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Weak-neutral-current effects in $e^- + e^+ \to q + \bar{q} \to (\Lambda, \Sigma, \Lambda_c, \Sigma_c, \ldots) + X$ and $e^- + e^+ \to q + \bar{q} \to (\rho, K^*, D^*, \ldots) + X$

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The forward-backward asymmetry A_B and longitudinal polarization P_B of a spin-1/2 baryon in the process $e^- + e^+ \to B + X$ are calculated in terms of the vector (a_q) and axial-vector (b_q) weak-neutral-current couplings of the quarks composing B, their electric charges Q_q, and their ($q \to B$) fragmentation probabilities. Using a theoretical argument for baryons composed of one heavy $c, b, \ldots,$ quark and two light u, d quarks, and SU(3) symmetry for baryons composed of light u, d, s quarks, A_B is expressed in terms of b_q and Q_q only. In similar fashion, some relations between the various P_B, independent of the fragmentation probabilities, are obtained. The results are discussed in detail for the strange and charmed baryons. The corresponding weak-neutral–electromagnetic interference effects for the vector mesons are also briefly discussed.

There is little doubt that the study of weak-electromagnetic interference effects will deepen our understanding of the weak interactions, in particular of the weak-neutral-current interactions, and will play an important role in discriminating between different weak-interaction models. A recent indication of this is found in the analysis of the polarized-electron–nucleus scattering experiment.[1] Here, as well as in the analysis of high-energy neutrino scattering data, the effects of the heavier $s, c, \ldots,$ quarks in the nucleon sea are always neglected, such effects being presumably small. In this way, the weak-neutral-current couplings of the u and d quarks have been determined. But, as long as the $s, c, \ldots,$ quarks in the nucleon sea are neglected, the determination of their weak-neutral-current couplings must proceed in some other direction. With this motivation in mind, we have previously considered[2] the weak-electromagnetic interference effects in the process $e^- + e^+ \to M + X$, where M is a pseudoscalar meson. In this process, the interference between the weak and electromagnetic amplitudes produces a forward-backward asymmetry A_M of M. Working in the framework of the quark-parton model, A_M was expressed in terms of the axial-vector weak-neutral-current couplings b_q of the quarks composing M, their electric charges Q_q, and the ratio $D(x)$ of their ($q \to M$) fragmentation probabilities. A_M is independent of the vector weak-neutral-current couplings a_q since the forward-backward asymmetry is an intrinsically parity-conserving effect so that terms of the form $b_e a_q$ (or $a_e b_q$) do not appear in A_M, while terms of the form $a_e a_q$ merely renormalize the purely electromagnetic cross section.

In this paper we discuss the interference effects in the process $e^- + e^+ \to B + X$, where B is a spin-$\frac{1}{2}$ baryon. Here, in addition to the forward-backward asymmetry A_B, a longitudinal polarization P_B is also expected. The longitudinal polarization is intrinsically parity nonconserving and, in contrast to the forward-backward asymmetry, is sensitive to both b_q and a_q. We also study the process $e^- + e^+ \to V + X$, where V is a vector meson, for which similar effects are expected. From an experimental point of view, the baryon process is much more useful than the vector-meson process even though the expected longitudinal polarization of B and V are of the same order and the production cross section for V is considerably larger than that for B. The reason for this is that the dominant decays of the vector mesons are parity conserving and hence unsuitable for the determination of longitudinal-polarization effects. Thus consider, for example, the mesons ρ, K^*, D^*. The dominant decay modes of these mesons are parity conserving, i.e., $\rho \to \pi\pi$, $K^* \to K\pi$, $D^* \to D\pi$ or $D\gamma$, and for such modes the angular distribution of the daughter pseudoscalar mesons is independent of the longitudinal polarization of the parent vector mesons. In contrast, the decay $B \to B' + M$ is parity nonconserving and sensitive to the longitudinal polarization of B. In what follows we calculate the longitudinal polarization and forward-backward asymmetry of B and apply the results to the strange and charmed baryons. For baryons composed of even heavier quarks the results are analogous to those for the charmed baryons. We work in the framework of the quark-parton model and, to describe the quark $\to$ hadron fragmentation process, introduce several polarized quark fragmentation probabilities; the number of independent fragmentation probabilities of this kind can then be reduced by use of space-inversion and SU(3)-invariance considerations. In this way, relations independent of the fragmentation probabilities can be obtained for the asymmetries and polarizations of the baryons. Finally, the vector mesons are briefly discussed in Appendix A.

