^{-1}$] show good fitting for a reasonable range of respective parameters. The fitting of (5) (which is theoretically interesting, see Sec. V) seems not so bad, although it has the $(q^2)^{-1}$ behavior. The cases of *BIM* and $BI\tilde{U}$ (which are[13] good for radiative decays of baryon resonances, see Sec. IV) are not shown there, since they seem completely wrong, having the asymptotic behavior of $(q^2)^0$ and $(q^2)^{+1}$, respectively.

Second, the status of deviations from the scaling law of our proton electric FF in comparison with experiments[21] is shown in Fig. 2 for the above

TABLE III. Pion form factor F^π.

		$F^\pi \propto (q^2)^{-n}$ for $q^2 \to \infty$	n
S coordinate			
	$F^\pi=\left[1+\frac{\lambda f_E}{2}\frac{q^2}{q^2+2m^2}+\frac{\lambda f_M}{2}\begin{pmatrix}\frac{q^2}{q^2+2m^2}-\frac{q^2}{q^2+4m^2}\\ \frac{q^2}{q^2+2m^2}\\ -\frac{q^2}{q^2+2m^2}\end{pmatrix}\right]\begin{pmatrix}1+\frac{q^2}{4m^2}\\ 1\\ 1+\frac{q^2}{2m^2}\end{pmatrix}F_{SD}$	$\tilde{\mathrm{U}}(12)$	1
		$\mathrm{SU}(6)_M\mathrm{I}$	2
		$\mathrm{SU}(6)_M\mathrm{II}$	1
B coordinate			
	$F^\pi=\left[1+\nu g_E\frac{q^2}{q^2+2m^2}+\nu g_M\begin{pmatrix}\frac{q^2}{q^2+2m^2}-\frac{q^2}{q^2+4m^2}\\ \frac{q^2}{q^2+2m^2}\\ -\frac{q^2}{q^2+2m^2}\end{pmatrix}\right]\begin{pmatrix}1+\frac{q^2}{4m^2}\\ 1\\ 1+\frac{q^2}{2m^2}\end{pmatrix}F_{BD}$	$\tilde{\mathrm{U}}(12)$	$0 \xrightarrow[\mathrm{FKR}]{} 1$
		$\mathrm{SU}(6)_M\mathrm{I}$	1
		$\mathrm{SU}(6)_M\mathrm{II}$	$0 \xrightarrow[\mathrm{FKR}]{} 1$
VD1			
	$F^\pi=\left(1+\frac{\lambda}{2}\frac{q^2}{q^2+2m^2}\right)\left(1+\frac{q^2}{4m^2}\right)\frac{m_\rho^2}{q^2+m_\rho^2}F_{SD}$	$\tilde{\mathrm{U}}(12)$	2
VD2			
	$F^\pi=\left[1+0+2\nu g_M\begin{pmatrix}0\\ \frac{q^2}{2m^2}\end{pmatrix}\right]\frac{m_\rho^2}{q^2+m_\rho^2}$	$\mathrm{SU}(6)_M\mathrm{I}$	1
		$\mathrm{SU}(6)_M\mathrm{II}$	0
	$F_{SD}=\left(1+\frac{q^2}{2m^2}\right)^{-2}\exp\left(-\frac{1}{4\kappa}\frac{q^2}{1+(q^2/2m^2)}\right)\xrightarrow[q^2\to\infty]{}(q^2)^{-2}$,		
	$F_{BD}=\left(1+\frac{q^2}{2m^2}\right)^{-1}\exp\left(-\frac{1}{\Omega}\frac{q^2}{1+(q^2/2m^2)}\right)\xrightarrow[q^2\to\infty]{}(q^2)^{-1}$		

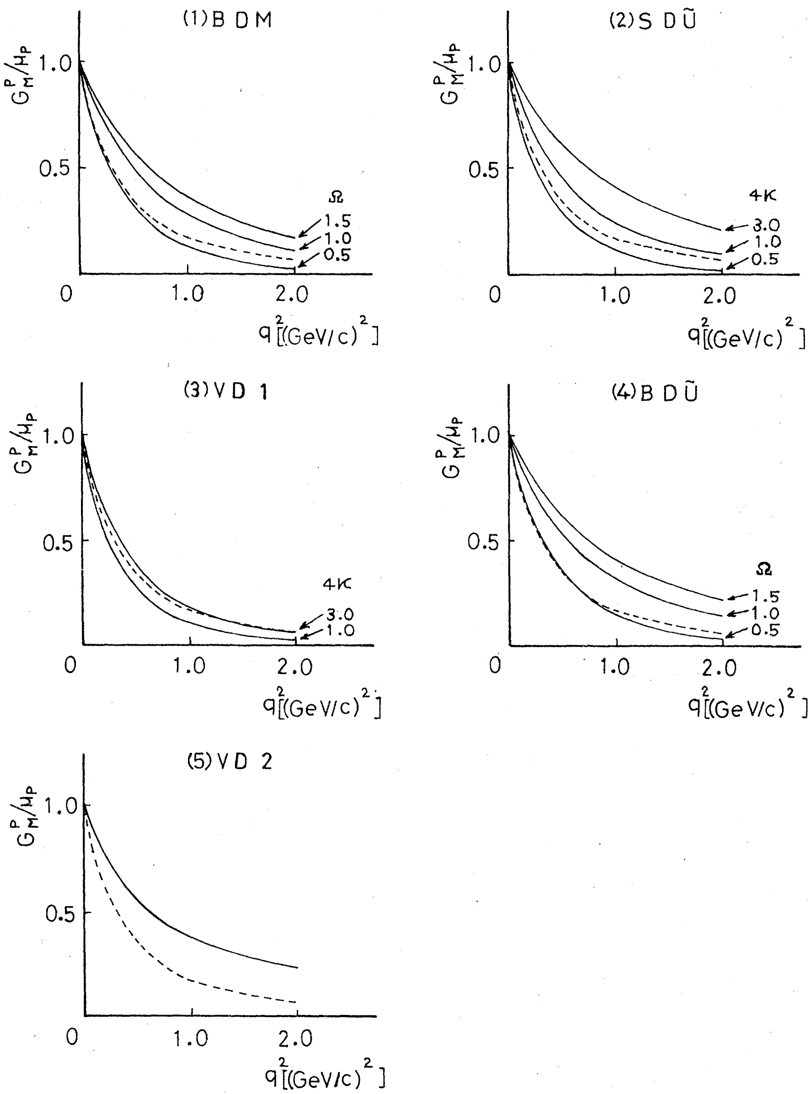

FIG. 1. Magnetic form factor of nucleon; theory (solid line) and experiment [dashed line $= 1/(1+q^2/0.71)^2$]. All cases (1) to (5) satisfy the scaling law $G_M^p \propto G_M^n$. The cases (1) to (3) [$\propto (q^2)^{-2}$ for $(q^2)\to\infty$] and the case (4) [$\propto (q^2)^{-1}$] show a good fitting for a reasonable range of parameters. The fitting of the case (5) [$\propto (q^2)^{-1}$] with no parameter seems not so bad.

five good cases. From this we see that (i) the cases (1) to (4) are not inconsistent with experiments for a proper region of respective parameters, and (ii) the case of (5) seems somewhat wrong, although this is not definite because of the experimental status.

