

Weak mass mixing, CP violation, and the decay of *b*-quark mesons

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Weak mixing and CP parameters for the $B_d-\bar{B}_d$ and $B_s-\bar{B}_s$ systems are computed in the Kobayashi-Maskawa (KM) model and found to exceed previous estimates. New values for the KM angles are presented. Explicit calculation of $\Delta\Gamma_B$ from the absorptive part of the $B^0-\bar{B}^0$ mixing amplitude and a formalism for large CP violation are included.

I. INTRODUCTION

Ellis, Gaillard, Nanopoulos, and Rudaz¹ were first to point out the possibility of large CP-violating effects in neutral *B* systems. Since the interpretation of the Υ as $b\bar{b}$, the prospect of observing weak mass mixing and CP violation in $B_d-\bar{B}_d$ decay has motivated several estimates of mixing and charge-asymmetry parameters, in the frameworks of both the Kobayashi-Maskawa (KM) model and Higgs exchange. Small predictions from these earlier estimates, especially in the context of the KM model, are not supported by the present work, which allows for larger effects.

Substantive methodological improvements over previous analyses contribute to these new findings. Section II presents an extension of the familiar mixing and CP formulas from kaon physics to accommodate the larger CP violation characteristic of *B* mesons. The unjustified use of an approximation for the CP impurity parameter ϵ valid near the origin is thereby avoided. In Sec. III, $\Delta\Gamma_{B_d}$ is calculated directly from the absorptive part of the $2W^\pm$ exchange box diagrams, eliminating large

errors from momentum mismatch in analyses based on decay channels. Section IV contains a reevaluation of the KM angles, based on precise expressions for kaon mixing. Section V summarizes $B^0-\bar{B}^0$ mixing and charge asymmetry with conclusions regarding their experimental detection. Details of the rarer $B_s-\bar{B}_s$ decay are contained in an appendix.

II. PHENOMENOLOGY

A phenomenological description of the $B^0-\bar{B}^0$ system is provided by the mass and decay matrices $M-i\Gamma/2$. The phenomenological Hamiltonian in the flavor eigenbasis can be written

$$\begin{pmatrix} M-\frac{1}{2}i\Gamma & M_{12}^*-\frac{1}{2}i\Gamma_{12}^* \\ M_{12}-\frac{1}{2}i\Gamma_{12} & M-\frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}. \quad (2.1)$$

The mass eigenstates

$$B_{1,2} = \frac{1}{[2(1+|\epsilon|^2)]^{1/2}} [(1+\epsilon)B^0 \pm (1-\epsilon)\bar{B}^0] \quad (2.2)$$

correspond to eigenvalues

$$M_{1,2} = M \pm \text{Re}[(M_{12}-\frac{1}{2}i\Gamma_{12})(M_{12}^*-\frac{1}{2}i\Gamma_{12}^*)]^{1/2} \quad (\approx M \pm \text{Re}M_{12}), \quad (2.3)$$

$$\frac{1}{2}\Gamma_{1,2} = \frac{1}{2}\Gamma \mp \text{Im}[(M_{12}-\frac{1}{2}i\Gamma_{12})(M_{12}^*-\frac{1}{2}i\Gamma_{12}^*)]^{1/2} \quad (\approx \frac{1}{2}\Gamma \pm \frac{1}{2}\text{Re}\Gamma_{12}).$$

The CP impurity parameter ϵ is given by

$$\epsilon = \frac{-\text{Re}M_{12} + \frac{1}{2}i\text{Re}\Gamma_{12} + [(M_{12}-\frac{1}{2}i\Gamma_{12})(M_{12}^*-\frac{1}{2}i\Gamma_{12}^*)]^{1/2}}{i\text{Im}M_{12} + \frac{1}{2}\text{Im}\Gamma_{12}} \quad \left(\approx \frac{\frac{1}{2}\text{Im}\Gamma_{12} + i\text{Im}M_{12}}{\frac{1}{2}i\Delta\Gamma - \Delta M} \right). \quad (2.4)$$