Our calculations are based on the following Lagrangian

$$\mathcal{L} = -e(J_\mu - \bar{e}\gamma_\mu e)A^\mu - g_Z(N_\mu + \bar{e}\Gamma^e_\mu e)Z^\mu ,$$

where

$$J_\mu = \sum_{\text{quarks}} Q_q \bar{q}\gamma_\mu q ,$$

$$N_\mu = \sum_{\text{quarks}} \bar{q}\Gamma^q_\mu q ,$$

and

$$\Gamma^f_\mu = \gamma_\mu(a_f + b_f\gamma_5) \quad f = e, q .$$

In the Weinberg-Salam[3] (WS) model we have the relations

$$g_Z = \frac{g}{2\cos\theta_W} = \frac{e}{2\cos\theta_W \sin\theta_W} ,$$

$$a_f = I_f^{(3)} - 2\sin^2\theta_W Q_f ,$$

$$\frac{G}{\sqrt{2}} = \frac{g_Z^2}{2m_Z^2} = \frac{g^2}{8m_W^2} = \frac{e^2}{8m_W^2\sin^2\theta_W} ,$$

$$b_f = -I_f^{(3)} ,$$

$$m_Z\cos\theta_W = m_W ,$$

where $I_f^{(3)}$ is the weak isospin of the (left-handed) fermion $(I_e^{(3)} = I_d^{(3)} = I_s^{(3)} = \cdots = -\frac{1}{2})$.

The cross section to produce a quark with momentum $\vec{p}_q$ and polarization $\hat{\lambda}_q$ in $e^- + e^+ \to q + \bar{q}$ is then

$$\frac{d\sigma_q(\vec{p}_q, \hat{\lambda}_q)}{d\Omega} = \frac{1}{2}\,\frac{\alpha^2 Q_q^2}{4s} f(\theta,\phi) \times [1 + A_q(s,\theta,\phi) + P_q(s,\theta,\phi)\hat{\lambda}_q \cdot \hat{p}_q] , \tag{1}$$

where $\cos\theta = \hat{p}_q \cdot \hat{p}_{e^-}$, ϕ is the azimuthal angle relative to the transverse polarization $\vec{\lambda}_{e^-} = -\vec{\lambda}_{e^+}$ of e^- $(\vec{\lambda}_{e^-} \cdot \hat{p}_{e^-} = 0)$, $\sqrt{s}$ is the total center-of-mass energy, $E_q = \frac{1}{2}\sqrt{s}$, and

$$\frac{\alpha^2 Q_q^2}{4s} f(\theta,\phi) \equiv \frac{\alpha^2 Q_q^2}{4s}\,(1 + \cos^2\theta - |\vec{\lambda}_e|^2 \sin^2\theta\cos 2\phi)$$

is the purely electromagnetic cross section. Further,

$$A_q(s,\theta,\phi) = \left(\frac{4B(s)b_e b_q}{Q_q}\right)\frac{2\cos\theta}{f(\theta,\phi)} , \tag{2}$$

$$P_q(s,\theta,\phi) = \frac{4B(s)}{Q_q}\left(a_e b_q + a_q b_e \frac{2\cos\theta}{f(\theta,\phi)}\right), \tag{3}$$

with

$$B(s) \equiv \left(\frac{g_Z^2 s}{2m_Z^2}\right)\left(\frac{-1}{4\pi\alpha}\right)\left(\frac{-1}{1 - s/m_Z^2}\right) .$$

In Eq. (1), we have neglected the square of the weak amplitude and a weak-electromagnetic interference term of the form $4B(s)a_e a_q/Q_q$ that merely renormalizes the electromagnetic cross section. In addition, we have set all masses equal to zero. To first order in $B(s)$, $A_q(s,\theta,\phi)$ and $P_q(s,\theta,\phi)$ are, respectively, the forward-backward asymmetry and the longitudinal polarization of the produced quark.