Third, concerning the neutron electric FF, it is interesting that the $SU(6)_M$ scheme gives[5] nonvanishing values for it, while it vanishes in all of the $\tilde{U}(12)$ cases as well as in the usual nonrelativistic models. The status of fitting for the $SU(6)_M$ cases of (1) and (5) are shown in Fig. 3.[22] It is seen that the case (1) of *BDM* is consistent with experiments for a range of parameter $0.5 \lesssim \Omega_B \lesssim 1.5\ \mathrm{GeV}^2$, while the case of (5) of VD2 is consistent only for the low-q^2 region.

Finally, the status for pion FF is shown in Fig. 4 for the previous five cases (1) to (5), where in the B-type cases [(1), (4), and (5)] the FKR-type choice ($g_E = 0$, but $\nu \neq 1$) of parameters is adopted. From this we see that the cases of (1) (in the second formulation), (2), and (5) are in agreement with experiments, while the cases of (1) (in the first formulation), (3), and (4) seem to deviate slightly from experiments.

As a summary of the above analysis we may

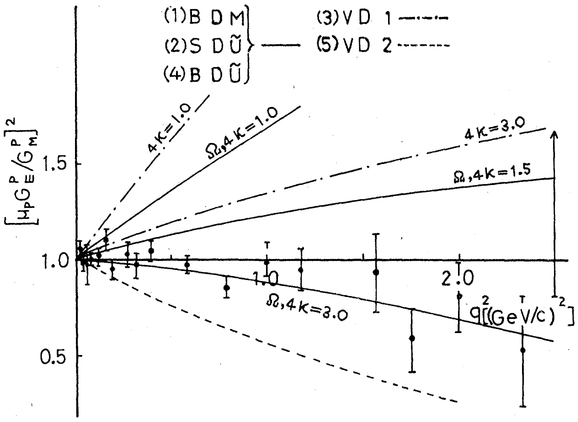

FIG. 2. Ratio of electric to magnetic form factors of proton; theory and experiment (Ref. 21). The cases (1) to (4) seem to be not inconsistent for a proper choice of parameters, while the case (5) is somewhat wrong.

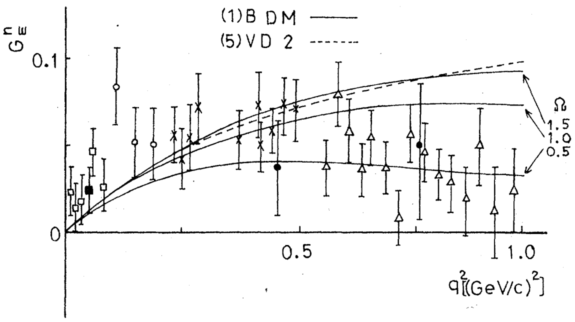

FIG. 3. Electric form factor of neutron; theory and experiment (Ref. 18). The $SU(6)_M$ cases (1) and (5) shown in the figure seem to be consistent with experiments, while all the $\tilde{U}(12)$ cases [(2), (4) and (3)] give vanishing values for the FF.

conclude on the respective models as follows. The case of (1) *BDM* can reproduce[6,23] satisfactorily experimental behaviors of all EMFF's of both nucleons and pions (in the second formulation). The good regions of the parameter Ω are $\Omega_B = 0.5-1.5$, $1.5-3$, $0.5-1.5$ GeV^2 for nucleon magnetic FF (G_M^n), proton electric FF (G_E^p), neutron electric FF (G_E^n) and $\Omega_M = 0.8-1.5$ GeV^2 for pion FF (F^π), respectively. It is interesting there is a common region of $\Omega_B \sim 1.5$ for nucleon, as it should be. The case[10] of (2) $SD\tilde{U}$ is in good agreement with experiments of G_M^n, G_E^p, ane F^π for regions of parameter $4\kappa_B = 0.5-3$, $1.5-3$ GeV^2, and $4\kappa_M = 0.5-3$ GeV^2, respectively, while it gives vanishing FF for G_E^n. The case[12] of (3) VD1 gives satisfactory results of G_M^n and G_E^p for $4\kappa_B \gtrsim 1$ and $4\kappa_B \gtrsim 3$ GeV^2, respectively, while it gives vanishing G_E^n and somewhat smaller F^π than the experimental value. The case[9,10] of (4) $BD\tilde{U}$ is in agreements with G_M^n and G_E^p for $\Omega_B = 0.5-1$ and $\Omega_B = 1.5-3$ GeV^2, respectively, while it gives the larger values of F^π. The case (5) VD2 seems to give poor results except for F^π.

C. Magnetic moments of baryons

The formulas and values of magnetic moments of octet baryons μ_i obtained from our currents are given in Table IV. In all the cases they are described by one parameter (which was fixed by the proton moment) and have the famous ratio $\mu_p/\mu_n = -\frac{3}{2}$ for nucleons, representing the SU(6) character of our scheme.

All the cases except the VD1 case give the same results and there is no difference between the use of symmetric and real masses, since there no form factor effect works. Thus the values in this table are obtained using all real masses for the remaining kinematical parts. For comparison the values in the SU(6) symmetry, where all baryon masses are taken to be the same, are also given there. It is interesting that a clear improvement[24] is seem in the case of VD1 compared with the SU(6)-symmetry case.

D. Radiative transitions of the ground-state mesons and baryons

The formulas and values obtained from our currents are given in Tables V and VI, respectively. With the use of symmetric masses all cases except the VD1 give, because of there being no form factor effects, the same results which are also identical with the ordinary SU(3)-symmetric values. Here both of the meson and the baryon processes are described by one parameter except for the VD1 case, which was fixed by the decay width of $\omega \to \pi^0\gamma$ and by the proton magnetic moment, respectively. With the use of real masses all the cases give the different values, and the difference is large for the meson processes reflecting the large symmetry breaking in mass values. In this case the amplitudes also depend upon another parameter (Ω or 4κ), which is tentatively chosen as given in Table VI. From this we see that the case of VD1 with symmetric mass seems getting some improvements[24] compared with the SU(3)-symmetric values. With the use of real masses the situation seems quite in confusion, and no meaningful conclusion might be obtained.

