PHYSICAL REVIEW D VOLUME 20, NUMBER 11 1 DECEMBER 1979

Quark fragmentation functions from high-energy nuclear collisions

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(Received 9 July 1979)

An additive quark model is used to derive relations which allow determination of fragmentation functions of "wounded" and spectator quarks from the A dependence of the spectra of particles produced in high-energy, low-momentum-transfer collisions of hadrons and nuclei. The method is applied to spectra of strange particles produced in 300-GeV proton-nucleus collisions. The implications for a fragmentation mechanism of high-energy hadrons are discussed.

I. INTRODUCTION

It was argued recently[1] that the quark model provides a useful way of understanding the A dependence of small-transverse-momentum particle production from nuclear targets. The spectra in the central rapidity region can be successsfully described as emission of hadrons by "wounded" quarks.[2-5] The fragmentation region of the projectile seems to be dominated by the fragmentation of the spectator quarks.[6-9] Since the data do not show any sharp separation between the two regions, however, to obtain a satisfactory description of the spectra it is necessary to develop an approach which would be valid simultaneously in both regions. In this paper we show that a straightforward extension of the quark-model ideas of Refs. 2–9 does indeed provide such a unified description of the spectra. The model gives the parametrization of the A dependence which may be valid everywhere outside the target-nucleus fragmentation region. There are (in general) two parameters in meson-induced and three parameters in baryon-induced reactions. Experimental verification of this parametrization gives a decisive test of the quark model as applied to hadron-nucleus collisions.[1-9]

As the next step, we investigate the possibility that emission of hadrons by wounded quarks is independent of the presence of the spectators so that the contributions from fragmentation of wounded quarks and from spectators simply add up to form the observed hadron spectrum. In this case all parameters describing the A dependence of the particle spectra have a definite physical meaning in terms of the quark fragmentation functions. This leads to the attractive conclusion that the quark fragmentation functions can be determined from the A dependence of the single-particle spectra. Thus it might be possible to obtain information which seems useful for understanding the multiple production processes and which cannot be obtained from the study of hadron-hadron collisions.

We applied our method to the strange-particle spectra in 300-GeV proton-nucleus collisions and found that it is indeed possible to obtain information on quark fragmentation functions. We thus feel confident that, with more data available, such an analysis should provide interesting and qualitatively new information on the fragmentation mechanism of high-energy hadrons.

The plan of the paper is as follows: In the next section we review briefly former applications of the quark model to hadron-nucleus collisions. Our formula for the spectrum is discussed in Secs. III and IV. In Sec. V, the quark fragmentation functions are derived from data on neutral-strange-particle production.[10] Our conclusions are listed in the last section.

II. APPLICATIONS OF THE ADDITIVE QUARK MODEL TO HADRON-NUCLEUS COLLISIONS AT HIGH ENERGIES

A. Central rapidity region

It was observed by Goldhaber[2] that if hadron-hadron collisions are described in terms of interaction of hadronic constiutents, one obtains a natural explanation of the increasing particle multiplicity in hadronic collisions with heavy nuclei. This argument was developed in Ref. 3, where it was shown that it leads to definite quantitative predictions. The idea is illustrated in Fig. 1. Let the incident hadron consist of N_h independent consitutents. Assuming that only these constituents which interacted with the target do produce particles in the central rapidity region and that production from a constituent does not depend on the target,[11] particle production in hadron-nucleus collisions is proportional to the number of wounded constituents, i.e., the constituents which interacted inelastically with the target (at least once). Consequently, we obtain for the ratio of multiplicities from nucleus A and hydrogen

$$R_A(\vec{p}) = \frac{n_A(\vec{p})}{n_H(\vec{p})} = \frac{w_A}{w_H}\ , \tag{2.1}$$

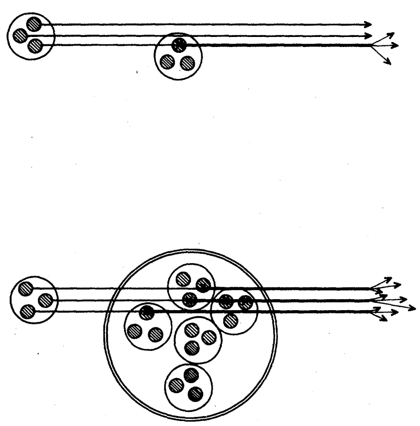

FIG. 1. Interaction of hadronic constituents in hydrogen and in nucleus.

