PHYSICAL REVIEW D VOLUME 20, NUMBER 11 1 DECEMBER 1979

Variation of mixing angles and masses with q^2 in the standard six-quark model

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(Received 29 June 1979)

The variation of mixing angles and masses with q^2 in the standard six-quark model is given in terms of a set of coupled differential equations. It is shown that to lowest order, these equations do not depend on the SU(2) gauge coupling, and, in the case of the mixing angles, not even on the U(1) gauge coupling. Experimental implications are briefly discussed.

I. INTRODUCTION

In quantum chromodynamics (QCD), the bare quark masses are the ones defined at infinite momentum, where the theory is asymptotically free.[1] The effective quark masses at a particular q^2 are then related to those at a different q^2 by a set of coupled differential equations.[2] However, these equations are also affected by the weak and electromagnetic interactions, which may very well be important for a large enough q^2. In this paper, we consider the theory of weak and electromagnetic interactions to be that of Weinberg[3] and Salam,[4] generalized to six quarks.[5] (We have already discussed the four-quark version[6] briefly in a previous paper.[7]) The parameters of this model are of course the six quark masses and the four angles which determine the charged-current mixing matrix connecting the three quarks (d,s,b) of charge $-\frac{1}{3}$ to the three quarks (u,c,t) of charge $\frac{2}{3}$. An explicit representation of this matrix is given by[5]

$$\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta} \end{pmatrix}, \quad (1.1)$$

where $c_1=\cos\theta_1$, $s_1=\sin\theta_1$, etc. In the following sections, we discuss in detail how this matrix (as well as the mass matrix) changes as a function of q^2. In particular, we show that to lowest order, the SU(2) gauge coupling g does not cause a change in either matrix, and the U(1) gauge coupling g' affects only the mass matrix. In so doing, we also derive a formula for the change of the Higgs vacuum expectation value v as a function of q^2. We then conclude with a brief discussion on the experimental implications of our results.

II. VARIATION OF YUKAWA COUPLINGS WITH q^2

Let us assume that there is only one Higgs doublet (ϕ^+,ϕ^0), and that all quarks are coupled to it in the most general way. Let the charged-current mixing matrix, after the proper diagonalization of the mass matrix, be given by the standard form (1.1). Then ϕ^0 is coupled to

$$g_d\bar{d}_Ld_R+g_s\bar{s}_Ls_R+g_b\bar{b}_Lb_R+g_u\bar{u}_Ru_L+g_c\bar{c}_Rc_L+g_t\bar{t}_Rt_L\,, \quad (2.1)$$

where $g_d=m_d/v=m_d/\langle\phi^0\rangle$, etc., and ϕ^+ is coupled to

$$\sum_{i,j}\lambda_{ij}(g_j\bar{q}_{iL}q_{jR}-g_i\bar{q}_{iR}q_{jL})\,, \quad (2.2)$$

where $i=(u,c,t)$, $j=(d,s,b)$, λ_{ij} is the matrix (1.1), and the subscripts L,R denote left-handed and right-handed projections, respectively.

Let us now consider the one-loop corrections to the Yukawa couplings. Since λ_{ij} is not diagonal, there will be induced couplings of the type $g_{ds}\bar{d}_Ls_R\phi^0$, and the resulting mass matrix will not be diagonal. Therefore, we have to rediagonalize this new mass matrix, in order to obtain the new quark masses and mixing angles. Let us first consider the diagonal coupling g_d. The one-loop corrections to g_d are given by[8]

$$16\pi^2\frac{dg_d}{dt}=g_d\left[\tfrac{3}{2}g_d^{\,2}-\tfrac{3}{2}\sum_i g_i^{\,2}|\lambda_{id}|^2+\sum_n g_n^{\,2}-\tfrac{3}{4}(3g^2+g'^2)+\tfrac{1}{3}g'^2-8g_{\rm QCD}^{\,2}\right], \quad (2.3)$$

where $i=(u,c,t)$ and n runs over all quarks, including colors, and leptons. The parameter t can be taken to be equal to $\ln q^2$. The corresponding equations for g_s and g_b are obtained by replacing d by s and b, respectively. As for g_u, and similarly g_c and g_t, the appropriate equation is

$$16\pi^2\frac{dg_u}{dt}=g_u\left[\tfrac{3}{2}g_u^{\,2}-\tfrac{3}{2}\sum_j g_j^{\,2}|\lambda_{uj}|^2+\sum_n g_n^{\,2}-\tfrac{3}{4}(3g^2+g'^2)-\tfrac{2}{3}g'^2-8g_{\rm QCD}^{\,2}\right], \quad (2.4)$$

where $j=(d,s,b)$ and the term $-\frac{2}{3}g'^2$ appears instead of the term $\frac{1}{3}g'^2$ as in Eq. (2.3) because the

u and d quarks are different in electric charge.

