Numerical upper bounds on the *CP* nonconservation of neutral-heavy-meson systems in the standard six-quark model

Ernest Ma, W. A. Simmons, and S. F. Tuan

Department of Physics and Astronomy, University of Hawaii at Manoa, Honolulu, Hawaii 96822

(Received 30 May 1979)

Using recently obtained bounds on the mixing angles in the standard six-quark model and our own calculations of $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$ as well as the standard expressions of $\mathrm{Im}M_{12}/\mathrm{Re}M_{12}$ for the heavy-meson systems D^0-$\bar{D}^0$, B^0-$\bar{B}^0$, and B_s^0-$\bar{B}_s^0$, we find upper limits of 4×10^{-3}, 5×10^{-4}, and 7×10^{-4} for the magnitude of the phase-convention-independent quantity $2\,\mathrm{Re}\epsilon/(1+|\epsilon|^2)$ as a measure of the size of *CP* nonconservation in these respective systems . Although our numerical results depend crucially on the specific values for the mixing angles taken from the work of others, our method is of general application and can be used to set upper limits on $2\,\mathrm{Re}\epsilon/(1+|\epsilon|^2)$, whatever values these angles might take.

I. INTRODUCTION

The role of *CP* nonconservation in elementary-particle physics has long been an outstanding problem of the utmost importance. At present, the most appealing model, which is also consistent with experimental data, is the six-quark model in which left-handed states form weak-isospin doublets and right-handed states are singlets. As pointed out by Kobayashi and Maskawa[1] several years ago, this model allows for exactly one *CP*-nonconserving phase, which would have been absent in a similarly constructed four-quark model.

In the six-quark model, the origin of *CP* nonconservation is in the charged-current mixing matrix which connects the three quarks (d,s,b) of charge $-\frac{1}{3}$ to the three quarks (u,c,t) of charge $\frac{2}{3}$. The explicit representation of this matrix as given by Ref. 1 is

$$\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta} \end{pmatrix}, \qquad (1.1)$$

where $c_1=\cos\theta_1$, $s_1=\sin\theta_1$, etc. Recently, numerical bounds on these angles have been obtained[2,3] by the use of currently available data and certain theoretical calculations for the K^0-$\bar{K}^0$ system. In this paper, we use these results as well as our own calculations of $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$, together with the standard expressions of $\mathrm{Im}M_{12}/\mathrm{Re}M_{12}$, to set upper limits on the size of *CP* nonconservation in the heavy-meson systems D^0-$\bar{D}^0$, B^0-$\bar{B}^0$, and B_s^0-$\bar{B}_s^0$.[4]

In Sec. II, we present a description of *CP* nonconservation for the systems K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, etc., which is independent of the choice of phase convention. In Sec. III, we present in detail a model calculation of $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$, the knowledge of which is necessary for the determination of the size of *CP* nonconservation. In Sec. IV, we put together all the ingredients and set forth the upper limits on the magnitude of the phase-convention-independent quantity $2\,\mathrm{Re}\epsilon/(1+|\epsilon|^2)$ for the heavy-meson systems D^0-$\bar{D}^0$, B^0-$\bar{B}^0$, and B_s^0-$\bar{B}_s^0$. Finally in Sec. V. we present some concluding remarks.

II. PHASE-CONVENTION-INDEPENDENT DESCRIPTION OF *CP* NONCONSERVATION

Consider the K^0-$\bar{K}^0$ system as an example. Let

$$K_S=\frac{1}{[2(1+|\epsilon|^2)]^{1/2}}[(1+\epsilon)K^0-(1-\epsilon)\bar{K}^0]\,, \qquad (2.1)$$

$$K_L=\frac{1}{[2(1+|\epsilon|^2)]^{1/2}}[(1+\epsilon)K^0+(1-\epsilon)\bar{K}^0]; \qquad (2.2)$$