The angular dependence of the forward-backward asymmetry $A_B(s, E_B, \theta, \phi)$ and longitudinal polarization $P_B(s, E_B, \theta, \phi)$ of the produced baryon will be the same as that of $A_q(s,\theta,\phi)$ and $P_q(s,\theta,\phi)$. It is thus convenient to write

$$A_B(s, E_B, \theta, \phi) = A_B(s, E_B)\,\frac{2\cos\theta}{f(\theta,\phi)} ,$$

$$P_B(s, E_B, \theta, \phi) = P_B^{(1)}(s, E_B) + P_B^{(2)}(s, E_B)\,\frac{2\cos\theta}{f(\theta,\phi)} , \tag{4}$$

with analogous equations for $A_q(s,\theta,\phi)$ and $P_q(s,\theta,\phi)$. From Eqs. (2) and (3) we then have

$$A_q(s) = \frac{4B(s)b_e b_q}{Q_q} ,$$

$$P_q^{(1)}(s) = \frac{4B(s)a_e b_q}{Q_q} , \tag{5}$$

$$P_q^{(2)}(s) = \frac{4B(s)b_e a_q}{Q_q} .$$

The corresponding quantities for μ^- in $e^- + e^+ \to \mu^- + \mu^+$ are obtained from Eq. (5) with the replacements $a_q \to a_\mu$, $b_q \to b_\mu$, $Q_q \to Q_\mu = -1$.

As already noted, the forward-backward asymmetry is intrinsically parity conserving so that there will be higher-order electromagnetic contributions to $A_B(s, E_B, \theta, \phi)$. On the other hand, the longitudinal polarization is intrinsically parity nonconserving and, if there are no directions available other than $\hat{p}_B$ and $\hat{p}_{e^-}$, its nonvanishing value is a clear signal for the presence of a parity-nonconserving amplitude. However, if the spins and/or momenta of final particles other than B are observed, or if the initial e^- and e^+ are polarized, there can be parity-conserving contributions to $P_B(s, E_B, \theta, \phi)$. In general, parity conservation implies

$$P_B(s, E_B, \theta, \phi) = -P_B(s, E_B, \theta, \pi - \phi) , \tag{6}$$

whence, for initially unpolarized e^-, e^+ beams for which there is no ϕ dependence, $P_B(s, E_B, \theta) = 0$; it is important to note that Eq. (6) holds even after inclusion of higher-order electromagnetic contributions to $P_B(s, E_B, \theta, \phi)$ and $P_B(s, E_B, \theta, \pi - \phi)$. These higher-order contributions have been calculated[4] for $e^- + e^+ \to \mu^- + \mu^+$, from which the corresponding contributions for $e^- + e^+ \to q + \bar{q}$ can be obtained with the replacement $Q_\mu \to Q_q$ and applied to Eqs. (2) and (3).

We proceed to treat the $q \to B$ fragmentation process; in this process, part of the quark polarization is transmitted to the baryon. In general, the resulting baryon polarization vector will depend on all the directions involved in the fragmentation. These are the momentum and polarization of the quark and the momentum of the baryon. As Eq. (1) shows, the first two coincide and, in the simplest version of the quark-parton model, the directions of the quark and baryon also coincide. Thus, the baryon is polarized along its direction of motion and to calculate its polarization we need only calculate the cross section for it to have a definite helicity. Denoting by $d\sigma_B(\vec{p}_B, \lambda_B)$ $(d\sigma_q(\vec{p}_q, \lambda_q))$ the cross section for production of $B(q)$ with momentum $\vec{p}_B$ $(\vec{p}_q)$ and helicity λ_B (λ_q), we have[5]

$$\frac{d\sigma_B(\vec{p}_B, \lambda_B)}{dx\, d\Omega} = \sum_{q\lambda_q} \left(\frac{d\sigma_q(1/x\, \vec{p}_B, \lambda_q)}{d\Omega}\right) D_{q\lambda_q}^{B\lambda_B}(x), \quad (7)$$

where $D_{q\lambda_q}^{B\lambda_B}(x)$ is the probability that a quark q with helicity λ_q will form a baryon B with helicity λ_B and momentum

$$\vec{p}_B = x\, \vec{p}_q = [E_B^2 - m_B^2)/(s/4 - m_q^2)]^{1/2}\, \vec{p}_q$$
$$\cong (2E_B/\sqrt{s})\, \vec{p}_q .$$