ϵ , $\Delta M \equiv M_1 - M_2$, and $\Delta\Gamma \equiv \Gamma_1 - \Gamma_2$ reduce to the familiar expressions in parentheses in the small- ϵ "kaon" limit, inappropriate for *b*-quark mesons where ϵ is *a priori* arbitrary.²

The experimental parameters for weak mixing and CP violation can be expressed in terms of the quantities r and \bar{r} due to Pais and Treiman³:

$$r \equiv \frac{\Gamma(B^0 \rightarrow \chi^- l^+ \nu_l)}{\Gamma(B^0 \rightarrow \chi^+ l^- \bar{\nu}_l)} = \left| \frac{1-\epsilon}{1+\epsilon} \right|^2 \frac{\Delta M^2 + (\Delta\Gamma/2)^2}{2\Gamma^2 + \Delta M^2 - (\Delta\Gamma/2)^2}, \quad (2.5)$$

$$\bar{r} \equiv \frac{\Gamma(\bar{B}^0 \rightarrow \chi^+ l^- \bar{\nu}_l)}{\Gamma(\bar{B}^0 \rightarrow \chi^- l^+ \nu_l)} = \left| \frac{1+\epsilon}{1-\epsilon} \right|^2 \frac{\Delta M^2 + (\Delta\Gamma/2)^2}{2\Gamma^2 + \Delta M^2 - (\Delta\Gamma/2)^2}.$$

Denoting by $l^{\pm\pm}$ the number of events of the type $(\chi^+ l^{\pm} \nu_l)(\chi^- l^{\pm} \bar{\nu}_l)$ coming from the production, sub-

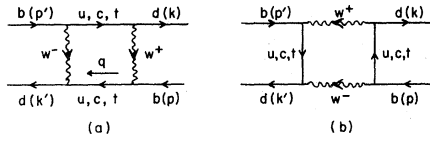


FIG. 1. Lowest-order diagrams contributing to $B_d - \bar{B}_d$ mass mixing.

sequent mixing, and decay of $B^0 - \bar{B}^0$, one defines the mixing parameter r_2 ,⁴

$$r_2 \equiv \frac{l^{++} + l^{--}}{l^{++} + l^{--} + l^{+-} + l^{-+}} = \frac{r + \bar{r}}{1 + r + \bar{r} + r\bar{r}}. \quad (2.6)$$

CP violation appears as a charge asymmetry in the number of these same-sign dilepton events. It is described by the parameter a ,⁴

$$a \equiv \frac{l^{++} - l^{--}}{l^{++} + l^{--}} = \frac{r - \bar{r}}{r + \bar{r}} = \frac{-4 \operatorname{Re} \epsilon (1 + |\epsilon|^2)}{(1 + |\epsilon|^2)^2 + 4(\operatorname{Re} \epsilon)^2}. \quad (2.7)$$

Finally, one can derive the *total* lepton charge asymmetry arising from the decay of conjugate mesons:

$$l^\pm \equiv \frac{l^+ - l^-}{l^+ + l^-} = \frac{r - \bar{r}}{2 + r + \bar{r}}. \quad (2.8)$$

Equations (2.1)–(2.8) summarize the notation

$$i\alpha = \sum_{i,j=u}^{c,t} \left(\frac{G_F}{\sqrt{2}} \right)^2 \int \frac{d^4 q}{(2\pi)^4} \bar{d}(k') \gamma^\mu (1 - \gamma_5) \gamma^\alpha \gamma^\nu (1 - \gamma_5) b(p) \times \bar{d}(k) \gamma_\nu (1 - \gamma_5) \gamma^\beta \gamma_\mu (1 - \gamma_5) b(p') \times q_\alpha (q - p - k)_\beta \left\{ \frac{\xi_i}{q^2 - m_i^2 + i\epsilon} \times \frac{\xi_j}{(q - p - k)^2 - m_j^2 + i\epsilon} \right\}, \quad (3.2)$$

$$\xi_u = -s_1 s_3 c_1, \quad \xi_c = s_1 c_2 (c_1 c_2 s_3 + s_2 c_3 e^{i\theta}), \quad \xi_t = s_1 s_2 (c_1 s_2 s_3 - c_2 c_3 e^{i\theta}).$$