then ϵ is exactly given by

$$\frac{1-\epsilon}{1+\epsilon}=\frac{2(\Gamma_{12}+iM_{12})}{\frac{1}{2}(\Gamma_S-\Gamma_L)+i\Delta m}, \qquad (2.3)$$

where $\Gamma_{12}=\langle K^0|\Gamma|\bar{K}^0\rangle$, and $\Delta m=m_S-m_L$, etc.[5] However, the phase of $(1-\epsilon)/(1+\epsilon)$ is not a measurable quantity, since it can always be absorbed by a redefinition of the relative phase between K^0 and $\bar{K}^0$. Therefore, an arbitrary phase convention has to be adopted[6] for the specification of the parameter ϵ. On the other hand, the magnitude of $(1-\epsilon)/(1+\epsilon)$ is a measurable quantity, and is certainly independent of phase convention. Define

$$\eta\equiv\left|\frac{1-\epsilon}{1+\epsilon}\right|; \qquad (2.4)$$

then it can easily be shown that $\eta=1$ is a necessary and sufficient condition for *CP* conservation. (This replaces the corresponding phase-convention-dependent statement of $\epsilon=0$.) If $\eta\neq1$, then

CP invariance is violated, and K_S, K_L are not orthogonal states. In fact,

$$\langle K_S | K_L \rangle = \frac{2\,\mathrm{Re}\,\epsilon}{(1+|\epsilon|^2)} = \frac{1-\eta^2}{1+\eta^2}, \tag{2.5}$$

which shows explicitly that the quantity $2\,\mathrm{Re}\,\epsilon/(1+|\epsilon|^2)$ is independent of phase convention.

Using[5]

$$[\tfrac{1}{2}(\Gamma_S - \Gamma_L) + i\Delta m]^2 = 4(\Gamma_{12} + iM_{12})(\Gamma_{12}^* + iM_{12}^*) \tag{2.6}$$

and Eq. (2.3), it is easily seen that

$$\frac{1-\eta^2}{1+\eta^2} = \frac{2\,\mathrm{Im}(\Gamma_{12}^* M_{12})}{|\Gamma_{12}|^2 + |M_{12}|^2}. \tag{2.7}$$

This means that *CP nonconservation is determined by the relative phase between* Γ_{12} *and* M_{12}. For a particular phase convention, such as that of Ref. 1, M_{12} may turn out to have a large phase for certain heavy-meson systems,[4] but without also knowing the phase of Γ_{12}, no conclusion can be reached as to the size of CP nonconservation.

In the following section, we will present model calculations of $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$, using the phase convention of Ref. 1. Then, using the inequality

$$\frac{2xy}{x^2(1+a^2)+y^2(1+b^2)} \le \frac{1}{[(1+a^2)(1+b^2)]^{1/2}}, \tag{2.8}$$

we will be able to set upper limits on $2\,\mathrm{Re}\,\epsilon/(1+|\epsilon|^2)$ according to

$$\frac{2|\mathrm{Re}\,\epsilon|}{1+|\epsilon|^2} < \frac{|a-b|}{[(1+a^2)(1+b^2)]^{1/2}} = |\sin(\phi_\Gamma - \phi_M)|, \tag{2.9}$$

where $a = \tan\phi_\Gamma = \mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$ and $b = \tan\phi_M = \mathrm{Im}M_{12}/\mathrm{Re}M_{12}$. Since $\phi_\Gamma - \phi_M$ is unchanged for any choice of phase convention, our results will again be phase-convention-independent.

III. MODEL CALCULATIONS OF $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$

Consider the D^0-$\bar{D}^0$ system as an example. Let D^0 be represented by the quark-antiquark combination $c\bar{u}$, and $\bar{D}^0$ by $\bar{c}u$. Then the process $D^0 \to f \to \bar{D}^0$, where f is a physical intermediate state, can be considered in terms of quarks as shown in Fig. 1,

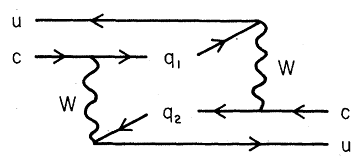

FIG. 1. Quark diagram for the D^0-$\bar{D}^0$ transition amplitude Γ_{12}.

where $q_1\bar{q}_2$ runs over $d\bar{d}$, $s\bar{d}$, $d\bar{s}$, and $s\bar{s}$.