The induced off-diagonal couplings can be obtained in a completely analogous way. For the $g_{ds}\bar{d}_L s_R \phi^0$ interaction, the corresponding equation is

$$16\pi^2 \frac{dg_{ds}}{dt} = \tfrac{3}{2} g_s \sum_i g_i{}^2 \lambda_{id}^* \lambda_{is}\,, \tag{2.5}$$

and for $g_{sd}\bar{s}_L d_R \phi^0$, it is

$$16\pi^2 \frac{dg_{sd}}{dt} = \tfrac{3}{2} g_d \sum_i g_i{}^2 \lambda_{is}^* \lambda_{id}\,. \tag{2.6}$$

The remaining equations for g_{db}, g_{uc}, etc., are obviously of the same form.

III. VARIATION OF QUARK MASSES WITH q^2

If the mass matrix M for the d, s, and b quarks is initially diagonalized at a particular q^2, then a change in q^2 will result in a new mass matrix $M+\Delta M$, which is given by

$$\frac{M+\Delta M}{v+\Delta v} = \begin{pmatrix} g_d(1+\Delta_{dd}) & g_s\Delta_{ds} & g_b\Delta_{db} \\ g_d\Delta_{ds}^* & g_s(1+\Delta_{ss}) & g_b\Delta_{sb} \\ g_d\Delta_{db}^* & g_s\Delta_{sb}^* & g_b(1+\Delta_{bb}) \end{pmatrix}, \tag{3.1}$$

where $v+\Delta v$ is the new ϕ^0 vacuum expectation value. To rediagonalize this new mass matrix, we multiply it on the left by the matrix

$$U_L = \begin{pmatrix} 1 & \epsilon_{ds} & \epsilon_{db} \\ -\epsilon_{ds}^* & 1 & \epsilon_{sb} \\ -\epsilon_{db}^* & -\epsilon_{sb}^* & 1 \end{pmatrix}, \tag{3.2}$$

where

$$\epsilon_{ds} = \frac{g_d{}^2 + g_s{}^2}{g_d{}^2 - g_s{}^2}\,\Delta_{ds}\,, \tag{3.3}$$

etc., and on the right by the matrix

$$V_R^\dagger = \begin{pmatrix} 1 & -\eta_{ds} & -\eta_{db} \\ \eta_{ds}^* & 1 & -\eta_{sb} \\ \eta_{db}^* & \eta_{sb}^* & 1 \end{pmatrix}, \tag{3.4}$$

where

$$\eta_{ds} = \frac{2 g_d g_s}{g_d{}^2 - g_s{}^2}\,\Delta_{ds}\,, \tag{3.5}$$

etc. As expected, the eigenvalues of $M+\Delta M$ are, to lowest order in Δ, just the diagonal entries $g_d(v+\Delta v + v\Delta_{dd})$, etc. However, the matrix U_L is still of importance because it rotates the (d,s,b) basis for the new charged-current mixing matrix $\lambda+\Delta\lambda$. (The matrix V_R plays no role because there are no right-handed charged currents in the standard six-quark model.) A similar procedure applies, of course, to the new (u,c,t) mass matrix as well.

To determine the variation of v with q^2, let us consider the Higgs potential given by

$$V^{(4)} = \tfrac{1}{2} f(\phi^+\phi^- + \phi^0\bar{\phi}^0)^2\,. \tag{3.6}$$

As $\phi^0 \to \phi^0 + v$, a cubic interaction of strength fv is induced. Therefore, the one-loop corrections to it can be compared with those to f, and the variation of v with q^2 extracted. As expected, the result corresponds exactly to the wave-function renormalization of ϕ^0, namely,

$$16\pi^2 \frac{dv}{dt} = \tfrac{3}{4}\Big(3g^2 + g'^2 - \tfrac{4}{3}\sum_n g_n{}^2\Big) v\,, \tag{3.7}$$

where n runs over all quarks, including colors, and leptons. Combining the above with Eqs. (2.3) and (2.4), we find

$$16\pi^2 \frac{dm_d}{dt} = m_d\Big(\tfrac{3}{2} g_d{}^2 - \tfrac{3}{2}\sum_i g_i{}^2 |\lambda_{id}|^2 + \tfrac{1}{3} g'^2 - 8 g_{\mathrm{QCD}}{}^2\Big) \tag{3.8}$$

and

$$16\pi^2 \frac{dm_u}{dt} = m_u\Big(\tfrac{3}{2} g_u{}^2 - \tfrac{3}{2}\sum_j g_j{}^2 |\lambda_{uj}|^2 - \tfrac{2}{3} g'^2 - 8 g_{\mathrm{QCD}}{}^2\Big). \tag{3.9}$$

The same equations are, of course, also obtained, if we consider instead the renormalization of the quark propagators directly. Notice that the SU(2) gauge coupling g does not appear. This is again expected because there are no right-handed charged currents in the standard six-quark model.