Space-inversion invariance of the fragmentation process implies the useful relation

$$D_{q-\lambda_q}^{B-\lambda_B}(x) = D_{q\lambda_q}^{B\lambda_B}(x), \quad (8)$$

and it is also convenient for what follows to define

$$D_{B/q}(x) \equiv D_{q+}^{B+}(x) + D_{q-}^{B+}(x),$$
$$\Delta D_{B/q}(x) \equiv D_{q+}^{B+}(x) - D_{q-}^{B+}(x). \quad (9)$$

Thus, substituting Eq. (1) in Eq. (7) we obtain

$$\frac{d\sigma_B(\vec{p}_B, \lambda_B)}{dx\, d\Omega} = \frac{1}{2}\left(\frac{d\sigma_B^{(0)}}{dx\, d\Omega}\right) \times [1 + A_B(s, x, \theta, \phi) + P_B(s, x, \theta, \phi)\lambda_B], \quad (10)$$

where

$$\frac{d\sigma_B^{(0)}}{dx\, d\Omega} = \frac{\alpha^2}{4s} f(\theta, \phi)\left[\sum_q Q_q^2 D_{B/q}(x)\right],$$
$$A_B(s, x, \theta, \phi) = \frac{\sum_q Q_q^2 D_{B/q}(x) A_q(s, \theta, \phi)}{\sum_q Q_q^2 D_{B/q}(x)} = A_B(s, x)\frac{2\cos\theta}{f(\theta, \phi)}, \quad (11)$$

$$P_B(s, x, \theta, \phi) = \frac{\sum_q Q_q^2 \Delta D_{B/q}(x) P_q(s, \theta, \phi)}{\sum_q Q_q^2 D_{B/q}(x)} = P_B^{(1)}(s, x) + P_B^{(2)}(s, x)\frac{2\cos\theta}{f(\theta, \phi)}, \quad (12)$$

and where Eqs. (4) and (5) have also been used.

Equations (11) and (12) have two limiting forms that we consider separately.

Case I. All the quarks contribute equally to the fragmentation process:

$$D_{B/q}(x) = D_B(x),$$
$$\Delta D_{B/q}(x) = \Delta D_B(x).$$

Defining

$$C_B(x) \equiv \frac{\Delta D_B(x)}{D_B(x)}, \quad (13)$$

Eqs. (11) and (12) yield

$$A_B(s) = 4B(s) b_e \frac{\sum_q Q_q b_q}{\sum_q Q_q^2},$$
$$P_B^{(1)}(s, x) = C_B(x)\left(\frac{a_e}{b_e}\right) A_B(s), \quad (14)$$
$$P_B^{(2)}(s, x) = C_B(x)[4B(s) b_e]\frac{\sum_q Q_q a_q}{\sum_q Q_q^2},$$

and reduce, in the WS model, to

$$\left(\frac{A_B(s)}{A_\mu(s)}\right)_{\rm WS} = \frac{\sum_q 2Q_q I_q^{(3)}}{\sum_q Q_q^2},$$
$$[P_B^{(1)}(s, x)]_{\rm WS} = C_B(x) B(s)(1 - 4\sin^2\theta_W)\left(\frac{A_B(s)}{A_\mu(s)}\right)_{\rm WS}, \quad (15)$$
$$[P_B^{(2)}(s, x)]_{\rm WS} = C_B(x) B(s)\left[\left(\frac{A_B(s)}{A_\mu(s)}\right)_{\rm WS} - 4\sin^2\theta_W\right],$$

where

$$[A_\mu(s)]_{\rm WS} = -B(s)$$

is the forward-backward asymmetry of the μ^-.