To evaluate this transition amplitude, we must sum over all intermediate states, and for each intermediate state, we must sum over all possible momenta and polarizations. The sum over polarizations is easily accomplished by using the identities

$$\sum_{\mathrm{pol}} \bar{u}(p_1)u(p_1) = \frac{\not{p}_1 + m_1}{2m_1} \tag{3.1}$$

and

$$\sum_{\mathrm{pol}} \bar{v}(p_2)v(p_2) = \frac{\not{p}_2 - m_2}{2m_2}. \tag{3.2}$$

Once this has been done, the effective Lagrangian for $D^0 \to f \to \bar{D}^0$ becomes a sum of terms, each one of which is of the form

$$[\bar{v}_u \gamma_\mu \not{p}_1 \gamma_\nu (1-\gamma_5) u_c][\bar{u}_u \gamma^\nu \not{p}_2 \gamma^\mu (1-\gamma_5) v_c], \tag{3.3}$$

where u_c refers to the wave function of an incoming c quark, etc. Using the identity

$$\gamma_\mu \not{p}_1 \gamma_\nu (1-\gamma_5) = (p_{1\mu}\gamma_\nu + p_{1\nu}\gamma_\mu - g_{\mu\nu}\not{p}_1 + i\epsilon_{\mu\alpha\nu\beta} p_1^\alpha \gamma^\beta) \times (1-\gamma_5) \tag{3.4}$$

and the relationship

$$\langle 0 | \bar{v}_u \gamma^\alpha (1-\gamma_5) u_c | D^0 \rangle = -i f_D p_D^\alpha, \tag{3.5}$$

we then obtain for the form of the transition amplitude an expression proportional to

$$f_D^2 (p_1 \cdot p_D)(p_2 \cdot p_D). \tag{3.6}$$

Now we must consider the sum over p_1 and p_2. From Fig. 1, it is clear that $p_c = p_1 + p_2 + p_u$; but both p_c and p_u are constrained to be components of D^0 and $\bar{D}^0$, hence the phase space in question is equivalent to that of a two-body decay from a parent particle of momentum $p_c - p_u$. Furthermore, if we assume that $m_D = m_c + m_u$, which holds well for constituent quark masses, then Γ_{12} is a sum of terms, each one of which is proportional to

$$\frac{f_D^2 m_D M^2}{64\pi}\left[1 - \left(\frac{m_1^2 - m_2^2}{M^2}\right)^2\right] \times \left[1 - \frac{2(m_1^2 + m_2^2)}{M^2} + \left(\frac{m_1^2 - m_2^2}{M^2}\right)^2\right]^{1/2}, \tag{3.7}$$

where $M = m_c - m_u$.

Using the charged-current mixing matrix (1.1), we are now ready to compute $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$. For the intermediate states $d\bar{d}$, $d\bar{s}$ (or $s\bar{d}$), and $s\bar{s}$, the relative interaction strengths are $s_1^2 c_1^2 c_2^2$, $-s_1^2 c_1 c_2 c_3 (c_1 c_2 c_3 - s_2 s_3 e^{-i\delta})$, and $s_1^2 c_3^2 (c_1 c_2 c_3 - s_2 s_3 e^{-i\delta})^2$, respectively.

TABLE I. Relative phase-space contributions to Γ_{12} for D^0-$\bar{D}^0$, B^0-$\bar{B}^0$, and B_s^0-$\bar{B}_s^0$.