Finally, in this section we add some remarks. As actual values of symmetric masses we have chosen our "SU(6)-averaged" values $m_B = 1.33$ GeV and $m_M = 0.79$ GeV for baryons and mesons, respectively, which have been also used in our previous analyses, showing remarkable regularity[25]

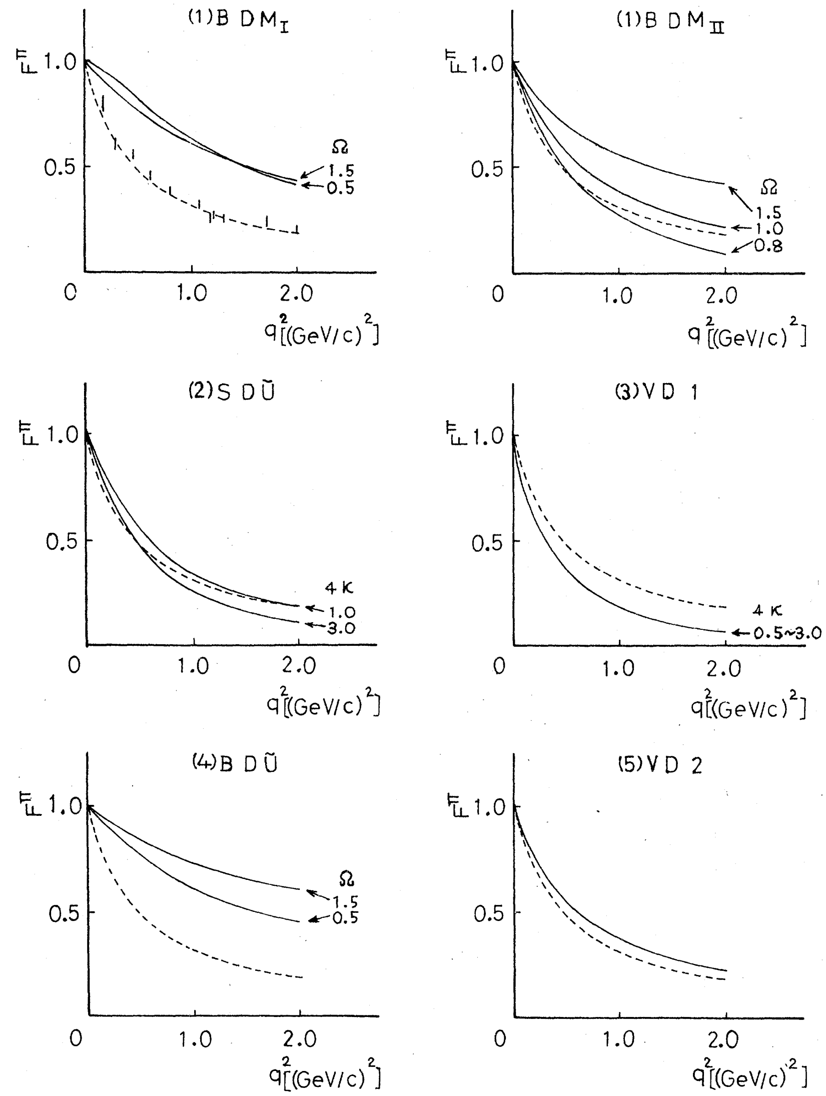

FIG. 4. Form factor of pion; theory (solid line) and experiment (Ref. 22) (dashed line = $1/[1+q^2/(0.671)^2]$). For mesons two types of formulation (denoted $M_{\rm I}$ and $M_{\rm II}$) are possible in the SU(6)$_M$ scheme. The cases (1) ($M_{\rm II}$), (2), and (5) can give good agreement with experiments, while the cases (1) ($M_{\rm I}$), (3), and (4) seem to deviate more or less.

of hadron interactions. Generally parameters for mesons and baryons may be different from each other, while they should be the same in the scheme of Ref. 3. In this connection it is notable that (i) the values in the $SD\tilde{U}$ case determined from G_M^N and F^{π} are the same as $4\kappa_B = 4\kappa_M = 0.5-3$ GeV^2, and (ii) the predicted values (from proton magnetic moment) of decay width for $\Gamma(\omega\to\pi^0\gamma)$ are close to the experimental one with the use of symmetric masses (see Table VI).

IV. RADIATIVE DECAYS OF BARYON RESONANCES

In this section we shall apply our EM interactions derived in Sec. II to radiative decays of baryon resonances [which are assigned to (SU(6) dimension, orbital momentum L) = ($\underline{56}$, 0), ($\underline{70}$, 1), and ($\underline{56}$, 2)], comparing our various models. These processes are usually described by the amplitudes[26] of photon-resonance helicity couplings $\tilde{A}_\lambda^{p,n,\Delta}$ ($\lambda = \frac{1}{2}, \frac{3}{2}$) which are related with the decay width of resonance Γ_γ as

$$\Gamma_\gamma = \frac{\vec{q}^{\,2}}{\pi}\frac{m_N}{m_R}\frac{2}{2j+1}\left(|\tilde{A}^i_{1/2}|^2 + |\tilde{A}^i_{3/2}|^2\right), \tag{14}$$

where $\vec{q}$ is the momentum of photon, m_R (m_N) is the mass of initial (final) resonance (nucleon), and j is the spin of the resonance. From the interactions (6), following our usual systematics with aid

TABLE IV. Magnetic moments of octet baryons; μ_i (in units of $e/2m_p$).

Particle	SU(6)		All minimal current and VD2		VD1 ($m_\rho = m_\omega \equiv m$)		Experiment
p	μ_p	input	μ_p	input	$\frac{1}{m} m\mu_p$	input	2.79
n	$-\frac{2}{3}\mu_p$	−1.86	$-\frac{2}{3}\mu_p$	−1.86	$\left(-\frac{2}{3m}\right)m\mu_p$	−1.86	−1.91
Λ	$-\frac{1}{3}\mu_p$	−0.93	$-\frac{1}{3}\mu_p$	−0.78	$\left(-\frac{1}{3m_\phi}\right)m\mu_p$	−0.70	−0.61± 0.03
Σ^+	μ_p	2.79	μ_p	$\times \frac{m_p}{m_i}$ 2.20	$\left(\frac{8}{9m}+\frac{1}{9m_\phi}\right)m\mu_p$	2.71	2.83± 0.25
Σ^0	$\frac{1}{3}\mu_p$	0.93	$\frac{1}{3}\mu_p$	0.73	$\left(\frac{2}{9m}+\frac{1}{9m_\phi}\right)m\mu_p$	0.85	⋯
Σ^-	$-\frac{1}{3}\mu_p$	−0.93	$-\frac{1}{3}\mu_p$	−0.73	$\left(-\frac{4}{9m}+\frac{1}{9m_\phi}\right)m\mu_p$	−1.01	−1.48± 0.37
Ξ^-	$-\frac{1}{3}\mu_p$	−0.93	$-\frac{1}{3}\mu_p$	−0.66	$\left(\frac{1}{9m}-\frac{4}{9m_\phi}\right)m\mu_p$	−0.63	−1.85± 0.75
Ξ^0	$-\frac{2}{3}\mu_p$	−1.86	$-\frac{2}{3}\mu_p$	−1.33	$\left(-\frac{2}{9m}-\frac{4}{9m_\phi}\right)m\mu_p$	−1.56	⋯