where $w_{A(\mathrm{H})}$ is the number of the wounded constituents in the incident hadron during interaction with the nucleus A (hydrogen). w_A can be calculated from the formula[3,12]

$$w_A = \frac{N_h \tilde{\sigma}_{qA}}{\tilde{\sigma}_{hA}}, \tag{2.2}$$

where $\tilde{\sigma}_{qA}$ and $\tilde{\sigma}_{hA}$ are inelastic, nondiffractive cross sections of the constituent q and hadron h on the target A. They are given by the formula[13]

$$\tilde{\sigma}_A = \int d^2 b \{1 - [1 - \tilde{\sigma}_{\mathrm{H}} D_A(b)]^A\}, \tag{2.3}$$

where $D_A(b) = \int_{\infty}^{\infty} dz\, \rho_A(\vec{b}, z)$ and $\rho_A(\vec{b}, z)$ is the nuclear density.

It was shown in Ref. 3 that at fixed w_A, R_A given by Eq. (2.1) depends mainly on N_h and thus may be used for determination of the number of constituents in the incident hadron. The data indicate $N_h = 3$ for incident nucleons[4,5] and $N_h = 2$ for incident pions,[1,14] thus suggesting validity of the quark model. To summarize, there are good reasons to believe that in the central rapidity region Eq. (2.1) describes correctly the data if one chooses quarks as hadronic constituents which independently emit the produced particles. In this case, the quark additivity implies

$$\tilde{\sigma}_{q\mathrm{H}} = \frac{1}{N_h} \tilde{\sigma}_{h\mathrm{H}} \tag{2.4}$$

and thus $w_{\mathrm{H}} = 1$. This leads to an even simpler formula

$$R_A^C(\vec{p}) = w_A, \tag{2.5}$$

where w_A is given by Eq. (2.2) and $\tilde{\sigma}_{qA}$ and $\tilde{\sigma}_{hA}$ can be calculated from Eq. (2.3).

B. Projectile fragmentation region

The predictions of the quark model are very different in the projectile fragmentation region, as was first observed by Anisovich *et al.*[6] (see also Ref. 7). They pointed out that in this region the contribution from spectator quarks which did not take part in the interaction might be actually much more important, since those quarks are not slowed down by emitting particles in the central region and thus are faster than wounded quarks.

If indeed the fragmentation of spectator quarks is dominant in this region, we obtain a simple prediction

$$R_A^F(\vec{p}) = s_A, \tag{2.6}$$

where

$$s_A = N_h - w_A \tag{2.7}$$

is the average number of spectator quarks.[15] Since w_A increases with increasing A, formula (2.6) gives an immediate explanation of the decrease of the projectile fragmentation multiplicity in collisions with heavy nuclei.

The simple formula (2.6) should be valid for incident mesons. However, for a hadron beam the situation is more complicated because it is necessary to consider the possibility that two spectator quarks do not fragment independently (for example, they may recombine into one final baryon[6-9,16]). More generally one can thus write

$$n_A^F(\vec{p}) = P_A^{(2)} n_{\mathrm{sp}}(\vec{p}) + P_A^{(1)} n_d(\vec{p}). \tag{2.8}$$

Here $P_A^{(1)}$ and $P_A^{(2)}$ are probabilities that exactly one ($P_A^{(1)}$) or two ($P_A^{(2)}$) quarks get wounded. These probabilities are given by[6]

$$P_A^{(1)} = 3(1 - \gamma_A) \tag{2.9}$$

and

$$P_A^{(2)} = s_A - 2P_1 = 6\gamma_A - 3 - w_A, \tag{2.10}$$

with

$$\gamma_A = \tilde{\sigma}_{\pi A} / \tilde{\sigma}_{NA}, \tag{2.11}$$

where $\tilde{\sigma}_{\pi A}$ and $\tilde{\sigma}_{NA}$ are given by Eq. (2.3) with $\tilde{\sigma}_{\pi N} = \frac{2}{3}\tilde{\sigma}_{NN}$. $n_{\mathrm{sp}}(\mathrm{p})$ is the average multiplicity in the fragmentation of one spectator quark and $n_d(\vec{p})$ is the average multiplicity in the fragmentation of a pair of spectator quarks. If all spectator quarks fragment independently, we have

$$n_d(\vec{p}) = 2n_{\mathrm{sp}}(\vec{p}). \tag{2.12}$$

In Fig. 2 $P_A^{(1)}$ and $P_A^{(2)}$ are plotted versus w_A.[17] One sees that they show a very different behavior for $w_A \lesssim 1.5$ ($A \lesssim 60$). However, for heavy nuclei they are almost equal to each other.[6] Consequently, the two terms in Eq. (2.8) should show a similar (rather weak) A dependence for scattering off heavy nuclei.