In Eqs. (3.8) and (3.9), the QCD contribution is obviously the most important[2] for moderate values of q^2. However, as q^2 becomes comparable to $M_W{}^2$, the g' term may start to contribute more, and it tends to increase m_d and decrease m_u. The Yukawa terms are normally negligible because they are proportional to m_q/M_W. As q^2 becomes even greater, the non-Abelian coupling g_{QCD} will decrease, but the Abelian coupling g' will increase. Therefore, Eqs. (3.8) and (3.9) are not expected to be valid as $q^2 \to \infty$.

IV. VARIATION OF MIXING ANGLES WITH q^2

The charged-current mixing matrix λ_{ij} given by (1.1) is changed by a redefinition of the mass eigenstates (d,s,b) and (u,c,t) as q^2 changes. The new matrix is given by

$$\lambda + \Delta\lambda = U_L' \lambda U_L^\dagger\,, \tag{4.1}$$

where U_L is given by Eq. (3.2), and U_L' is the corresponding unitary matrix for the u, c, and t quarks. To lowest order in ϵ as defined by Eq. (3.3), we find

$$\Delta\lambda_{ij} = \sum_{k\neq i} \epsilon_{ik}\lambda_{kj} - \sum_{l\neq j} \lambda_{il}\epsilon_{lj}\,, \tag{4.2}$$

where it is understood that $\epsilon_{sd} = -\epsilon_{ds}^*$, etc. Notice that $\Delta\lambda_{ij}$ is in general complex. This means that we have to redefine the phases of the quarks, too, in order that $\lambda + \Delta\lambda$ remains of the form (1.1).

Let us now consider in detail $\lambda_{ud} = c_1$, which is of course the cosine of the generalized Cabibbo angle. Using Eq. (4.2) and redefining the relative phase between the u and d quarks, we find

$$\begin{aligned}16\pi^2 \frac{dc_1}{dt} = &-\tfrac{3}{2}s_1^2 c_1 \Big\{ (s_2^2 g_t^2 + c_2^2 g_c^2 - g_u^2)\left[c_3^2\left(\frac{m_s^2+m_d^2}{m_s^2-m_d^2}\right) + s_3^2\left(\frac{m_b^2+m_d^2}{m_b^2-m_d^2}\right)\right] \\ &+ (s_3^2 g_b^2 + c_3^2 g_s^2 - g_d^2)\left[c_2^2\left(\frac{m_c^2+m_u^2}{m_c^2-m_u^2}\right) + s_2^2\left(\frac{m_t^2+m_u^2}{m_t^2-m_u^2}\right)\right]\Big\} \\ &-\tfrac{3}{2}s_1^2 c_2 s_2 c_3 s_3 \cos\delta \left[(g_t^2-g_c^2)\left(\frac{m_s^2+m_d^2}{m_s^2-m_d^2} - \frac{m_b^2+m_d^2}{m_b^2-m_d^2}\right) + (g_b^2-g_s^2)\left(\frac{m_c^2+m_u^2}{m_c^2-m_u^2} - \frac{m_t^2+m_u^2}{m_t^2-m_u^2}\right)\right],\end{aligned} \tag{4.3}$$

which reduces in the four-quark model[6] to

$$16\pi^2 \frac{dc}{dt} = -\tfrac{3}{2}s^2 c\left[(g_c^2-g_u^2)\frac{m_s^2+m_d^2}{m_s^2-m_d^2} + (g_s^2-g_d^2)\frac{m_c^2+m_u^2}{m_c^2-m_u^2}\right], \tag{4.4}$$

in agreement with Eq. (11) of Ref. 7. Note that Eq. (4.3), and in general Eq. (4.2), depends not at all on the gauge couplings g, g', or $g_{\rm QCD}$. Therefore, if the Yukawa couplings are indeed proportional to m_q/M_W, the variation of the mixing angles with q^2 is substantially smaller than that of the quark masses, as shown by Eqs. (3.8) and (3.9).