Case II. The contribution of one quark (call it Q) to the fragmentation process is much larger than the contribution of the other two quarks. Defining

$$C_{B/Q}(x) \equiv \frac{\Delta D_{B/Q}(x)}{D_{B/Q}(x)}, \quad (16)$$

Eqs. (11) and (12) yield in this case

$$A_B(s) = A_Q(s),$$
$$P_B^{(1)}(s, x) = C_{B/Q}(x) P_Q^{(1)}(s), \quad (17)$$

where $A_Q(s)$ and $P_Q^{(i)}(s)$ are given by Eq. (5) with b_q, a_q, Q_q replaced by b_Q, a_Q, Q_Q. In the WS model these equations reduce to

$$\left(\frac{A_B(s)}{A_\mu(s)}\right)_{\mathrm{WS}}=\frac{2I_Q^{(3)}}{Q_Q},$$

$$(P_B^{(1)}(s,x))_{\mathrm{WS}}=C_{B/Q}(x)B(s)(1-4\sin^2\theta_W)\left(\frac{A_B(s)}{A_\mu(s)}\right)_{\mathrm{WS}}, \tag{18}$$

$$(P_B^{(2)}(s,x))_{\mathrm{WS}}=C_{B/Q}(x)B(s)\left[\left(\frac{A_B(s)}{A_\mu(s)}\right)_{\mathrm{WS}}-4\sin^2\theta_W\right].$$

We have considered these two particular cases because we believe that Case I applies to baryons composed of the u, d, and s quarks, while Case II applies to baryons containing c, or even heavier b, ..., quarks. A further discussion of the physical assumptions underlying these two cases is given in Appendix B.

Let us now consider the strange baryons Λ, Σ^+, and Σ^-. Assuming SU(2) invariance of the strong-interaction Hamiltonian we obtain

$$D_{\Lambda/u}(x)=D_{\Lambda/d}(x),$$
$$D_{\Sigma^+/u}(x)=D_{\Sigma^-/d}(x);\quad D_{\Sigma^+/s}(x)=D_{\Sigma^-/s}(x), \tag{19a}$$

with similar relations for the functions $\Delta D_{B/q}(x)$. In addition, if the quark $\rightarrow$ baryon fragmentation process is viewed as the decay of the originally produced quark into the quarks contained in B, we further obtain

$$D_{\Lambda/u}(x)=D_{\Lambda/s}(x),\quad D_{\Sigma^+/u}(x)=D_{\Sigma^+/s}(x),$$
$$D_{\Sigma^-/d}(x)=D_{\Sigma^-/s}(x), \tag{19b}$$

which together with Eq. (19a) corresponds to the relations of Case I:

$$D_{\Sigma^+/u}(x)=D_{\Sigma^-/d}(x)=D_{\Sigma^-/s}(x)=D_{\Sigma^+/s}(x)\equiv D_\Sigma(x),$$
$$D_{\Lambda/u}(x)=D_{\Lambda/d}(x)=D_{\Lambda/s}(x)\equiv D_\Lambda(x). \tag{19c}$$

Thus, using Eqs. (19a)–(19c) and remembering that $Q_u=-2Q_d=-2Q_s=\frac{2}{3}$, Eq. (15) gives

$$[A_\Lambda(s)/A_\mu(s)]_{\mathrm{WS}}=2,$$
$$[A_{\Sigma^+}(s)/A_\mu(s)]_{\mathrm{WS}}=\tfrac{5}{3}, \tag{20}$$
$$[A_{\Sigma^-}(s)/A_\mu(s)]_{\mathrm{WS}}=3.$$

Since [on the basis of Eq. (19c)] the same function $C_\Sigma(x)$ describes the longitudinal polarizations of Σ^+ and Σ^- we also obtain, from Eqs. (14) and (15),

$$\frac{P_{\Sigma^+}^{(1)}(s,x)}{P_{\Sigma^-}^{(1)}(s,x)}=\left(\frac{2Q_ub_u+Q_sb_s}{2Q_u^2+Q_s^2}\right)\left(\frac{2Q_d^2+Q_s^2}{2Q_db_d+Q_sb_s}\right)=\frac{5}{9}, \tag{21}$$