Letting

$$(p+k)^\mu \approx P_B^\mu \quad [\text{or } (p-k)^\mu \approx P_B^\mu \text{ in the case of Fig. 1(b)}], \quad (3.3)$$

$$i\alpha = \sum_{i,j=u}^{c,t} \left(\frac{G_F}{\sqrt{2}} \right)^2 \bar{d}(k') \gamma^\mu (1 - \gamma_5) \gamma^\alpha \gamma^\nu (1 - \gamma_5) b(p) \bar{d}(k) \gamma_\nu (1 - \gamma_5) \gamma^\beta \gamma_\mu (1 - \gamma_5) b(p') \times \xi_i \xi_j \int_0^1 dx \left\{ \frac{i g_{\alpha\beta}}{32\pi^2} [x(1-x)m_B^2 - (1-x)m_i^2 - xm_j^2] \ln[-x(1-x)m_B^2 + (1-x)m_i^2 + xm_j^2 - i\epsilon] + \frac{i P_{B\alpha} P_{B\beta}}{16\pi^2} [x(1-x)] \ln[-x(1-x)m_B^2 + (1-x)m_i^2 + xm_j^2 - i\epsilon] \right\}. \quad (3.4)$$

In considering the Dirac structure, we find

$$\bar{d}(k') \gamma^\mu (1 - \gamma_5) \gamma^\alpha \gamma^\nu (1 - \gamma_5) b(p) \bar{d}(k) \gamma_\nu (1 - \gamma_5) \gamma^\beta \gamma_\mu (1 - \gamma_5) b(p') = 16 \bar{d}(k') \gamma^\beta (1 - \gamma_5) b(p) \bar{d}(k) \gamma^\alpha (1 - \gamma_5) b(p').$$

It is now useful to introduce the vacuum-intermediate-state assumption in the evaluation of matrix element (3.1), and notice that terms proportional to $m_B^2 g_{\alpha\beta}$ and $P_{B\alpha} P_{B\beta}$ give rise to identical contributions in the meson rest frame. Summing contributions from both diagrams with proper attention to color statistics then yields⁷

$$M_{12}^{Bd} - \frac{i}{2} \Gamma_{12}^{Bd} = \frac{4}{3} \left(\frac{-1}{2m_B} \right) \frac{G_F^2}{4\pi^2} f_B^2 m_B^2 \sum_{i,j=u}^{c,t} \xi_i \xi_j \int_0^1 dx [3x(1-x)m_B^2 - (1-x)m_i^2 - xm_j^2] \times \ln[-x(1-x)m_B^2 + (1-x)m_i^2 + xm_j^2]. \quad (3.5)$$

and phenomenological description of the neutral B system. Details concerning detection and background are found elsewhere in the literature.^{5,6}

III. $B_d - \bar{B}_d$ MIXING AMPLITUDE

In the KM model, mixing results from the $2W^+$ exchange box diagrams (Fig. 1) which can be thought of as giving rise to an effective $\Delta B=2$ term in the Lagrangian. M_{12} and Γ_{12} are related to the dispersive and absorptive parts of the mixing amplitude⁵ obtained by taking the matrix element of \mathcal{L}_{eff} between conjugate mesons:

$$M_{12}^B - \frac{i}{2} \Gamma_{12}^B = \frac{1}{2m_B} \langle \bar{B}^0 | -\mathcal{L}_{\text{eff}} | B^0 \rangle. \quad (3.1)$$

Estimates of M_{12} are usually obtained by saturating the matrix element implicit in (3.1) by the intermediate vacuum state. This amounts to using a valence quark wave function for quark operators in \mathcal{L}_{eff} , and is probably an overestimate of (3.1).⁶ Evaluation of Figs. 1(a) and 1(b) is simplified by ignoring the momentum dependence of the gauge propagators. In this case we have, from Fig. 1(a),