The precise prediction of A dependence is possible only if one adopts specific assumptions about

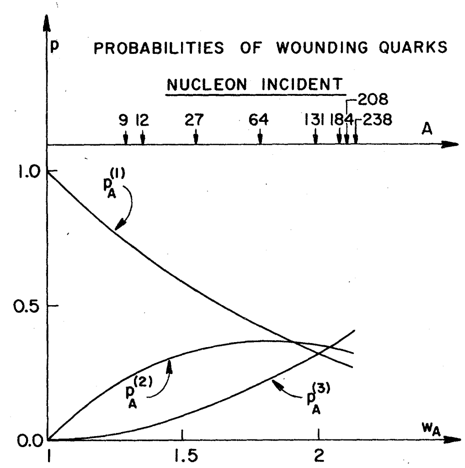

FIG. 2. Probabilities of wounding one ($P_A^{(1)}$), two ($P_A^{(2)}$), and three ($P_A^{(3)}$) quarks in collision of a high-energy proton with a nucleus.

the fragmentation functions $n_{sp}(\vec{p})$ and $n_d(\vec{p})$ (they are, in general, different for different beam and observed final particles). There exist several proposals.[6-9] As indicated by the arguments which lead to formula (2.8), it is necessary to distinguish (at least) between the production of baryons which have two common quarks with the projectile (we call them "favored" baryons) and production of other particles.

For production of favored baryons Anisovish *et al.*[6] assume that[18]

$$n_{sp}(p) \ll n_d(p) \tag{2.13}$$

so that $n_{sp}(\vec{p})$ can be neglected. This leads to the simple formula

$$n_A^F(\vec{p}) = P_A^{(1)} n_H(\vec{p}) . \tag{2.14}$$

The same assumption is made in Refs. 7 and 8, whereas in Ref. 9 both terms are kept.[19]

For meson production it is assumed in Ref. 6 (on the basis of the specific quark counting rules) that $n_{sp}(\vec{p}) = \frac{4}{5} n_d(\vec{p})$, which gives

$$n_A^F(\vec{p}) = (P_A^{(1)} + \tfrac{4}{5} P_A^{(2)}) n_H(\vec{p}) . \tag{2.15}$$

In papers by Nikolaev and collaborators[7,8] it is assumed that

$$n_d(\vec{p}) = \alpha n_{sp}(\vec{p}) , \tag{2.16}$$

where α is a parameter depending on quantum numbers of the beam and produced particles.

Finally, Dar and Takagi[9] assume an independent quark fragmentation into mesons [Eq. (2.12)] which gives

$$n_A^F = s_A n_{sp}(\vec{p}) , \tag{2.17}$$

i.e., the same formula as for incident mesons [Eq. (2.6)].

Good agreement with experiment is claimed in all Refs. 6–9. This indicates that (i) Eq. (2.8) is flexible enough to describe the present data, and (ii) more refined analysis and/or better data are needed to pin down which choice of the parameters (if any) is the correct one.

Our point of view, which we shall describe in more detail in the next two sections, is that instead of trying to guess the quark fragmentation functions (for which there is a little basis except for general intuition) it might be more useful to determine them from experimentally observed A dependence of the spectra. In this way one can obtain interesting information on the dynamics of the quark fragmentation and thus learn about the underlying elementary processes.

III. GENERAL FORMULA FOR A DEPENDENCE OF THE SPECTRA

We argued in the previous section that the quark model gives an adequate description of the data in central and projectile fragmentation regions. However, the application of these ideas is difficult because these regions are not very well defined. The data actually show a rather large intermediate region in which neither formula (2.5) nor (2.8) is applicable.[20,21]

We would like to point out now that the arguments of the previous section can be extended to obtain a general formula for the A dependence of the spectra, which is expected to be valid in the forward hemisphere ($x>0$) and possibly even everywhere outside the target-nucleus fragmentation region.[11]