$$\frac{P_{\Sigma^+}^{(2)}(s,x)}{P_{\Sigma^-}^{(2)}(s,x)}=\left(\frac{2Q_ua_u+Q_sa_s}{2Q_u^2+Q_s^2}\right)\left(\frac{2Q_d^2+Q_s^2}{2Q_da_d+Q_sa_s}\right)=\frac{\frac{5}{3}-4\sin^2\theta_W}{3-4\sin^2\theta_W},$$

where, in both equations, the last equality holds in the WS model. Since $C_\Lambda(x)\neq C_\Sigma(x)$, there is no definite relation between the longitudinal polarization of Λ and the longitudinal polarizations of Σ^+ and Σ^-. The only relation available is

$$\frac{P_\Lambda^{(1)}(s,x)}{P_\Lambda^{(2)}(s,x)}=\left(\frac{a_e}{b_e}\right)\frac{Q_ub_u+Q_db_d+Q_sb_s}{Q_ua_u+Q_da_d+Q_sa_s}=\frac{1-4\sin^2\theta_W}{1-2\sin^2\theta_W}, \tag{22}$$

where again the last equality holds in the WS model. On the other hand, if we assume that Case II applies to these baryons, Eq. (17) gives, with $Q=s$,

$$\left[\frac{A_\Lambda(s)}{A_\mu(s)}\right]_{\mathrm{WS}}=\left[\frac{A_{\Sigma^+}(s)}{A_\mu(s)}\right]_{\mathrm{WS}}=\left[\frac{A_{\Sigma^-}(s)}{A_\mu(s)}\right]_{\mathrm{WS}}=3,$$

in contrast to Eq. (20). However, this last assumption cannot be justified for the strange baryons.

In Ref. 2 an argument was given that indicated that for mesons composed of a heavy quark $Q=c,b,\ldots$, and a light antiquark $\bar{q}=\bar{u},\bar{d}$, the contribution of Q to the fragmentation is much larger than the contribution of $\bar{q}$; i.e., $D_{M/Q}\gg D_{M/\bar{q}}$. The same argument indicates that a similar situation exists for a baryon composed of a heavy quark and two light quarks: $D_{B/Q}\gg D_{B/q}$, $\Delta D_{B/Q}\gg\Delta D_{B/q}$. Assuming then that Case II (with $Q=c$) applies to the charmed baryons $\Lambda_c(udc)$, $\Sigma_c^{++}(uuc)$, $\Sigma_c^0(ddc)$, we obtain from Eqs. (17) and (18),

$$\frac{A_{\Lambda_c}(s)}{A_\mu(s)}=\frac{A_{\Sigma_c^{++}}(s)}{A_\mu(s)}=\frac{A_{\Sigma_c^0}(s)}{A_\mu(s)}=\frac{-1}{Q_c}\left(\frac{b_c}{b_\mu}\right)=\frac{3}{2},$$

$$\frac{P_{\Lambda_c}^{(1)}(s,x)}{P_{\Lambda_c}^{(2)}(s,x)}=\frac{P_{\Sigma_c^{++}}^{(1)}(s,x)}{P_{\Sigma_c^{++}}^{(2)}(s,x)}=\frac{P_{\Sigma_c^0}^{(1)}(s,x)}{P_{\Sigma_c^0}^{(2)}(s,x)}=\frac{a_eb_q}{a_qb_e}=\frac{1-4\sin^2\theta_W}{1-\frac{8}{3}\sin^2\theta_W}. \tag{23}$$

In both equations the last equality holds in the WS model. Further, Case II (with $Q=c$) yields, independently of the weak-interaction model,

$$\frac{P^{(1)}_{\Sigma_c^{++}}(s,x)}{P^{(1)}_{\Sigma_c^0}(s,x)}=\frac{P^{(2)}_{\Sigma_c^{++}}(s,x)}{P^{(2)}_{\Sigma_c^0}(s,x)}=1\,,$$
$$\frac{P^{(1)}_{\Lambda_c}(s,x)}{P^{(1)}_{\Sigma_c^{++}}(s,x)}=\frac{P^{(2)}_{\Lambda_c}(s,x)}{P^{(2)}_{\Sigma_c^{++}}(s,x)}=\frac{C_{\Lambda/c}(x)}{C_{\Sigma/c}(x)}\,, \tag{24}$$

which can be used as a test of the assumption of c-quark dominance in the fragmentation.