from the tables in our previous works,[3,6] we can express these amplitudes in terms of parameters in the respective models. The results are given in Table VII [parts (a) and (c)]. It is notable that the kinematical relations among respective amplitudes given in part (a) are, aside from the explicit expressions in part (c), common to all our relativistic models and also to nonrelativistic models,[27] which are based on the SU(6)⊗O(3) scheme and supposing the additive photon interactions with constituent quarks. Fixing parameters from general restrictions, Regge slopes, proton charge, and proton magnetic moment [see part (c)], all the amplitudes in Table VII are represented by one parameter (chosen 4κ and Ω, respectively, for the S-type and B-type coordinate models) in each model. With a tentative choice of these parameters, numerical values of the amplitudes are given in part (b) of Table VII for the definite-metric cases, (1) $SD\tilde{U}$ (SDM), (2) $BD\tilde{U}$ (BDM), (3) VD1, using symmetric masses for baryons (see below); and for the indefinite-metric cases, (4) VD2 = $BIM = SIM$ ($\approx BI\tilde{U} = SI\tilde{U}$), where there are no differences between the B-coordinate model and the S-coordinate one (both are related by an interchange of $\Omega \to 6\kappa$), and between the use of symmetric masses and physical ones. The experimental values[24] are also given in part (b). The status of fitting our results for some range of parameters (Ω, $4\kappa = 1$–3 GeV^2) to experiments is schematically shown in Figs. 5(a) and 5(b) and in Figs. 5(c) and 5(d) for $L=1$ baryons (only for ones without possibility of mixing states) and for $L=2$ baryons, respectively.

By inspecting these tables and figures we may conclude that the results of all the above cases are quite similar and reproduce well,[28] except for a few resonances [$\tilde{A}^p_{3/2}(D_{13}(1520))$, $\tilde{A}^p_{1/2}(S_{11}(1535))$, and $\tilde{A}^p_{3/2}(F_{15}(1688))$], the general experimental behaviors, although more accurate data are needed for some definite conclusions.

In the following we describe the results of our analysis in more detail and give some remarks.

Definite or indefinite metric. There appears an essential difference in the parameter dependence of the amplitudes between the cases of definite-metric H.O. and indefinite one. In the former (latter) there is a (no) form-factor effect (denoted F in the table) due to internal overlapping integrals, and the amplitudes have a (no) maximal values for some (any) value of the parameter (κ or Ω).

Symmetric or physical mass. As was mentioned in Sec. III, the symmetric mass ought to be used for the parts of effective interactions except for the ones being considered as kinematical. In this table the symmetric mass is used for the factors; d and F in the SU(6)$_M$ scheme, and d, F, and U^2 in the $\tilde{U}$(12) scheme.

SU(6)$_M$ or $\tilde{U}$(12). The difference between the

TABLE V. Transition currents and decay widths Γ.

		Minimal current and VD2 S coord.	B coord.	VD1
$V \to P\gamma$ $(P \to V\gamma)$				
$J_\mu = a\frac{i}{2}\epsilon_{\mu\upsilon\kappa\lambda}q_\upsilon[P(p')V_\lambda(p)]^{a'}_{Da}M^a_{a'}F \times \begin{cases} \frac{P_\kappa}{m_V}+\frac{P'_\kappa}{m_P}, & \tilde{\mathrm{U}}(12) \\ \frac{2P_\kappa}{m_V}\left(\frac{2P_\kappa}{m_P}\right), & \mathrm{SU}(6)_M\mathrm{I} \end{cases}$	a	$\frac{\lambda f_M}{2}$	$2\nu g_M$	$-C_M(m_V+m_P)$
$\Gamma = a^2 U_{VP\gamma}{}^2 \frac{e^2}{4\pi}F^2\vert\vec{q}\vert^3 \times [\frac{1}{12}\ (\frac{1}{4})] \times \begin{cases} \left(\frac{1}{m_V}+\frac{1}{m_P}\right)^2, & \mathrm{U}(12) \\ \frac{4}{m_V^2}\ \left(\frac{4}{m_P^2}\right), & \mathrm{SU}(6)_M\mathrm{I} \end{cases}$	$M^a_{a'}$	$Q^a_{a'}$	$Q^a_{a'}$	$V^a_{ia'}\frac{1}{m_{V_i}}\langle V_i Q\rangle$
$F = \begin{cases} \left(\frac{2m_Vm_P}{m_V^2+m_P^2}\right)^2\exp\left[-\frac{1}{4\kappa}\frac{(m_V^2-m_P^2)^2}{m_V^2+m_P^2}\right] & (S\ \text{coordinate}) \\ \left(\frac{2m_Vm_P}{m_V^2+m_P^2}\right)\exp\left[-\frac{1}{2\Omega}\frac{(m_V^2-m_P^2)^2}{m_V^2+m_P^2}\right] & (B\ \text{coordinate}) \end{cases}$				
$\Delta \to N\gamma$				
$J_\mu = a(-i\epsilon_{\mu\upsilon\kappa\lambda}q_\upsilon)\epsilon^{a'bs}\bar{N}^c_s(p')D_{\lambda\ abc}\ (p)M^a_{a'}F \times \begin{cases} \frac{P_\kappa}{m_\Delta}+\frac{P'_\kappa}{m_N}, & \tilde{\mathrm{U}}(12) \\ \frac{2P_\kappa}{m_\Delta}, & \mathrm{SU}(6)_M\mathrm{I} \end{cases}$	a	$\frac{\lambda f_M}{2}$	$3\nu g_M$	$-C_M(m_\Delta+m_N)$
$\Gamma = a^2 U_{DN\gamma}{}^2\frac{e^2}{4\pi}F^2\frac{(m_\Delta+m_N)^2}{12m_\Delta m_N}\vert\vec{q}\vert^3 \times \begin{cases} \left(\frac{1}{m_\Delta}+\frac{1}{m_N}\right)^2, & \tilde{\mathrm{U}}(12) \\ \frac{4}{m_N^2}, & \mathrm{SU}(6)_M\mathrm{I} \end{cases}$	$M^a_{a'}$	$Q^a_{a'}$	$Q^a_{a'}$	$V^a_{ia'}\frac{1}{m_{V_i}}\langle V_i Q\rangle$
$F = \begin{cases} \left(\frac{2mm_N}{m_\Delta^2+m_N^2}\right)^3\exp\left[-\frac{1}{4\kappa}\frac{(m_\Delta^2-m_N^2)^2}{m_\Delta^2+m_N^2}\right] & (S\ \text{coordinate}) \\ \left(\frac{2mm_N}{m_\Delta^2+m^2}\right)^2\exp\left[-\frac{1}{\Omega}\frac{(m_\Delta^2-m_N^2)^2}{m_\Delta^2+m_N^2}\right] & (B\ \text{coordinate}) \end{cases}$				
$\Sigma \to \Lambda\gamma$				
$J_\mu = a(\bar{N}i\sigma_{\mu\upsilon}q_\upsilon N)_{[D+\ 2/3\ F]^{a'}_a}M^a_{a'}F \times \begin{cases} \frac{(m_\Sigma+m_\Lambda)^2}{4m_\Sigma m_\Lambda}, & \tilde{\mathrm{U}}(12) \\ 1, & \mathrm{SU}(6)_M\mathrm{I} \end{cases}$	a	$\frac{\lambda f_M}{2}$	$3\nu g_M$	$-iC_M\frac{2m_\Sigma m_\Lambda}{m_\Sigma+m_\Lambda}$
$\Gamma = a^2 U_{NN\gamma}{}^2\frac{e^2}{4\pi}F^2\frac{\vert\vec{q}\vert^3}{m_\Sigma m_\Lambda} \times \begin{cases} \frac{(m_\Sigma+m_\Lambda)^4}{16m_\Sigma^2 m_\Lambda^2}, & \tilde{\mathrm{U}}(12) \\ 1, & \mathrm{SU}(6)_M I \end{cases}$	$M^a_{a'}$	$Q^a_{a'}$	$Q^a_{a'}$	$V^a_{ia'}\frac{1}{m_{V_i}}\langle V_i Q\rangle$
$F = \begin{cases} \left(\frac{2m_\Sigma m_\Lambda}{m_\Sigma^2+m_\Lambda^2}\right)^3\exp\left[-\frac{1}{4\kappa}\frac{(m_\Delta^2-m_N^2)^2}{m_\Delta^2+m_N^2}\right] & (S\ \text{coordinate}) \\ \left(\frac{2m_\Sigma m_\Lambda}{m_\Sigma^2+m_\Lambda^2}\right)^2\exp\left[-\frac{1}{\Omega}\frac{(m_\Delta^2-m_N^2)^2}{m_\Delta^2+m_N^2}\right] & (B\ \text{coordinate}) \end{cases}$				