Consider first the meson-induced processes. In this case the intermediate state after the collision (but before fragmentation into final hadrons) may consist either of one wounded and one spectator quark or of two wounded quarks. Thus the A dependence of the spectrum can be parametrized as

$$n_A(\vec{p}) = P_A^{(1)} n_{WS}(\vec{p}) + P_A^{(2)} n_{WW}(\vec{p}) , \tag{3.1}$$

where $P_A^{(1)}$ and $P_A^{(2)}$ are probabilities that exactly one or two quarks get wounded. The parameters $n_{WS}(\vec{p})$ and $n_{WW}(\vec{p})$ are particle densities in the fragmentation of the wounded-quark–spectator-quark system (n_{WS}) or of two wounded quarks (n_{WW}). $P_A^{(1)}$ and $P_A^{(2)}$ can be easily calculated in terms of w_A:

$$P_A^{(1)} = 2 - w_A , \tag{3.2}$$

$$P_A^{(2)} = w_A - 1 . \tag{3.3}$$

The important point to realize is that both $P_A^{(1)}$ and $P_A^{(2)}$ are *linear* in w_A. Consequently, $n_A(\vec{p})$ is also a linear function of w_A. Thus, despite the generality of the argument, we obtain a surprisingly simple result which should be easy to test experimentally.

Consider now the more complicated case of nucleon-induced reactions. Here we can have three possible intermediate states: one wounded quark and two spectators, two wounded quarks and one spectator, and three wounded quarks. Consequently, the formula for the A dependence of the spectrum consists of three terms:

$$n_A(\vec{p}) = P_A^{(1)} n_{WSS}(\vec{p}) + P_A^{(2)} n_{WWS}(\vec{p}) + P_A^{(3)} n_{WWW}(\vec{p}), \tag{3.4}$$

where $P_A^{(3)}$ is the probability of having exactly three wounded quarks. The parameters $n_{WSS}(\vec{p})$, $n_{WWS}(\vec{p})$, and $n_{WWW}(\vec{p})$ are the particle densities in the fragmentation of these intermediate systems of quarks. $P_A^{(1)}$ and $P_A^{(2)}$ are given by Eqs. (2.9)–(2.11), and $P_A^{(3)} = 1 - P_A^{(1)} - P_A^{(3)}$.

Although the three-parameter formula (3.4) is slightly more complicated than the two-parameter formula (3.1) describing the meson-induced reactions, it still provides a relatively simple description of the spectrum and, consequently, an important test of the quark model.

We should add that Eqs. (3.1) and (3.4) are not entirely new. They were used implicitly in Ref. 6, and a formula similar to our Eq. (3.4) was considered in Ref. 8 and applied to the projectile fragmentation region, as described in the previous section. Our main point is, however, that Eqs. (3.1) and (3.4) are very general and are expected to be valid in a fairly large region of longitudinal momenta, possibly even everywhere outside the target-nucleus fragmentation region. Thus they constitute the basic test of the applications of the quark model to hadron-nucleus collisions. The crucial assumption which leads to this conclusion is that *particle production by wounded quarks does not depend on how many times the wounded quarks interacts inside the nucleus*. This assumption seems to be confirmed by existing experiments at least for particles produced in the forward hemisphere.[1-5]

IV. DETERMINATION OF THE QUARK FRAGMENTATION FUNCTIONS

Although the parametrization of the A dependence of the spectra given by Eqs. (3.1) and (3.4) seems quite useful, the physical meaning of the parameters n_{WS}, n_{WW}, $n_{WSS}, \ldots$ is not particularly appealing because they refer to the fragmentation of rather complicated systems of quarks. It is therefore of interest to try to simplify further these formulae in order to obtain a better physical understanding of their content. In this section we explore one such possibility which is an extension of the ideas presented in Refs. 2–9: We shall assume that the fragmentation of wounded quarks is independent of the presence of spectators, so that contributions from wounded quarks and from spectators simply add up.[22] We thus obtain

$$n_A(\vec{p}) = w_A n_W(\vec{p}) + n_A^F(\vec{p}), \tag{4.1}$$

where $n_A^F(\vec{p})$ is given by Eq. (2.6) for incident mesons and by Eq. (2.8) for incident baryons. Of course Eq. (4.1) is a special case of the more general Eqs. (3.1) and (3.4). For incident mesons we have

$$n_A(\vec{p}) = w_A n_W(\vec{p}) + (2 - w_A) n_{\rm sp}(\vec{p}). \tag{4.2}$$

The important feature of Eq. (4.2) is that now the parameters $n_W(\mathrm{p})$ and $n_{\rm sp}(\mathrm{p})$ have a straightforward physical meaning, being the fragmentation functions of wounded and spectator quarks. Thus by comparing Eq. (4.2) to the data it should be possible to determine both n_W and $n_{\rm sp}$. In this way one can obtain interesting information on fragmentation of quarks which is not available from hadron-hadron experiments.