$\Gamma_{12}^{B_d}$ is obtained by replacing the logarithm by $-i\pi$ and integrating over the domain of negative argument:

$$\begin{aligned}\Gamma_{12}^{B_d} &= -\frac{G_F^2 f_B^2 m_B}{6\pi} \left\{ \xi_c^2 m_B^2 \left(1 - 4\frac{m_c^2}{m_B^2}\right)^{1/2} + 2\xi_c \xi_u \left[m_B^2 \left(1 - 3\frac{m_c^4}{m_B^4} + 2\frac{m_c^6}{m_B^6}\right) - m_c^2 \left(1 - \frac{m_c^2}{m_B^2}\right)^2 \right] + \xi_u^2 m_B^2 \right\} \\ &= -\frac{G_F^2 f_B^2 m_B}{6\pi} \left[\xi_t^2 m_B^2 + 2\xi_c \xi_t m_c^2 + O\left(\frac{m_c^4}{m_B^2}\right) \right],\end{aligned}\quad (3.6)$$

where the leading correction to the dominant contribution (proportional to ξ_t^2) has been maintained for reasons to follow. The dispersive calculation is most easily accomplished by expanding the logarithms in Eq. (3.5). One obtains

$$M_{12}^{B_d} = \frac{G_F^2 f_B^2 m_B}{12\pi^2} \left[\xi_t^2 \left(m_t^2 + \frac{1}{3} m_B^2 + m_B^2 \ln \frac{m_t^2}{m_B^2} \right) + O\left(m_c^2, \frac{m_B^4}{m_t^2} \right) \right]. \quad (3.7)$$

We are led to the following conclusions:

(1) $\Gamma_{12}^{B_d}$ is significantly smaller than previous estimates based upon an analysis of decay modes common to B_d and \bar{B}_d .

(2) The leading contributions to $\Gamma_{12}^{B_d}$ and $M_{12}^{B_d}$ share a common phase. Their contribution to ϵ [Eq. (2.4)] is therefore *purely imaginary*—hence no contribution to a [Eq. (2.7)]. This results in a substantial reduction in charge asymmetry, which is even more severe in the case of the B_s - \bar{B}_s (see Appendix).

(3) Although the expressions for $\Gamma_{12}^{B_d}$ and $M_{12}^{B_d}$ as stated depend upon the intermediate vacuum state, the ratio $\Gamma_{12}^{B_d}/M_{12}^{B_d}$, relevant to CP violation, *does not*. It does depend on the relation (3.3), which is a reasonable approximation in light of the b -quark mass.

Lastly, the decay width Γ_B is estimated in the parton model by analogy with μ decay:

$$\Gamma_B \simeq \frac{G_F^2 m_b^5}{192\pi^3} \left[\frac{1}{3} |c_1 c_2 s_3 + s_2 c_3 e^{i\delta}|^2 (2 + 3c_1^2 + 3s_1^2 c_3^2) + s_1^2 s_3^2 (2\frac{1}{3} + 3c_1^2 + 3s_1^2 c_3^2 + \frac{2}{3} |c_1 c_2 c_3 - s_2 s_3 e^{i\delta}|^2 + \frac{2}{3} s_1^2 c_2^2) \right]. \quad (3.8)$$

Angular coefficients are obtained by summing over decay channels and including a phase-space suppression factor of $\frac{1}{3}$ for final states with charm or taon. The common belief that gluon radiative corrections to Γ_B are small⁵ extends immediately to $\Gamma_{12}^{B_d}$. The same is undoubtedly true of $M_{12}^{B_d}$.

IV. EXPERIMENTAL RESTRICTIONS ON KM ANGLES

The original calculation of the K^0 - \bar{K}^0 mixing amplitude by Ellis, Gaillard, and Nanopoulos³ is quoted in various places throughout the literature. The results of a rederivation with more specific attention to the angular dependence are now presented. These results are used to more accurately determine the allowed regions for the KM angles.