System	Intermediate state	Relative phase space
D^0-$\bar{D}^0$	$d\bar{d}$	1.000
$(M=m_c-m_u)$	$d\bar{s}$ or $s\bar{d}$	0.838
	$s\bar{s}$	0.638
B^0-$\bar{B}^0$	$u\bar{u}$	1.000
$(M=m_b-m_d)$	$u\bar{c}$ or $c\bar{u}$	0.875
	$c\bar{c}$	0.738
B_s^0-B_s^0	$u\bar{u}$	1.000
$(M=m_b-m_s)$	$u\bar{c}$ or $c\bar{u}$	0.862
	$c\bar{c}$	0.707

Combining these with the phase-space factor (3.7) and summing over all intermediate states, we then obtain an expression for Γ_{12}. However, the magnitude of Γ_{12} is likely to be changed by a significant factor if hadronic corrections[7] are taken into account. Therefore, we will use only the phase of Γ_{12}, since any hadronic correction to the phase of Γ_{12} is likely to be compensated for in the phase of M_{12}, so that CP nonconservation remains a weak-interaction phenomenon, and Eq. (2.9) is unchanged.

In Table I, we list the relative phase-space contributions of the various intermediate states to Γ_{12} for the three heavy-meson systems D^0-$\bar{D}^0$, B^0-$\bar{B}^0$, and B_s^0-$\bar{B}_s^0$. We use $m_u = m_d = 0.3$ GeV, $m_s = 0.5$ GeV, $m_c = 1.5$ GeV, and $m_b = 4.7$ GeV in these calculations. However, to obtain a value for $\text{Im}\Gamma_{12}/\text{Re}\Gamma_{12}$, we must also choose a set of values for s_1, s_2, s_3, and δ. To this end, we will use the results of Refs. 2 and 3, and present in the next section upper bounds on $2|\text{Re}\epsilon|/(1+|\epsilon|^2)$ according to Eq. (2.9).

IV. UPPER BOUNDS ON THE CP NONCONSERVATION OF HEAVY-MESON SYSTEMS IN THE STANDARD SIX-QUARK MODEL

As indicated in Sec. II, both $\text{Im}\Gamma_{12}/\text{Re}\Gamma_{12}$ and $\text{Im}M_{12}/\text{Re}M_{12}$ have to be known before any conclusion can be drawn with regard to the size of CP nonconservation. In Sec. III, we presented a method of obtaining $\text{Im}\Gamma_{12}/\text{Re}\Gamma_{12}$. Now we must calculate $\text{Im}M_{12}/\text{Re}M_{12}$. However, it is well known[8] that M_{12} is proportional to

$$\sum_{i,j} \lambda_i \lambda_j A_{ij}, \tag{4.1}$$

where, using the notation $x_i = m_i^2/M_W^2$, we have

$$A_{ij} = \frac{1}{(1-x_i)(1-x_j)} + \frac{1}{(x_i - x_j)}\left[\frac{x_i^2 \ln x_i}{(1-x_i)^2} - \frac{x_j^2 \ln x_j}{(1-x_j)^2}\right], \tag{4.2}$$

and, in the case of the D^0-$\bar{D}^0$ system, $\lambda_d = s_1 c_1 c_2$, $\lambda_s = -s_1 c_3 (c_1 c_2 c_3 - s_2 s_3 e^{-i\delta})$, and $\lambda_b = -s_1 s_3 (c_1 c_2 s_3 + s_2 c_3 e^{-i\delta})$. Therefore, we can also compute $\text{Im}M_{12}/\text{Re}M_{12}$, once we are given a set of values for s_1, s_2, s_3, and δ.

From Refs. 2 and 3, we find that $s_1 = 0.23$, s_3 is between 0.0 and 0.5, and for a given s_3, s_2 and δ are constrained to lie within certain ranges of values, depending on the mass of the t quark. More

TABLE II. Calculated values of $\text{Im}\Gamma_{12}/\text{Re}\Gamma_{12}$, $\text{Im}M_{12}/\text{Re}M_{12}$, and max $[2|\text{Re}\epsilon|/(1+|\epsilon|^2)]$ for various allowed values of s_3, s_2, and δ, as given in Refs. 3 and 9. We assume that $m_t = 14$ GeV.