This feature makes Eq. (4.2) particularly useful for phenomenology. The parametrization given by (4.2) is not more complicated than the generally used formula

$$n_A(\vec{p}) = C(\vec{p}) A^{\alpha(\vec{p})}, \tag{4.3}$$

whereas it has a much clearer physical meaning. We thus feel that the precise measurements of particle production in meson-nucleus collisions are of particular interest, in view of the simplicity of the quark description in this case. They certainly should provide crucial tests of quark fragmentation and recombination ideas in low-momentum-transfer hadron physics.

The description of particle production by incident baryons is more complicated, as we discussed already in the previous sections. Adding contributions from wounded and spectator quarks we obtain using Eqs. (2.4) and (2.8)

$$n_A(\vec{p}) = w_A n_W(\vec{p}) + P_A^{(2)} n_{\rm sp}(\vec{p}) + P_A^{(1)} n_d(\vec{p}). \tag{4.4}$$

This formula reveals similar properties as the simpler Eq. (4.2). Thus it can also be used for determination of the quark fragmentation functions $n_W(p)$, $n_{\rm sp}(p)$, and $n_d(\vec{p})$. However, Eq. (4.4) is more complicated and contains three unknown functions. Thus much better data are needed in this case to disentangle all quark fragmentation functions.

In the next section we discuss the data of Ref. 10.

V. QUARK FRAGMENTATION FUNCTIONS FROM 300-GeV PROTON-NUCLEUS DATA

Recently the A dependence of single-particle spectra was measured for Λ, $\bar{\Lambda}$, and K^0 produced in 300-GeV proton-nucleus interactions.[10] We attempt now an interpretation of this data in terms of the quark fragmentation functions using ideas developed in the previous section.

First, let us observe that the measurements were performed only for Be, Cu, and Pb targets. This means that the three-parameter formula (3.4) cannot be tested by these data. At most, we can hope to determine the quark fragmentation functions by solving the system of three linear equations which arises when Eq. (4.4) is applied consecutively to Be, Cu, and Pb. To do this, we need values of w_A, $P_A^{(1)}$, and $P_A^{(2)}$ for different targets. We calculated them using Eqs. (2.2)–(2.4) and (2.9)–(2.11) and the Saxon-Woods nuclear density

$$\rho(r)=\rho_0/\{1+\exp[(r-R)/a]\}\,.$$

Two sets of nuclear parameters were employed to test the sensitivity of the results. The following discussion and all the figures are obtained using parameters of Ref. 20. Another set was that of Ref. 23. The results from these two calculations are slightly different in details, but none of our conclusions are influenced by this uncertainty. The values of all parameters used are summarized in Tables I and II. Since the data are given in the form of inclusive cross sections, we had to divide them by inelastic nuclear cross sections to obtain particle densities. The values of these cross sections were calculated from Eq. (2.3). They are also given in Tables I and II.

Let us start with a discussion of Λ production. In Fig. 3 the quark fragmentation functions derived from these data are plotted versus scaled Feynman momentum and at fixed transverse momentum = 0.[24] One sees that the fragmentation function of the pair of spectators is quite well determined by the data. It shows a broad maximum for $x \sim 0.5$, indicating that direct recombination of spectator pairs plays an important role in Λ production. It is also seen that the contribution from wounded quarks is small (consistent with zero) in the considered region, as expected by most authors.[6-9] The errors are large enough, however, to allow a nonvanishing contribution. The fragmentation function of the single spec-

TABLE I. Nuclear parameters from Ref. 20. $R = (0.978+0.0206A^{1/3})A^{1/3}$, $a=0.54$, $\tilde{\sigma}_{pp}=32.3$ mb.

	Be	Cu	Pb
w_A	1.294	1.785	2.102
$P_A^{(1)}$	0.738	0.429	0.284
$P_A^{(2)}$	0.229	0.357	0.330
σ_{pA} (mb)	194.2	748.7	1651.2

TABLE II. Nuclear parameters from Ref. 23. $R = 1.14A^{1/3}$, $a=0.545$, $\tilde{\sigma}_{pp}=30$ mb.

	Be	Cu	Pb
w_A	1.246	1.694	2.03
$P_A^{(1)}$	0.776	0.473	0.308
$P_A^{(2)}$	0.202	0.360	0.354
σ_{pA} (mb)	191.9	782.8	1703.6

tator quark n_{sp} cannot be reliably determined from the data and is not shown in Fig. 3. The reason for this is that both w_A and $P_A^{(2)}$ are increasing functions of A. Consequently, the contributions of n_W and n_{sp} are difficult to separate. On the other hand, $P_A^{(1)}$ is a decreasing function of A and thus n_d is easily singled out.