From a diagram analogous to Fig. 1(a),

$$\begin{aligned}\alpha_{K\bar{K}} \simeq \frac{-G_F^2}{8\pi^2} [\bar{s}\gamma_\mu(1-\gamma_5)d]^2 s_1^2 \left\{ (c_1 c_3 s_2^2 + c_2 s_2 s_3 e^{-i\delta})^2 m_t^2 \left[1 + \frac{m_c^2}{m_t^2} \left(3 - 2 \ln \frac{m_t^2}{m_c^2} \right) \right] \right. \\ \left. + 2c_1 c_3 (c_1 c_3 s_2^2 + c_2 s_2 s_3 e^{-i\delta}) m_c^2 \left(\ln \frac{m_t^2}{m_c^2} - 1 \right) + c_1^2 c_3^2 m_c^2 \left(1 - 3 \frac{m_c^2}{m_t^2} \right) \right\}.\end{aligned}\quad (4.1)$$

For the purposes of the present analysis,

$$\begin{aligned}\alpha_{K\bar{K}} \simeq \frac{-G_F^2}{8\pi^2} [s\gamma_\mu(1-\gamma_5)d]^2 s_1^2 \left[(c_1 c_3 s_2^2 + c_2 s_2 s_3 e^{-i\delta})^2 m_t^2 \right. \\ \left. + 2(c_1 c_3 s_2^2 + c_2 s_2 s_3 e^{-i\delta}) m_c^2 \left(\ln \frac{m_t^2}{m_c^2} - 1 \right) + c_1^2 c_3^2 m_c^2 \right].\end{aligned}\quad (4.2)$$

ΔM_K follows trivially in the valence-quark approximation:

$$\begin{aligned}\Delta M_K (\simeq 2 \operatorname{Re} M_{12}^K) \simeq \frac{G_F^2}{6\pi^2} f_K^2 m_K s_1^2 \left[c_1^2 c_3^2 m_c^2 + (c_1 c_3 s_2^2 + c_2 s_2 s_3 \cos\delta)^2 m_t^2 + 2(c_1 c_3 s_2^2 + c_2 s_2 s_3 \cos\delta) m_c^2 \left(\ln \frac{m_t^2}{m_c^2} - 1 \right) \right] \\ \equiv \frac{G_F^2}{6\pi^2} f_K^2 m_K m_c^2 s_1^2 \mathcal{F} \left(s_2, s_3, \cos\delta, \frac{m_t^2}{m_c^2} \right).\end{aligned}\quad (4.3)$$

This is nominally consistent with the experimental value⁹ of $\Delta M_K = 3.52 \times 10^{-12}$ MeV for \mathcal{F} equal to its minimum (slightly less than 1).¹⁰ Shrock, Treiman, and Wang¹¹ evaluated $\langle K | -\mathcal{L}_{\text{eff}} | \bar{K} \rangle$ in the framework of the MIT bag model with very reasonable results equal to about one-half those based on the intermediate vacuum. Within this framework, consistency of (4.3) with experiment requires $\mathcal{F} \approx 2$. Shrock *et al.* favor a factor of 2 uncertainty in their evaluation, suggesting an upper limit of $\mathcal{F} \approx 4$.

Experimental data on CP violation sets $\text{Re}\epsilon_K = (1.64 \pm 0.06) \times 10^{-3}$.¹² This, along with known values for ΔM_K and $\Delta\Gamma_K$, implies that

$$\frac{\text{Im}M_{12}^K}{\text{Re}M_{12}^K} \approx 6 \times 10^{-3} (\text{exp}) \approx \frac{2s_2s_3c_2 \sin\delta((c_1c_3s_2^2 + c_2s_2s_3 \cos\delta)(m_t^2/m_c^2) + c_1c_3[\ln(m_t^2/m_c^2) - 1])}{\mathcal{F}} \quad (4.4)$$

with \mathcal{F} as defined in Eq. (4.3). This expression imposes a constraint on the KM angles which is independent of the vacuum-intermediate-state assumption. Equation (4.4), together with $\mathcal{F} \leq 4$ and $s_3 \leq 0.3$ from Cabibbo universality,¹³ comprise the known constraints upon θ_2 , θ_3 , and δ .

Equation (4.4) is solved implicitly for s_2 and plotted against s_3 in Fig. 2. If θ_1 , θ_2 , and θ_3 are taken to lie in the first quadrant¹⁴ then $\sin\delta$ is constrained to positive values¹⁵ [Eq. (4.4)]. The two regions of physical interest are then distinguished by the sign of c_δ .