System	s_3	s_2	δ	$\text{Im}\Gamma_{12}/\text{Re}\Gamma_{12}$	$\text{Im}M_{12}/\text{Re}M_{12}$	$\max\left(\frac{2\|\text{Re}\epsilon\|}{1+\|\epsilon\|^2}\right)$
D^0-$\bar{D}^0$	0.35	0.55	0.0042	-3.73×10^{-3}	6.4×10^{-5}	3.8×10^{-3}
	0.40	0.58	0.0036	-3.10×10^{-3}	5.5×10^{-5}	3.2×10^{-3}
	0.45	0.62	0.0031	-2.57×10^{-3}	4.8×10^{-5}	2.6×10^{-3}
	0.50	0.66	0.0027	-2.14×10^{-3}	4.1×10^{-5}	2.2×10^{-3}
B^0-$\bar{B}^0$	0.35	0.55	0.0042	6.16×10^{-3}	6.61×10^{-3}	4.5×10^{-4}
	0.40	0.58	0.0036	5.00×10^{-3}	5.38×10^{-3}	3.8×10^{-4}
	0.45	0.62	0.0031	4.02×10^{-3}	4.34×10^{-3}	3.2×10^{-4}
	0.50	0.66	0.0027	3.24×10^{-3}	3.50×10^{-3}	2.6×10^{-4}
B_s^0-$\bar{B}_s^0$	0.35	0.55	0.0042	1.889×10^{-2}	1.858×10^{-2}	3.1×10^{-4}
	0.40	0.58	0.0036	1.821×10^{-2}	1.785×10^{-2}	3.6×10^{-4}
	0.45	0.62	0.0031	1.670×10^{-2}	1.632×10^{-2}	3.8×10^{-4}
	0.50	0.66	0.0027	1.530×10^{-2}	1.491×10^{-2}	3.9×10^{-4}

TABLE III. Calculated values of $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$, $\mathrm{Im}M_{12}/\mathrm{Re}M_{12}$, and max $[2|\mathrm{Re}\epsilon|/(1+|\epsilon|^2)]$ for various allowed values of s_3, s_2, and δ, as given in Refs. 3 and 9. We assume that m_t = 30 GeV.

System	s_3	s_2	δ	$\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$	$\mathrm{Im}M_{12}/\mathrm{Re}M_{12}$	$\max\left(\frac{2\lvert\mathrm{Re}\epsilon\rvert}{1+\lvert\epsilon\rvert^2}\right)$
D^0-$\bar{D}^0$	0.35	0.48	0.0024	-2.17×10^{-3}	3.7×10^{-5}	2.2×10^{-3}
	0.40	0.52	0.0020	-1.75×10^{-3}	3.1×10^{-5}	1.8×10^{-3}
	0.45	0.56	0.0017	-1.44×10^{-3}	2.6×10^{-5}	1.5×10^{-3}
	0.50	0.60	0.0014	-1.14×10^{-3}	2.2×10^{-5}	1.2×10^{-3}
B^0-$\bar{B}^0$	0.35	0.48	0.0024	3.61×10^{-3}	3.98×10^{-3}	3.7×10^{-4}
	0.40	0.52	0.0020	2.85×10^{-3}	3.16×10^{-3}	3.1×10^{-4}
	0.45	0.56	0.0017	2.28×10^{-3}	2.54×10^{-3}	2.6×10^{-4}
	0.50	0.60	0.0014	1.75×10^{-3}	1.96×10^{-3}	2.1×10^{-4}
B_s^0-$\bar{B}_s^0$	0.35	0.48	0.0024	1.578×10^{-2}	1.531×10^{-2}	4.7×10^{-4}
	0.40	0.52	0.0020	1.492×10^{-2}	1.438×10^{-2}	5.4×10^{-4}
	0.45	0.56	0.0017	1.432×10^{-2}	1.370×10^{-2}	6.2×10^{-4}
	0.50	0.60	0.0014	1.325×10^{-2}	1.259×10^{-2}	6.6×10^{-4}