For an experimental test of the relations in Eqs. (20)–(24) the longitudinal polarization and the forward-backward asymmetry have to be measured. For the strange baryons it is well known how to proceed.[6] The two-body decays of these baryons are of the form

$$S\to N+\pi\,,$$

where S is the strange baryon and N is a nucleon. By measuring the angular distribution of the decays, the combination $\alpha_S P_S$ is determined, which in turn determines the longitudinal polarization P_S since the decay parameter α_S is known. The same idea can be applied to the charmed baryon. The decays

$$\Sigma_c^{++}\to\Sigma^++\pi^+\,,\quad \Sigma_c^0\to\Sigma^-+\pi^+$$

produce two charged particles in the final state. If the particles are detected in coincidence, the forward-backward asymmetry and the combination $\alpha_C P_C$ $(C=\Sigma_c^{++},\Sigma_c^0)$ can be measured. Here the decay parameter α_C is not known and must be measured separately to determine the longitudinal polarization P_C. This can be done by measuring $(P_\Sigma)_{\text{final}}$, the polarization of the Σ in the final state; $(P_\Sigma)_{\text{final}}$ is itself determined by the angular distribution in the decay $\Sigma\to N+\pi$. The procedure then for the charmed baryons is to look for the decays

$$\begin{array}{l} C\to\Sigma+\pi \\ \qquad\searrow \\ \qquad\; N+\pi' \end{array}$$

and to measure the angular distribution of the $N\pi'$ and of the $\Sigma\pi$. This is in fact the procedure used to determine the longitudinal polarization and decay parameter for the cascade baryons through the decays[6]

$$\begin{array}{l} \Xi^-\to\Lambda+\pi^- \\ \qquad\searrow \\ \qquad\; p+\pi^-\,. \end{array}$$

The same procedure can be followed for

$$\begin{array}{l} \Lambda_c\to\Lambda+\pi^+ \\ \qquad\searrow \\ \qquad\; p+\pi^-\,. \end{array}$$

Although the dominant decay modes of the charmed baryons are expected to contain more than one pion in the final state, the two-body decays required for the above analysis should be a nonnegligible fraction of all the decays.

In Eqs. (21), (22), and (23) we have focused our attention on quantities that (subject to the assumptions made) are independent of the fragmentation probabilities $C_B(x)$, $C_{B/Q}(x)$, defined by Eqs. (9), (13), and (16). In contrast we can focus our attention on quantities that depend explicitly on the $C_B(x)$, $C_{B/Q}(x)$, i.e., on the longitudinal polarizations $P_B^{(i)}(s,x)$ of Eqs. (14)–(18), and regard measurements of these longitudinal polarizations as determinations of the $C_B(x)$, $C_{B/Q}(x)$. We also note that the same $C_B(x)$ or $C_{B/Q}(x)$ appear in any process in which B is formed, e.g., $\nu_\mu+p\to\nu_\mu+B+X$, and if they have been determined from the $e^-+e^+\to B$ process, can be used to predict the longitudinal polarization of the B in the $\nu_\mu\to\nu_\mu+B$ process.[7] In other B-formation processes, e.g., $p+p\to B+X$ and $\pi+p\to B+X$, the calculated (weak-strong) interference effects will also depend on the details of the model used to describe the quark-quark collisions, so that, if the quark→hadron fragmentation probabilities are known, comparison of theory with experiment for the interference effects can be used to discriminate between different models of the quark-quark collisions.[8] From this point of view, the measurement of the asymmetries and longitudinal polarizations calculated here would also be very useful.

Finally we mention that the results for the charmed baryons can be extended to baryons containing the heavier $b,t,\ldots,$ quarks, where we expect the Case II assumption to be even more accurate.

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APPENDIX A

In this appendix we discuss the interference effects in $e^-+e^+\to V+X$, where V is a vector meson. We begin by establishing Eq. (7) on the basis of a density-matrix argument.