To obtain more information on n_{sp} in absence of better data, one has to fix some other parameters. To compare our results with those of other authors,[6-9] we assumed that wounded quarks do not contribute in the considered region $x \gtrsim 0.2$, i.e.,

$$n_W(\vec{p})=0\,. \tag{5.1}$$

n_d and n_{sp} can then be calculated using only Be and Pb data. The Cu data were used to test hypothesis (5.1). (χ^2 turns out to be very good: $\chi^2=62$ for 75 degrees of freedom.) The results of this exercise are shown in Fig. 4, where both n_{sp} and n_d are plotted versus x. The most striking feature seen in this figure is a dramatic difference between the behavior of n_{sp} and n_d. n_{sp} dominates over n_d for $x \lesssim 0.4$. At higher momenta n_d becomes bigger and

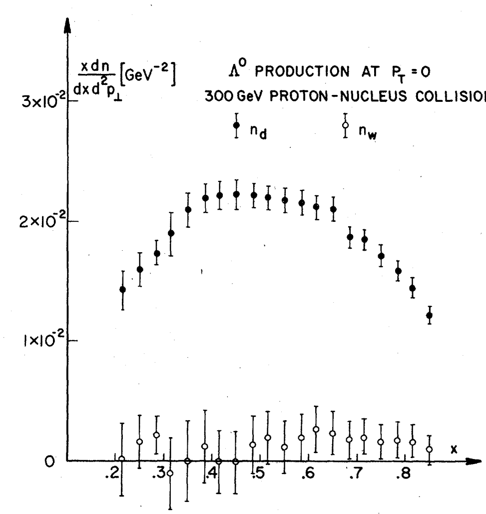

FIG. 3. Fragmentation functions of a wounded quark and of a pair of spectator quarks for Λ production.

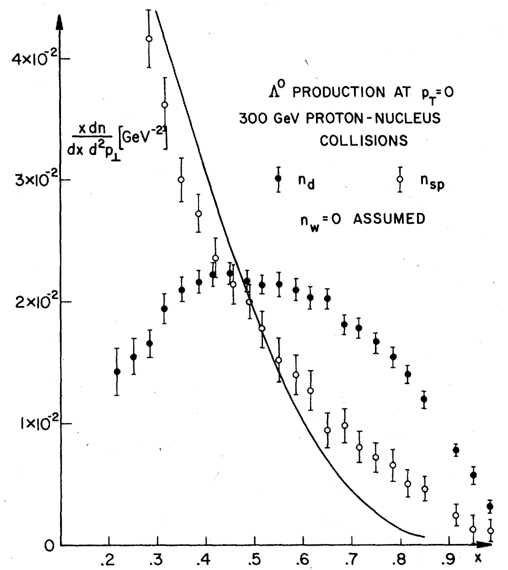

FIG. 4. Fragmentation functions of spectator quark and of pair of spectator quarks for Λ production obtained assuming the condition $n_w = 0$. The line is the elementary valence quark momentum distribution from Ref. 9 normalized to the data at $x = 0.5$.

exceeds $2n_{sp}$ for $x \gtrsim 0.6$. As we already observed, the condition $n_d = 2n_{sp}$ [Eq. (2.12)] is valid if the quarks in the diquark fragment independently. It is seen from Fig. 4 that this condition is badly violated for $x \lesssim 0.4$, thus giving evidence that some collective fragmentation of spectator quark pairs is present.[25] The most natural interpretation of the small values of $n_d(x)$ for $x \lesssim 0.4$ is that spectator pairs recombine directly into favored baryons of higher momentum (as indicated by large values of n_d for $x \gtrsim 0.6$), and thus only a fraction of them is available below $x \approx 0.4$.

The results shown in Fig. 4 agree only qualitatively with those of other authors who discussed the projectile fragmentation region.[6,9] Although we do find the dominance of the direct quark pair recombination into Λ, it occurs at substantially higher momenta than suggested in Ref. 6. The n_s obtained from the data does follow qualitatively the shape suggested by elementary valence quark spectrum[9] (see Fig. 4) but there seems to be a definite tendency for the data to extend for larger x. This could be understood if the objects emitting Λ's are not elementary valence quarks but rather constituent quarks[6,7] which are expected to have harder momentum spectrum (they share the total momentum of the incident particle).

The shape and magnitude of the spectator-pair fragmentation function n_d is not substantially affected by the approximation (5.1). However, as we already noted, n_{sp} is quite sensitive to it. We thus feel that the determination of n_{sp} below $x \approx 0.4$ is not very good and the large values obtained in this region may well be simulating the nonvanishing contribution from wounded quarks, i.e., of the tail from the central region. With the present data we were unable to resolve this problem.