The dots in Fig. 2 represent the quantity \mathcal{F} , the

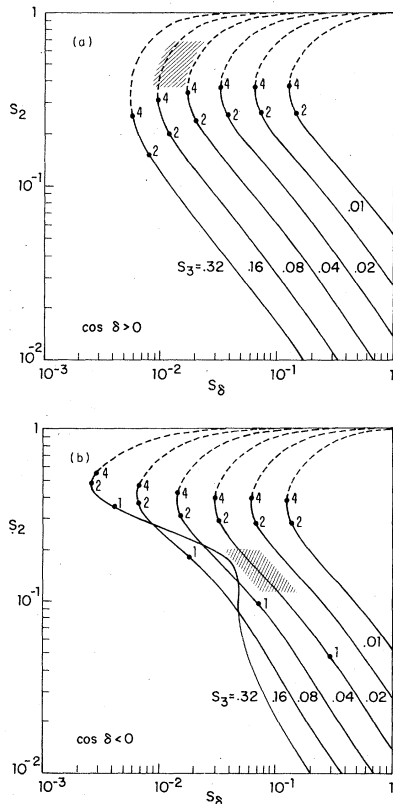


FIG. 2. The Kobayashi-Maskawa angles lie on a family of lines parametrized by s_3 , \mathcal{F} , and the sign of c_δ . Dashes ($\mathcal{F} > 4$) are inconsistent with ΔM_K . Shaded are values of s_2 and s_3 in an O(10) grand unified theory.

dashed portions ($\mathcal{F} > 4$) being inconsistent with Eq. (4.3). Thus ΔM_K provides an upper bound on s_2 of 0.4 for $c_\delta > 0$ and 0.6 for $c_\delta < 0$ and values of s_3 consistent with Cabibbo universality. There is, however, no lower bound imposed on s_2 from ΔM_K unless $\langle \bar{K} | \mathcal{L}_{\text{eff}} | K \rangle^{\text{vac}}$ is definitively shown to be too large (and therefore \mathcal{F} bounded above 1). The lower bound on s_2 proposed by Shrock *et al.*^{11,16} is rather sensitive to variations in their upper limit for $\langle \bar{K} | \mathcal{L}_{\text{eff}} | K \rangle^{\text{bag}}$. There is, nonetheless, a weak lower bound on s_2 from Eq. (4.4) and Cabibbo universality (Fig. 2) of $s_2 > 0.001$.

δ is bound from below at 0.002 (0.005 if $c_\delta > 0$), although large values of s_δ are consistent with the known physics. This last fact justifies the use of the extended CP formalism of Sec. II for b -quark mesons.

Shaded in Fig. 2 are values of s_2 and s_3 predicted by an O(10) grand unified model due to Georgi and Nanopoulos.¹⁷ The model would seem to require $c_\delta < 0$, since $c_\delta > 0$ corresponds to a region where $\mathcal{F} \approx 12$, requiring for consistency a six-fold reduction in $\langle \bar{K} | \mathcal{L}_{\text{eff}} | K \rangle$ from that of the MIT bag model. A comparison between the value of s_δ predicted by O(10) and that implied by Fig. 2 would provide a powerful test for the O(10) model. Unfortunately, this comparison is complicated by an extreme sensitivity of the O(10) δ to minor variations in quark masses.

V. CONCLUSIONS

Figures 3 and 4 display $B_d - \bar{B}_d$ mixing and charge asymmetry. Values for a are independent of f_B and the vacuum-intermediate-state assumption, and are hence reliable up to variations in m_t . The figures shown for mixing are valid in the vacuum-intermediate-state assumption with $f_B = 500$ MeV. A different assumption with regard to the matrix element will lead to correspondingly smaller values.

$B_d - \bar{B}_d$ mixing is potentially complete. It is large throughout most of the $c_\delta < 0$ region, especially where the popular decay mode into charm proportional to $|c_1c_2s_3 + s_2c_3e^{i\delta}|^2$ is somewhat suppressed (Fig. 4).¹⁸ The comparatively smaller mixing characteristic of $c_\delta > 0$ provides a plausible means of

experimentally determining the sign of c_6 .