recently, an analysis of charmed-meson decay has shown[9] that s_3 must be restricted further to be between 0.35 and 0.5, and that δ must be in the first quadrant, in the notation of Ref. 3. (Our notation differs somewhat from that of Ref. 3. To make use of the results there, we have to make c_1 negative in all our calculations.) Notice that according to Ref. 3, for $s_3>0.35$ and δ in the first quadrant, $\sin\delta$ must be less than about 5×10^{-3}. Therefore, the size of CP nonconservation is expected not to exceed 10^{-2} for all heavy-meson systems.

Since there is still no experimental information on the t quark, we will arbitrarily choose two values, 14 and 30 GeV, for its mass in our calculations. As noted earlier, the other quark masses are set at $m_u=m_d=0.3$ GeV, $m_s=0.5$ GeV, $m_c=1.5$ GeV, and $m_b=4.7$ GeV. According to Ref. 3, depending on whether $m_t=14$ or 30 GeV, s_2 and δ have somewhat different values for a given s_3. Therefore, our calculations of $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$ and $\mathrm{Im}M_{12}/\mathrm{Re}M_{12}$ will change as well, even though the t quark is not directly involved in some of these expressions. Furthermore, we will use those values of s_2 and δ, corresponding to a calculation of the K_L-K_S mass difference with a hadronic bag correction[7] factor of 0.4. In Table II, we list for m_t = 14 GeV the values of $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$, $\mathrm{Im}M_{12}/\mathrm{Re}M_{12}$, and $\max[2|\mathrm{Re}\epsilon|/(1+|\epsilon|^2)]$, as a function of the allowed values of s_3, s_2, and δ, for the three heavy-meson systems D^0-$\bar{D}^0$, B^0-$\bar{B}^0$, and B_s^0-$\bar{B}_s^0$. In Table III, we repeat everything for $m_t=30$ GeV. Notice the almost complete cancellation of $\mathrm{Im}\Gamma_{12}/\mathrm{Re}\Gamma_{12}$ with $\mathrm{Im}M_{12}/\mathrm{Re}M_{12}$ for the B^0-$\bar{B}^0$ and B_s^0-$\bar{B}_s^0$ systems. This points out explicitly the unreliability of calculating only $\mathrm{Im}M_{12}/\mathrm{Re}M_{12}$, and claiming it as an indication of the size of CP nonconservation.[4]

From Tables II and III, it can then be concluded that the upper bounds on the phase-convention-independent quantity $2|\mathrm{Re}\epsilon|/(1+|\epsilon|^2)$ for D^0-$\bar{D}^0$, B^0-$\bar{B}^0$, and B_s^0-$\bar{B}_s^0$, are 4×10^{-3}, 5×10^{-4}, and 7×10^{-4}, respectively.

V. CONCLUDING REMARKS

The question of CP nonconservation in physical systems containing c and b quarks is likely to be thoroughly explored at the new e^+e^- colliding-beam facilities. Of particular interest as an experimental test is the process $e^+e^-\to D^0\bar{D}^0$ (or $B^0\bar{B}^0$, $B_s^0\bar{B}_s^0$) $\to l^\pm l^\pm +$ anything.[10] The charge asymmetry is then a measure of CP nonconservation according to the formula

$$\frac{N^{++}-N^{--}}{N^{++}+N^{--}}=\frac{1-\eta^4}{1+\eta^4}, \tag{5.1}$$

where $(1-\eta^2)/(1+\eta^2)=2\,\mathrm{Re}\epsilon/(1+|\epsilon|^2)$ as given by Eqs. (2.4) and (2.5). Using the results of the preceding section, we find the above charge asymmetry to be no more than 8×10^{-3}, 1.0×10^{-3}, and 1.4×10^{-3} respectively for D^0-$\bar{D}^0$, B^0-$\bar{B}^0$, and B_s^0-$\bar{B}_s^0$.