However, it is also possible that there are two Higgs doublets, but only one of them is coupled to the quarks.[9] Then the Yukawa couplings are of the order m_q/v', where v' can be a much smaller vacuum expectation value, say on the order of 1 GeV. In that case, the variation of the mixing angles may in fact be measurable. In the four-quark model, using Eq. (4.4), we find[7] that θ_C can change as much as 10% if q^2 is changed by a factor of 100. In the six-quark model, because of the large mass of the t quark, this change can be even greater. Another possible effect of a small v' is in the ratio of two quark masses of the same charge, for example,

$$16\pi^2 \frac{d}{dt}\frac{m_s}{m_d} = \frac{3}{2}\frac{m_s}{m_d}\left[g_s^2 - g_d^2 + \sum_i g_i^2(|\lambda_{id}|^2 - |\lambda_{is}|^2)\right], \tag{4.5}$$

which will change very slowly as a function of q^2 if there is only one Higgs doublet.

V. CONCLUDING REMARKS

Since quark masses vary with q^2, which is defined by the effective distance resolution of the probe, it is important to know exactly how they behave in a given theory of the strong, weak, and electromagnetic interactions. In the standard six-quark model, to the extent that we can use the one-loop approximation, the appropriate differential equations are given by Eqs. (3.8) and (3.9), and their obvious generalizations. For large enough values of q^2, where $g_{\rm QCD}$ can be neglected, g' is potentially the dominant term in these equations, and its effect is to decrease m_u and increase m_d. Although this does not explain why m_u is less than m_d at any particular q^2, it is still a rather interesting result. For small enough values of q^2, the QCD contribution is the most important, and explicit mass effects in the renormalization must also be taken into account.[2] Therefore, as q^2 decreases from about 10^4 GeV^2 to about 1 GeV^2, the quark masses will increase in magnitude from their "current" values to their "constituent" values.

As for the weak mixing angles, they are not affected by QCD or even the weak and electromagnetic gauge couplings, as shown by Eq. (4.2). Therefore, their variation with q^2 depends crucially on the magnitudes of the Yukawa couplings, which are adjustable with two or more Higgs doublets. As a quantitative example, let us consider Eq. (4.3) with $g_i = m_i/v'$, where $v' = 1$ GeV, then using $m_t = 14$ GeV, $s_1 = 0.23$, $s_2 = 0.55$, $s_3 = 0.35$, and $\delta = 4.2\times 10^{-3}$ as a typical set of values[10] for these parameters, we find

$$\frac{1}{s_1}\frac{ds_1}{dt} \simeq 0.5\,, \tag{5.1}$$

which means that a 20% change in q^2 will result in a 10% change in the generalized Cabibbo angle. However, since the weak-interaction q^2 is equal to $Q^2 + M_W^2$, where Q^2 is the square of the actual momentum transfer of the process in question, a 20% increase in q^2 corresponds to a change in Q^2 from zero to about 10^3 GeV^2. Therefore, such an effect is difficult to measure experimentally, even if we allow for the absurdly small value of 1 GeV for v'.

Our discussion can be easily extended to include more quarks, as long as each generation is repeated in the same way. However, as the number of generations is increased, there are more masses and mixing angles to contend with, and the interplay among the various differential equations which govern their q^2 behavior is much more involved. As a result, even with six quarks, we are unable to disentangle all ten such equations and make simple predictions as we did[7] for the four-quark case. Nevertheless, the generalized Cabibbo angle θ_1 is expected to rise, albeit very slowly, as a function of q^2. Of course, such a variation should be looked for experimentally, without regard to the predictions of any particular theory, whether it is this one or some other.[11]

ACKNOWLEDGMENT

This work was supported in part by the U. S. Department of Energy under DOE Contract No. EY-76-C-03-0511.

[1]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, *ibid.* 30, 1346 (1973).

[2]H. Georgi and H. D. Politzer, Phys. Rev. D 14, 1829 (1976).

[3]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); Phys. Rev. D 5, 1412 (1972).

[4]A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[5]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[6]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[7]E. Ma and S. Pakvasa, Phys. Lett. 86B, 43 (1979). See also C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B147, 277 (1979); L. Maiani, G. Parisi, and R. Petronzio, *ibid.* B136, 115 (1978).

[8]T. P. Cheng, E. Eichten, and L.-F. Li, Phys. Rev. D 9, 2259 (1974).

[9]P. Ramond and G. G. Ross, Phys. Lett. 81B, 61 (1979); H. Haber, G. L. Kane, and T. Sterling, Univ. of Michigan report, 1978 (unpublished).

[10]V. Barger, W. F. Long, and S. Pakvasa, Phys. Rev. Lett. 42, 1585 (1979); R. E. Shrock, S. B. Treiman, and L.-L. Wang, *ibid.* 42, 1589 (1979);

[11]T. Matsuki, Univ. of Tokyo Report No. INS-Rep.-333, 1979 (unpublished).