$\mathrm{SU}(6)_M$ and $\tilde{\mathrm{U}}(12)$ scheme exists in a factor

$$S_{\tilde{U}}/S_M = [\bar{u}(p')u(p)]^2 = U^2 = (m_N+m_R)^2/4m_Nm_R\,,$$

which is very close to 1 (actually, 1.02 and 1.04, respectively, for $L=1$ and 2 baryons with symmetric masses). Thus we have neglected the difference in the table.

S-coordinate or B-coordinate H.O. In the case

TABLE VI. Radiative decay widths of ground-state mesons and baryons. The underlined values are input.

Process	Decay widths Γ (keV): Symmetric mass				Real mass						Experiment
	All minimal currents and VD2		VD1		BDM_I	$SD\tilde{U}$	VD1	$BD\tilde{U}$	VD2	
	$2\nu g_M^M = \frac{\lambda f_M^M}{2} = 2.01$; $3\nu g_M^B = \frac{\lambda f_M^B}{2} = \mu_p$		$C_M = \frac{\mu_p}{2m_P} m$		$2\nu g_M^M = 7.96$; $3\nu g_M^B = \mu_p$; $\Omega = 1.0\ \mathrm{GeV}^2$	$\frac{\lambda f_M^M}{3} = 6.38$; $\frac{\lambda f_M^B}{3} = \mu_p$; $4\kappa = 3.0\ \mathrm{GeV}^2$	$C_M = \frac{\mu_p}{2m_p} m$; $4\kappa = 3.0\ \mathrm{GeV}^2$	$2\nu g_M^M = 2.34$; $3\nu g_M^B = \mu_p$; $\Omega = 1.0\ \mathrm{GeV}^2$	$2\nu g_M^M = 2.01$; $3\nu g_M^B = \mu_p$	
	U_i		U_i							
$\omega \to \pi^0\gamma$	1	$\frac{880}{(1696)^{a}}$	$\frac{1}{m_\rho}$	1181	$\underline{880}$	$\underline{880}$	39.0	$\underline{880}$	$\underline{880}$	880 ± 50
$\omega \to \eta\gamma$	$\frac{\sqrt{3}}{9}$	4.7	$\frac{\sqrt{3}}{9}\frac{1}{m_\omega}$	4.4	58.4	50.6	4.7	7.4	4.7	seen
$\phi \to \pi^0\gamma$	0	0	0	0	0	0	0	0	0	5.6 ± 2.0
$\phi \to \eta\gamma$	$\frac{2\sqrt{6}}{9}$	225	$\frac{2\sqrt{6}}{9}\frac{1}{m_\phi}$	172	996	1009	77.0	170	133	64 ± 8
$\rho \to \pi\gamma$	$\frac{1}{3}$	91.5	$\frac{1}{3}\frac{1}{m_\omega}$	119	108	109	4.7	98.6	94.8	36 ± 10
$\rho \to \eta\gamma$	$\frac{1}{\sqrt{3}}$	36.0	$\frac{1}{\sqrt{3}}\frac{1}{m_\rho}$	34.2	473	405	38.3	59.1	37.2	seen
$\eta' \to \rho^0\gamma$	$\frac{\sqrt{6}}{3}$	157	$\frac{\sqrt{6}}{3}\frac{1}{m_\rho}$	211	1459	1155	188	160	93.0	$<(29.8 \pm 1.7) \times 10$
$\eta' \to \omega\gamma$	$\frac{\sqrt{6}}{9}$	14.2	$\frac{\sqrt{6}}{9}\frac{1}{m_\omega}$	18.6	136	105	16.8	14.5	9.6	$<(2.1 \pm 0.4) \times 10$
$K^{*\pm} \to K^\pm\gamma$	$\frac{1}{3}$	52.4	$\frac{1}{3}\frac{1}{m_K^*}$	52.6	340	340	26.5	57.9	40.4	<80
$\bar{K}^0 \to K^0\gamma$	$\frac{2}{3}$	214	$\frac{2}{3}\frac{1}{m_K^*}$	214	1368	1368	106	232	163	75 ± 35
$\Delta^+ \to p\gamma$	$\frac{1}{\sqrt{3}}$	253	$\frac{1}{\sqrt{3}}\frac{1}{m_\rho}$	509	180	281	377	242	294	590–609
$\Sigma^0 \to \Lambda\gamma$	$\frac{1}{\sqrt{3}}$	5.8	$\frac{1}{\sqrt{3}}\frac{1}{m_\rho}$	11.6	5.6	5.7	8.6	5.6	5.8	11.3 ± 2.0

[a] Determined from $2\nu g_M^M = \frac{\lambda f_M^M}{2} = \mu_p$.

TABLE VII. Helicity couplings of photon to baryon resonances $\tilde{A}^i_\lambda$.