Although our numerical results depend crucially on the specific values for the mixing angles taken from the work of others, our method is of general application and can be used to set upper limits on $2\,\mathrm{Re}\epsilon/(1+|\epsilon|^2)$, whatever values these angles might take. It can also be easily extended, if further theoretical refinement is desired. For example, the inclusion of "penguin" diagrams[11] for gluon exchange means an additional set of amplitudes to be summed over in both the Γ_{12} and M_{12} calculations. Of course, if such a program is to be followed, then s_2, s_3, and δ must be redetermined, using the appropriate expression for the K_L-K_S mass difference corrected for gluon exchange and so forth. If the magnitudes of M_{12} and Γ_{12} are also reliably calculated, then $2\,\mathrm{Re}\epsilon/(1+|\epsilon|^2)$ will be uniquely determined.

After the completion of this paper, it came to our attention that a similar phase-convention-independent treatment of CP nonconservation has been proposed by Wu Dan-di.[12] In addition, there is a paper by J. S. Hagelin[13] which deals with the same topic within the framework of a more conservative set of bounds on the mixing angles.

ACKNOWLEDGMENTS

We thank Professor V. Barger and Professor S. Pakvasa for discussions with regard to their work on determining the mixing angles. This work was supported in part by the U. S. Department of Energy under Contract No. DE-AC03-76-ER00511 and Decision Research Corporation.

APPENDIX: GEOMETRIC INTERPRETATION OF THE DEPENDENCE OF ϵ ON THE CHOICE OF PHASE CONVENTION

Let us define as in Eq. (2.4)

$$\eta=\left|\frac{1-\epsilon}{1+\epsilon}\right|; \qquad (A1)$$

then a little algebra will show that

$$\left(\mathrm{Re}\epsilon-\frac{1+\eta^2}{1-\eta^2}\right)^2+(\mathrm{Im}\epsilon)^2=\left(\frac{2\eta}{1-\eta^2}\right)^2, \qquad (A2)$$

which is, of course, the equation of a circle in the complex ϵ plane. Therefore, choosing a phase convention for ϵ amounts to picking a point on the circumference of this circle.

[1]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[2]R. E. Shrock and L.-L. Wang, Phys. Rev. Lett. 41, 1692 (1978); R. E. Shrock, S. B. Treiman, and L.-L. Wang, *ibid.* 42, 1589 (1979).
[3]V. Barger, W. F. Long, and S. Pakvasa, Phys. Rev. Lett. 42, 1585 (1979).
[4]J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. B131, 285 (1977); A. Ali and Z. Z. Aydin, *ibid.* B148, 165 (1979).
[5]See, for example, E. D. Commins, *Weak Interactions* (McGraw-Hill, New York, 1973), p. 242, or T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. 16, 511 (1966).
[6]T. T. Wu and C. N. Yang, Phys. Rev. Lett. 13, 380 (1964); L. Wolfenstein, in *Theory and Phenomenology in Particle Physics, Proceedings of the 1968 International School of Physics "Ettore Majorana,"* edited by A. Zichichi (Academic, New York, 1969), p. 218.
[7]R. E. Shrock and S. B. Treiman, Phys. Rev. D 19, 2148 (1979).
[8]M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974); E. Poggio and H. Schnitzer, *ibid.* 15, 1973 (1977).
[9]V. Barger and S. Pakvasa, Phys. Rev. Lett. 43, 812 (1979).
[10]L. B. Okun, V. I. Zakharov, and B. M. Pontecorvo, Lett. Nuovo Cimento 13, 218 (1975).
[11]A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Zh. Eksp. Teor. Fiz. Pis'ma Red. 22, 123 (1975) [JETP Lett. 22, 55 (1975)]; F. J. Gilman and M. B. Wise, Phys. Lett. 83B, 83 (1979).
[12]Wu Dan-di, Inst. of High Energy Phys., Peking University, report, 1979 (unpublished).
[13]J. S. Hagelin, Phys. Rev. D 20, 2893 (1979).