To summarize, the data are giving firm evidence that (a) Λ production in the fragmentation region is indeed dominated by fragmentation of the spectator quarks, (b) the fragmentation of single spectator quarks into Λ gives a dominant contribution for $x \lesssim 0.4$, and (c) pairs of spectator quarks do recombine directly into Λ at high values of the longitudinal momenta $x \gtrsim 0.6$. Although the data are consistent with no production from wounded quarks, some contribution for $x \lesssim 0.5$ is acceptable and actually desirable to obtain a neat physical interpretation of the results.

Let us now turn to $\bar{\Lambda}$ production. Since $\bar{\Lambda}$ are more difficult to produce than Λ, the statistics of $\bar{\Lambda}$ data are limited and errors considerably larger. Consequently, the errors for quark fragmentation functions obtained by inverting Eq. (4.4) are so large that the whole procedure becomes meaningless. Nevertheless, the data do provide interesting information on the fragmentation mechanism: Since $\bar{\Lambda}$ density decreases[10] with increasing A in the measured region $0.2 < x < 0.4$, it follows that the data cannot be described by the contribution from wounded quarks alone. Thus we conclude that fragmentation of the spectator quarks must be an important source of $\bar{\Lambda}$ production.

Let us add one remark. As noticed in Ref. 10, Λ and $\bar{\Lambda}$ production show a rather similar A dependence for $0.2 < x < 0.4$. As we have seen, the quark model can account for this observation but only below $x \approx 0.5$, where no strong contribution from recombination of spectator quark pairs into Λ is expected. At larger x the model predicts that the A dependence of Λ should be stronger than that of $\bar{\Lambda}$.

Finally, let us consider K^0 data. In Fig. 5 n_d and n_W are plotted versus x for $p_\perp = 0$ [they were obtained by inverting Eq. (4.4)]. We see that K^0 production is also dominated by the fragmentation of the spectator quarks. The behavior of n_d is, however, rather different from that observed in Λ production (Fig. 3): n_d is steadily falling with increasing x. This indicates that, as expected, there is no (or very little) collective quark-pair fragmentation into K^0.

To compare our results with the ideas of the other authors, we carried out the analysis assuming that the wounded quarks do not contribute at all [Eq. (5.1)]. The results are shown in Fig. 6, where the ratio $R_{d/s} \equiv n_d/n_{sp}$ is plotted versus x. It is seen that $R_{d/s}$ has quite a small value ($R_{d/s} \sim 0.6$) in the region $x \sim 0.3$ and indicates a tendency

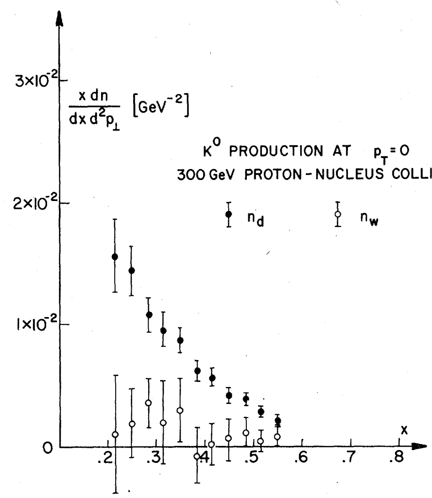

FIG. 5. Fragmentation functions of pair of spectator quarks and of wounded quark for K^0 production.

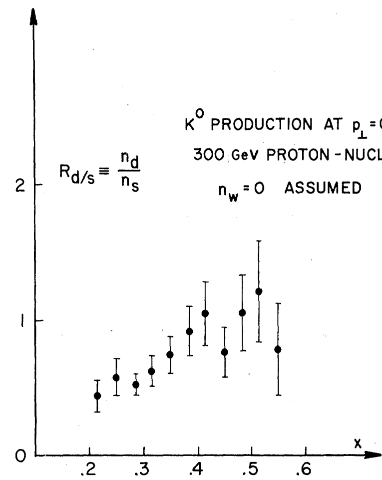

FIG. 6. Ratio $R_{d/s}=n_d/n_{sp}$ for K^0 production derived under the assumption $n_w=0$.

to rise for larger x. This result seems to agree neither with the suggestion of Ref. 6 ($R_{d/s}=\frac{5}{4}$) nor that of Ref. 9 ($R_{d/s}=2$). However, we would like to emphasize that the results shown in Fig. 6 are very sensitive to the assumption (5.1) used here. Thus the relations (2.15) of Ref. 6 and (2.17) of Ref. 9 may well be valid provided a significant contribution from wounded quarks is present for $x > 0.2$

VI. CONCLUSIONS

We have shown that the additive quark model gives a simple parametrization of the A dependence of the spectra of low-transverse-momentum particles produced in hadron-nucleus interactions at high energies. The main features of this result are as follows:

(i) The description is valid in a fairly large interval of longitudinal momenta of the secondary particles. This large region of validity is essential in the discussion of the data because it removes the ambiguities related to the choice of the proper region of applicability of the model (as was required in previous discussions[2,3,5-9]).