Resonance	i	λ	(a) Expressions of $\tilde{A}^i_\lambda$	(b) Numerical values of $\lvert\tilde{A}^i_\lambda\rvert$ (10^{-3} GeV$^{-1/2}$) $SD\tilde{U}$ $4\kappa=1$ GeV^2	$BD\tilde{U}$ $\Omega=2$ GeV^2	VD1 $4\kappa=2$ GeV^2	VD2 $\Omega=2$ GeV^2	Experiment (Ref. 24)
$P_{33}(1232)$	Δ	$\frac{3}{2}$	$-\sqrt{54}a_M^{(0)}$	188	188	188	188	-254 ± 5
	Δ	$\frac{1}{2}$	$-\sqrt{18}a_M^{(0)}$	110	110	110	110	-137 ± 7
$D_{13}(1520)$	p	$\frac{3}{2}$	$\sqrt{3}a_E^{(1)}$	84	88	69	103	166 ± 11
	p	$\frac{1}{2}$	$a_E^{(1)}-3a_M^{(1)}$	34	36	50	22	-8 ± 9
	n	$\frac{3}{2}$	$-\sqrt{3}a_E^{(1)}$	84	88	69	103	-130 ± 20
	n	$\frac{1}{2}$	$-a_E^{(1)}+a_M^{(1)}$	21	22	10	32	74 ± 16
$S_{11}(1535)$	p	$\frac{1}{2}$	$\sqrt{2}a_E^{(1)}+\sqrt{9/2}a_M^{(1)}$	129	135	120	143	64 ± 19
	n	$\frac{1}{2}$	$-\sqrt{2}a_E^{(1)}-\sqrt{1/2}a_M^{(1)}$	88	92	77	103	61 ± 34
$D_{33}(1670)$	Δ	$\frac{3}{2}$	$-\sqrt{3}a_E^{(1)}$	73	76	60	91	69 ± 50
	Δ	$\frac{1}{2}$	$-a_E^{(1)}-a_M^{(1)}$	78	82	76	89	67 ± 48
$S_{31}(1650)$	Δ	$\frac{1}{2}$	$-\sqrt{2}a_E^{(1)}+\sqrt{1/2}a_M^{(1)}$	36	37	21	50	43 ± 50
$D_{15}(1670)$	p	$\frac{3}{2}$	0	0	0	0	0	20 ± 13
	p	$\frac{1}{2}$	0	0	0	0	0	19 ± 14
	n	$\frac{3}{2}$	$-\sqrt{9/5}a_M^{(1)}$	49	51	55	49	-60 ± 33
	n	$\frac{1}{2}$	$-\sqrt{9/10}a_M^{(1)}$	35	36	39	34	-33 ± 25
$D_{13}(1710)$	p	$\frac{3}{2}$	0	0	0	0	0	5 ± 29
	p	$\frac{1}{2}$	0	0	0	0	0	-10 ± 38
	n	$\frac{3}{2}$	$\sqrt{27/10}a_M^{(1)}$	64	68	75	65	10 ± 55
	n	$\frac{1}{2}$	$\sqrt{1/10}a_M^{(1)}$	13	13	14	12	-11 ± 60
$S_{11}(1700)$	p	$\frac{1}{2}$	0	0	0	0	0	44 ± 24
	n	$\frac{1}{2}$	$\sqrt{1/2}a_M^{(1)}$	27	28	31	27	-18 ± 26
$F_{15}(1688)$	p	$\frac{3}{2}$	$-\sqrt{4/5}\,a_E^{(2)}$	31	41	25	44	132 ± 23
	p	$\frac{1}{2}$	$\sqrt{2/5}\,a_E^{(2)}+\sqrt{9/10}\,a_M^{(2)}$	12	15	37	3	-5 ± 17
	n	$\frac{3}{2}$	0	0	0	0	0	-18 ± 20
	n	$\frac{1}{2}$	$-\sqrt{2/5}a_M^{(2)}$	22	29	25	23	23 ± 14
$P_{13}(1810)$	p	$\frac{3}{2}$	$\sqrt{1/5}a_E^{(2)}$	17	22	13	24	-34 ± 50
	p	$\frac{1}{2}$	$-\sqrt{3/5}a_E^{(2)}-\sqrt{3/5}a_M^{(2)}$	68	88	67	81	26 ± 44
	n	$\frac{3}{2}$	0	0	0	0	0	-17 ± 65
	n	$\frac{1}{2}$	$\sqrt{4/15}a_M^{(2)}$	26	30	30	26	-6 ± 39
$F_{37}(1950)$	Δ	$\frac{3}{2}$	$\sqrt{2/7}a_M^{(2)}$	38	50	46	38	-71 ± 27
	Δ	$\frac{1}{2}$	$\sqrt{6/35}a_M^{(2)}$	30	39	36	30	-71 ± 22
$F_{35}(1890)$	Δ	$\frac{3}{2}$	$-\sqrt{18/35}a_M^{(2)}$	44	57	52	44	-1 ± 57
	Δ	$\frac{1}{2}$	$-\sqrt{1/21}a_M^{(2)}$	13	17	16	13	28 ± 25
$P_{33}(1690)$	Δ	$\frac{3}{2}$	$\sqrt{1/5}a_M^{(2)}$	17	21	18	17	-7 ± 35
	Δ	$\frac{1}{2}$	$\sqrt{1/15}a_M^{(2)}$	10	12	10	10	-8 ± 35
$P_{31}(1910)$	Δ	$\frac{1}{2}$	$\sqrt{1/15}a_M^{(2)}$	16	19	19	16	-12 ± 20

TABLE VII. *(Continued)*

(c) Parameters $a_E^{(L)}$ and $a_M^{(L)}$

S coordinate

$$a_E^{(L)}=\frac{N\omega}{\sqrt{18}}(qd)^{L-1}\left(\frac{1}{\kappa}\right)^{L/2}F_S\times\begin{cases}U^3, & \tilde{\mathrm{U}}(12)\\ U, & \mathrm{SU}(6)_M\end{cases}\qquad a_M^{(L)}=\frac{N\mu_p}{9}\sqrt{2}\,(m_R-m_N)(qd)^L\left(\frac{1}{\kappa}\right)^{L/2}F_S\times\begin{cases}U^3, & \tilde{\mathrm{U}}(12)\\ U, & \mathrm{SU}(6)_M\end{cases}$$

B coordinate

$$a_E^{(L)}=\frac{N\omega}{\sqrt{18}}(qd)^{L-1}\left(\frac{6}{\Omega}\right)^{L/2}F_B\times\begin{cases}U^3, & \tilde{\mathrm{U}}(12)\\ U, & \mathrm{SU}(6)_M\end{cases}\qquad a_M^{(L)}=\frac{N\mu_p}{9}\sqrt{2}\,(m_R-m_N)(qd)^L\left(\frac{6}{\Omega}\right)^{L/2}F_B\times\begin{cases}U^3, & \tilde{\mathrm{U}}(12)\\ U, & \mathrm{SU}(6)_M\end{cases}$$

VD1

$$a_E^{(L)}=\frac{N\omega}{\sqrt{18}}U^3(qd)^{L-1}\left(\frac{1}{\kappa}\right)^{L/2}F_S\qquad a_M^{(L)}=\frac{N\mu_p}{9}\sqrt{2}\,(m_R-m_N)\,\frac{m_R+m_N}{2m_p}\,U^3(qd)^L\left(\frac{1}{\kappa}\right)^{L/2}F_S$$