CP violation may also be appreciable—the charge asymmetry in the same-sign dilepton events can be large in favor of l^{++} (Fig. 3). Chances of detection are unfortunately small since the largest CP violation occurs in regions of relatively little mixing. The total lepton charge asymmetry $|l^\pm|$ is consequently bound below 0.01 if $c_6 < 0$, and is an order of magnitude smaller if $c_6 > 0$. Similarly, the $B_s - \bar{B}_s$ mixes completely everywhere except where decay into charm is strongly suppressed—the *only* region for which CP violation is appreciable. Its contribution to the total charge asymmetry is consequently small.

In summary, although CP violation is potentially large, its experimental detection remains unlikely—a lack of overlap between regions of large mixing and CP violation restricts the total lepton charge asymmetry. The $B_s - \bar{B}_s$, despite complete mixing, contributes little to the total charge asymmetry. Thus an observation of a total lepton-

charge asymmetry in excess of 0.01, or in the direction of positive charge, would be evidence for an alternate means of CP violation.

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APPENDIX: THE $B_s - \bar{B}_s$ MIXING AMPLITUDE

The calculation of $M_{12}^{B_s}$ and $\Gamma_{12}^{B_s}$ follows that of $M_{12}^{B_d}$ and $\Gamma_{12}^{B_d}$ in Sec. III. $M_{12}^{B_s}$ is obtained from the dispersive part of diagrams analogous to Fig. 1. Defining new angular factors

$$\xi_u^l = s_1^2 s_3 c_3,$$

$$\xi_c^l = (c_1 c_2 c_3 - s_2 s_3 e^{-i\delta})(c_1 c_2 s_3 + s_2 c_3 e^{i\delta}),$$

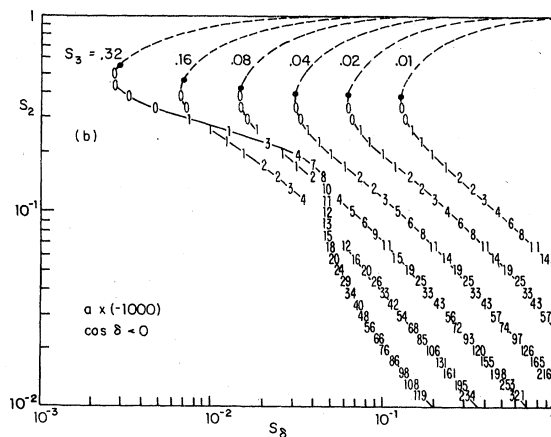
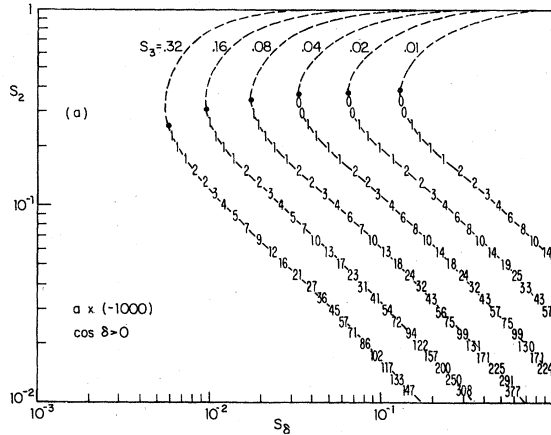


FIG. 3. The same-sign dilepton charge asymmetry for $B_d - \bar{B}_d$ decay is computed in the allowed angular regions. Numbers on curves represent values of a multiplied by -10^3 .