(ii) The parameters describing the A dependence of the spectra have very clear physical meaning. They are linear combinations of different quark fragmentation functions. This allows to determine the quark fragmentation functions from the data and thus obtain information on transition from quarks to hadrons which cannot be obtained from hadron-hadron experiments.

(iii) The parametrization is particularly simple for meson-induced reactions. Thus precise measurements of hadron production by incident high-energy mesons should prove very interesting. Not only shall they provide decisive test of the model, but they should also greatly facilitate the determination of quark fragmentation functions from the data.

We have applied our method to data on strange-particle production by 300-GeV protons.[10] The fragmentation functions of quarks and quark pairs were obtained for Λ and K^0 production. We found that in the measured region ($x > 0.2$) the production of all strange particles (including $\overline{\Lambda}$) is strongly dominated by fragmentation of spectator quarks. Evidence was also found for direct recombination of pairs of spectator quarks into Λ, whereas no such process seems to be present for $\overline{\Lambda}$ and K^0 production. Finally, the data suggest that the constituent quarks rather than the elementary valence quarks may be relevant in the process of hadron emission.

These results indicate that our method of analysis of A dependence of the spectra can indeed be useful. With more data available (particularly for incident mesons), one may hope to establish general features of the mechanism of transition from hadronic constituents to hadrons and thus to obtain qualitatively new information on high-energy phenomena.

ACKNOWLEDGMENT

We would like to thank Hannu Miettinen for helpful discussions.

*Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration.

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[11]We consider this assumption as a tentative first approximation. It is likely to be a good approximation in the forward hemisphere, $x>0$ (where the influence of the target is limited), and it is confirmed by the data in this region. (Refs. 1–5). It is not excluded, however, that it may also be valid over the entire central region of rapidity (Ref. 3) (target fragmentation region excluded). This uncertainty is related to the fact that we do not know how far (in rapidity) the target can influence particle production. The experimental measurement of the deviations from our formula (2.1), as one moves from the point $x=0$ towards the target fragmentation region, would provide valuable information on this problem. A particularly interesting question is if (and how fast) the region of target influence (i.e., the correlation length) expands with the increasing energy of the projectile.

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[13]The diffractive (coherent) scattering is not discussed here. If it is included, $\tilde{\sigma}_{qA}$ and σ_{hA} given by Eq. (2.3) are not exactly the nondiffractive cross sections. See L. Bertocchi and D. Treleani, J. Phys. G 3, 147 (1977); T. Jaroszewicz *et al.*, Z. Phys. C 1, 181 (1979).

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[15]The averaging over different kinds of quarks is not necessary. Formula (2.7) is easily generalized to include different quarks.

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[17]Throughout this paper we follow Ref. 20 for the choice of nuclear parameters. See Sec. V for more details.

[18]In Ref. 6 relations (2.13) and (2.15) were proposed only for restricted regions of phase space.

[19]In notation of Ref. 8, $n_d=(\sigma^1+\sigma^2)/\sigma_{pp}$ and $n_{sp}=\frac{1}{2}\sigma^1/\sigma_{pp}$.

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[22]It is by no means obvious that contributions from wounded and spectator quarks can be considered independent in this intermediate region (Refs. 1 and 4). In the bag models, for example, there seems to be no particular reason for such a separation (Ref. 8). In general, one expects the possible interactions between the quarks to influence mostly the projectile fragmentation region, where hadrons are produced a long time after the collision takes place and thus there is plenty of time for interaction between the constituents. It is, however, an open question how important such collective effects are. The semiquantitative successes of the spectator quark counting (Refs. 6–9) seem to indicate that they may be relatively small. We feel, however, that this is now an entirely open question which should be decided by experiment. Particularly strong collective effects can be expected at the end of phase-space, where energy and momentum conservation is an important constraint. An alternative approach, where all quarks recombine together in the projectile fragmentation region, was developed in Ref. 4.

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[25]We checked that the hypothesis $n_d=2n_{sp}$ is not satisfied by the data: $\chi^2=122$ for 75 degrees of freedom.