VD2

$$a_M^{(L)}=\frac{N\omega}{\sqrt{18}}Uq^{L-1}\left(\frac{6}{\Omega}\right)^{L/2}\qquad a_M^{(L)}=\frac{N\mu_p}{9}\sqrt{2}\,(m_R-m_N)Uq^L\left(\frac{6}{\Omega}\right)^{L/2}$$

$$U=\frac{m_R+m_N}{(4m_Rm_N)^{1/2}},\qquad N=\left(\frac{\pi\alpha}{2qm_Rm_N}\right)^{1/2},\quad q\equiv|\vec{q}|=\frac{m_R^2-m_N^2}{2m_R}$$

$$d_{\mathrm{def}}=\frac{2m_R^2}{m_R^2+m_N^2},\qquad F_{SD}=\left(\frac{2m_Rm_N}{m_R^2+m_N^2}\right)^3\exp\left[-\frac{1}{4\kappa}\,\frac{(m_R^2-m_N^2)^2}{m_R^2+m_N^2}\right],\qquad F_{BD}=\left(\frac{2m_Rm_N}{m_R^2+m_N^2}\right)^2\exp\left[-\frac{1}{\Omega}\,\frac{(m_R^2-m_N^2)^2}{m_R^2+m_N^2}\right]$$

$$d_{\mathrm{ind}}=1,\qquad F_{SI}=1,\qquad F_{BI}=1,$$

Symmetric mass; $m_{L=0}=1.33$, $m_{L=1}=1.69$, $m_{L=2}=1.99$ GeV, $m_L^2=m_0^2+L\omega$

of definite-metric H.O., the difference between the S type and the B type exists, apart from an exponential factor (depending upon a parameter κ or Ω) in a factor $(-m_Nm_R/pp')$ which is close to 1 (actually 0.96 and 0.91, respectively, for $L=1$ and 2 baryons with symmetric masses). Thus the difference is rather small. On the other hand, there is no difference in the case of indefinite metric between the amplitudes for the two types, which are mutually related by an interchange of their parameters $\Omega\leftrightarrow 6\kappa$.

Failure of definite-metric case with physical masses. To see characteristic features of our respective models we have shown the parameter dependence of the amplitudes

$$\tilde{A}^{\Delta}_{3/2}(D_{33}(1670))\propto a_E^{(1)}$$

and

$$\tilde{A}^{n}_{3/2}(D_{15}(1670))\propto a_M^{(1)}$$

in Figs. 6(a) and 6(b), respectively. From this we see that all cases of definite-metric H.O. with physical masses give[29] too small values of the amplitudes compared with experiments, although this conclusion is not definite because of large experimental error bars, while all the other cases have a good region of parameter fitting to experiments. These features are common to general $L=1$ amplitudes. Thus we have omitted this case in part (b) of Table VII. For the $L=2$ amplitudes the difference between physical and symmetric mass cases becomes rather small and both cases seem to be consistent with experiments except for $\tilde{A}^p_{3/2}(F_{15}(1688))$.

Effects of mixing states. In the table the values are obtained without considering possible mixing effects. We have investigated the possible effects for the amplitudes $A^p_{3/2}$, $A^p_{1/2}$, $A^n_{3/2}$, and $A^n_{1/2}$ of mixing between the $D_{13}(1520)$ and $D_{13}(1710)$ states, getting no improvement for any mixing angle. We have also investigated the effects for the amplitudes $A^p_{1/2}$ and $A^n_{1/2}$ of mixing between the $S_{11}(1535)$ and $S_{11}(1700)$ states, and obtained satisfactory results for the mixing angle $\theta\approx 40°$, which is close to some solutions obtained from the analyses[30] of strong decay of the resonances.

Relativistic effects. We can easily obtain the nonrelativistic limits of our amplitudes, and the relativistic effects are estimated to be, aside from the exponential factor, at most 10% and 20% for $L=1$ and $L=2$ baryons, respectively. The expon-