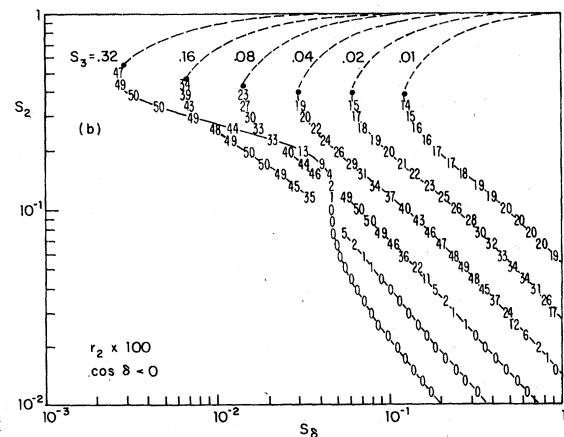
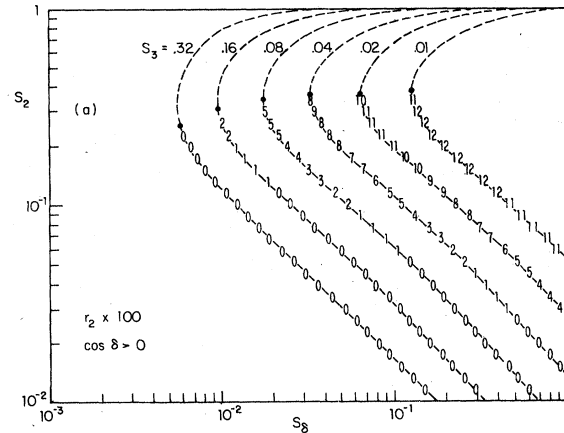


FIG. 4. Mass mixing for the $B_d - \bar{B}_d$ is computed in the allowed angular regions. Numbers on curves represent values of r_2 multiplied by 10^2 .

and

$$\xi'_t = (c_1 s_2 c_3 + c_2 s_3 e^{-i\delta})(c_1 s_2 s_3 - c_2 c_3 e^{i\delta}),$$

one obtains

$$M_{12}^{B_s} \simeq \frac{G_F^2 f_B^2 m_B \xi_t'^2}{12\pi^2} \left(m_t^2 + \frac{1}{3} m_B^2 + m_B^2 \ln \frac{m_t^2}{m_B^2} \right). \quad (\text{A1})$$

$\Gamma_{12}^{B_s}$ may be read directly from Eq. (3.6). Letting $\xi_i \rightarrow \xi'_i$, it follows that

$$\Gamma_{12}^{B_s} \simeq -\frac{G_F^2 f_B^2 m_B}{6\pi} (\xi_t'^2 m_b^2 + 2\xi_c' \xi_t' m_c^2). \quad (\text{A2})$$

The absence of strong angular suppression in the leading terms of these equations results in large mixing for the B_s - \bar{B}_s . No deviation from complete mixing is observed except where ξ'_t and ξ'_c are highly suppressed—in regions where the B^0 decays predominantly into pions. The nearly identical phases of (A1) and (A2) severely reduce CP violation, except where decay into pions is dominant. Since this region is one of restricted mixing, the total lepton charge asymmetry remains very small (<0.007). The B_s - \bar{B}_s adds little to the total charge asymmetry.

¹J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. **B131**, 285 (1977).

²It should be noted that the nonvanishing of ϵ is a necessary but insufficient condition for CP violation. In fact, the magnitude of ϵ is somewhat peripheral, as all physical observables are invariant under $\epsilon \rightarrow 1/\epsilon$; see Eqs. (2.5)–(2.8). These ambiguities reflect an arbitrary choice of phase for B^0 [Eq. (2.2)]. See Wu Dan-di, Inst. of High Energy Physics, Peking report (unpublished), as well as E. Ma *et al.*, Phys. Rev. D **20**, 2888 (1979).

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⁹Particle Data Group, Phys. Lett. **75B**, 1 (1978).

¹⁰For the purposes of this section I make the following assignments: $s_1^2 = 0.05$, $m_c = 1.5$ GeV, $m_b = 4.7$ GeV, $m_t = 15$ GeV, $f_K = 160$ MeV, $f_B = 500$ MeV.

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¹²K. Kleinknecht, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. III-23.

¹³R. E. Shrock and L.-L. Wang, Phys. Rev. Lett. **41**, 1692 (1978).

¹⁴This is always possible through a redefinition of the quark phases.

¹⁵Strictly speaking, this is true only if $s_3 \lesssim 2m_c/m_t$ ($(\ln m_t^2/m_c^2 - 1)^{1/2} \approx 0.38$).

¹⁶See also V. Barger *et al.*, Phys. Rev. Lett. **42**, 1585 (1979).

¹⁷H. Georgi and D. V. Nanopoulos, Nucl. Phys. **B155**, 52 (1979).

¹⁸In these regions, the B decays predominantly into pions.