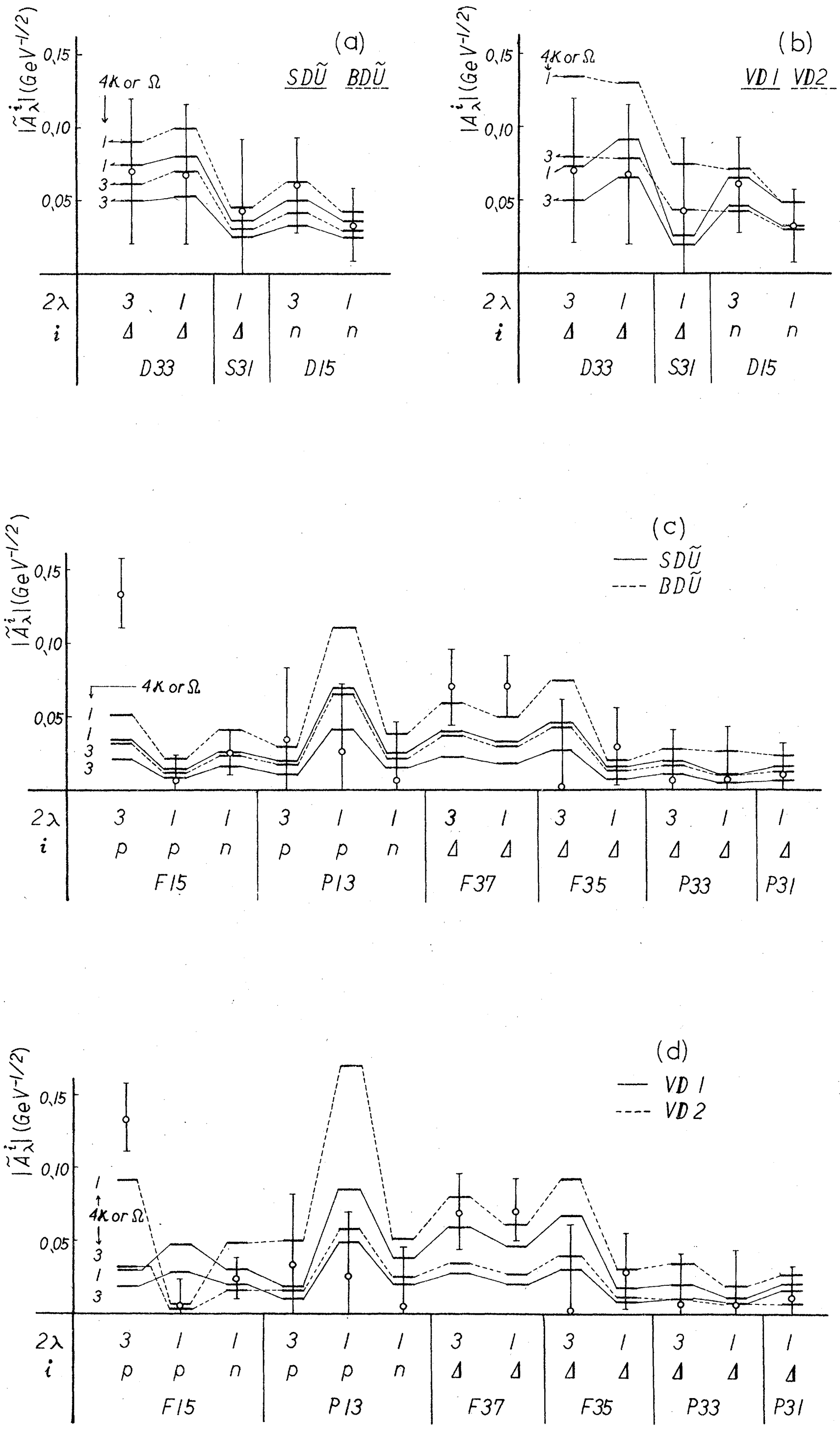

FIG. 5. Amplitudes of photon–baryon-resonance helicity coupling. Status of comparison with experiments (Ref. 24) of our theory is schematically shown for the resonances (which has no possibility of mixing states) assigned to (SU(6), L) = ($\underline{70}$, 1) and ($\underline{56}$, 2) in Figs. 5(a), 5(b) and 5(c), 5(d), respectively. The parameters are arbitrarily chosen as Ω = 1–3 GeV^2 (4κ = 1–3 GeV^2) for the B-coordinate (S-coordinate) case. The cases *SDM* and *BDM* (which are not shown in the figure) are almost the same as the cases $SD\tilde{U}$ and $BD\tilde{U}$ (given in the figure), respectively. The not shown cases *BIM* and *SIM* ($BI\tilde{U}$ and $SI\tilde{U}$) are exactly (nearly) the same as the case VD2 (given in the figure). The results of all cases are quite similar and seem to reproduce, except for a few resonances, well the general experimental behavior.

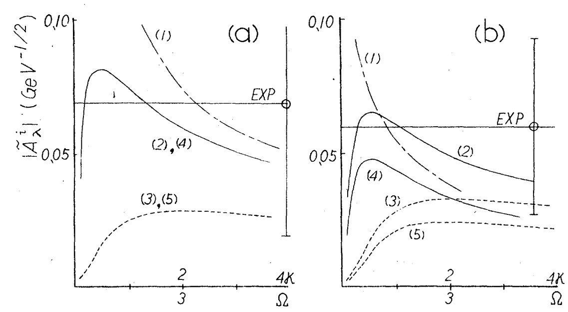

FIG. 6. Dependence of the amplitudes $\tilde{A}^{\Delta}_{3/2}(D_{33}(1670))$ and $\tilde{A}^{n}_{3/2}(D_{15}(1670))$ on the parameter (Ω for the B-coordinate case, 4κ for the S-coordinate case) are shown in (a) and (b), respectively. A dot-dashed line (1) represents the indefinite-metric cases VD2, *SIM*, and *BIM*. The other indefinite-metric cases *SIŨ* and *BIŨ* are quite close to (1): A solid line (2) [(4)] represents the definite-metric cases with symmetric mass VD1 [*SDŨ*], a dashed line (3) [(5)] corresponds to the definite-metric case with real mass VD1 [*SDŨ*]. The difference between corresponding $SU(6)_M$ and $\tilde{U}(12)$ cases is very small. It seems that all the *definite-metric cases* with *real mass* give too small values, while all the other cases have a good region of parameter fitting to experiments (Ref. 24).

ential factor for the S-coordinate (B-coordinate) definite-metric H.O. is

$$\exp\left(-\frac{\vec{q}^2}{4\kappa}\frac{4m_R^2}{m_R^2+m_N^2}\right)$$

(expression $4\kappa\to\Omega$), which is quite similar to the usual nonrelativistic factor $\exp[-(1/c)\vec{q}^2]$ (c: constant parameter), while it is identical to 1 for the indefinite-metric H.O.

FKR amplitudes. The FKR amplitudes[2] are obtained from ours in the *BDŨ* case by replacement of

$$d\to 1,\quad \omega\to\Omega,\quad g_p(=2.79)\to 3,\quad U^3\to 1,$$

$$F_{\rm BD}=\left(\frac{2m_Rm_N}{m_R^2+m_N^2}\right)^2\exp\left[-\frac{1}{\Omega}\frac{(m_R^2-m_N^2)^2}{m_R^2+m_N^2}\right] \quad (15)$$

$$\to\exp\left[-\frac{1}{4\Omega}\frac{(m_R^2-m_N^2)^2}{m_R^2+m_N^2}\right].$$

Note that in the FKR case Ω is restricted as $\Omega=\omega=1.1$ GeV^2, while $\nu\Omega=\omega$ in our case.

Sign of the amplitudes. In this work we have not been concerned with signs of the amplitudes. For this problem it is necessary to investigate the whole process of $\gamma+N\to B^*\to\Delta(N)+\pi$. In this paper we have investigated the interactions concerning the first step, while the strong interactions on the second step had been treated in our previous works.[31] So it is possible to treat this problem. Here we only note that our scheme is more flexible than the usual model such as the FKR model because we consider the more general interactions for both steps in our scheme.

V. CONCLUDING REMARKS

In this paper we have examined systematically electromagnetic properties of ground-state and excited hadrons by applying general minimal currents or newly introduced vector-meson-dominant ones, comparing various relativistic H.O. quark models. As a result, it was especially shown that a minimal current in the $SU(6)_M$ scheme with definite-metric B-coordinate H.O. (the *BDM* case) explains[32] quite well EMFF's of nucleons (including the nonvanishing neutron electric one) and helicity couplings of photon-baryon resonances (except for a few couplings) unifiedly for a value of $\Omega_B\approx 1.5$ GeV^2 and EMFF of pions for $\Omega_M=0.8$–1.5 GeV^2 (in the second formulation). A minimal current in the $\tilde{U}(12)$ scheme with definite-metric S-coordinate H.O. (the *SDŨ* case) also gives satisfactory results for EMFF's of nucleons and pions and for the couplings of photon-baryon resonances unifiedly for $4\kappa=0.5$–3 GeV^2. A newly introduced conserved vector-dominance current (VD1) gives interesting results for EM properties of baryons (EMFF's of nucleons and magnetic moments of baryons and the couplings of photon-baryon resonances) unifiedly for $4\kappa\approx 3$ or $4\kappa=0.5$–3 GeV^2 (aside from G_E^p for which $4\kappa=1.5$–3 GeV^2). Previously, we made[31] and analysis of strong decay vertices for the baryon resonances in the similar theoretical framework getting good results for $4\kappa\approx 1$ in the S-coordinate H.O. and for $\Omega=1$–2 GeV^2 in the B-coordinate H.O., which are similar to the above corresponding (the *BDM* case and the *SDŨ* case) values.

All the above good cases are with the definite-metric H.O. However, from the viewpoint of obtaining a unified theoretical scheme for a higher-order process,[33] as was discussed[13] previously, the models with indefinite metric and with normal ordering might be more interesting. In these cases we lose a good explanation due to the internal overlapping integral of FF's, although they give good results for the real-photon couplings with ground-state and excited mesons and baryons.

The results of our various models are, aside from those for the FF's not so much different from each other and from that of the usual nonrelativistic model, and explain equally well the photon couplings with baryon resonances assigned to $L=0, 1$, and 2. This comes from the fact that for these resonances nonrelativistic approximations are still valid. (We must treat resonances with the higher excitation to discriminate the respective models.) However, we should still note the universal characteristics of our relativistic models, such as concerning current conservation and a relation to the slope parameter, which are lacking in the usual nonrelativistic models.

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