Let $\rho_H^{\lambda\lambda'}=(\rho_H^{\lambda'\lambda})^*$ and $\rho_q^{\kappa\kappa'}=(\rho_q^{\kappa'\kappa})^*$ be density matrices for the hadron H and quark q, respectively, where λ, λ', κ, κ' are helicity indices. Wave-function superposition then yields

$$\rho_H^{\lambda\lambda'}=\sum_{q\kappa\kappa'}D^{H\lambda\lambda'}_{q\kappa\kappa'}(x)\rho_q^{\kappa\kappa'}\,. \tag{A1}$$

We normalize ρ_H and ρ_Q so that

$$\frac{d\sigma_H(\vec{p}_H,\lambda)}{dx\,d\Omega}=\rho_H^{\lambda\lambda}\,, \tag{A2}$$

$$\frac{d\sigma_q(\vec{p}_q,\kappa)}{d\Omega}=\rho_q^{\kappa\kappa}\,. \tag{A3}$$

In the limit $m_q \to 0$ the $V-A$ and $V+A$ amplitudes do not interfere. In this limit the polarization of the quark is purely longitudinal and $\rho_q^{\kappa\kappa'}$ is diagonal. We then obtain from Eqs. (A1) and (A3),

$$\rho_H^{\lambda\lambda'} = \sum_{q\kappa} D_{q\kappa\kappa}^{H\lambda\lambda'}(x)\left(\frac{d\sigma_q(\vec{p}_q,\kappa)}{d\Omega}\right). \tag{A4}$$

In the simplest version of the quark-parton model, the transverse momentum $p_{H\perp}$ of the hadron relative to the quark direction is neglected. Then the three directions available in the fragmentation process, namely the polarization and momentum of the quark and the momentum of the hadron, coincide. Considered as a matrix in the indices $\lambda\lambda'$, $D_{q\kappa\kappa}^{H\lambda\lambda'}(x)$ must then commute with $\vec{J}_H \cdot \hat{p}_H = \vec{S}_H \cdot \hat{p}_H$ since it is invariant under rotations about $\hat{p}_H$. Thus the matrix $D_{q\kappa\kappa}^{H\lambda\lambda'}(x)$ is diagonal in $\lambda\lambda'$ and so is $\rho_H^{\lambda\lambda'}$. According to Eqs. (A2) and (A4), the nonzero elements of $\rho_H^{\lambda\lambda'}$ are then given by

$$\rho_H^{\lambda\lambda} = \frac{d\sigma_H(\vec{p}_H,\lambda)}{dx\, d\Omega} = \sum_{q\kappa} D_{q\kappa\kappa}^{H\lambda\lambda}(x)\left(\frac{d\sigma_q(\vec{p}_q,\kappa)}{d\Omega}\right), \tag{A5}$$

which, with $H=B$, is just Eq. (7). The argument we have used to arrive at Eq. (A5) depends crucially on the assumption that there is only one direction available in the fragmentation process. Thus, if we take into account the transverse polarization of the quark due to the nonzero quark mass, and the transverse component $p_{H\perp}$ of the hadron momentum, our argument does not hold and $\rho_H^{\lambda\lambda'}$ and $\rho_q^{\kappa\kappa'}$ have off-diagonal elements. However, these elements are of order (m_q^2/s) and $(p_{H\perp}^2/s)$ relative to the diagonal elements and are negligible for practical purposes.

Let us consider the vector mesons in the light of this discussion. The angular distribution for the decay $V \to M + M'$, where M and M' are two pseudoscalar mesons, is (θ, ϕ are polar angles of $\hat{p}_M$ relative to $\hat{p}_V$)

$$\begin{aligned} W(\theta,\phi) = \frac{1}{4\pi}[1 + \tfrac{1}{2}(1-3\rho_V^{00})(1-3\cos^2\theta) - 3\sqrt{2}\cos\theta\sin\theta\cos\phi\,\mathrm{Re}(\rho_V^{10}-\rho_V^{0-1}) - 3\sin^2\theta\cos 2\phi\,\mathrm{Re}\,\rho_V^{1-1} \\ + 3\sqrt{2}\cos\theta\sin\theta\sin\phi\,\mathrm{Im}(\rho_V^{10}-\rho_V^{0-1}) + 3\sin^2\theta\sin 2\phi\,\mathrm{Im}\,\rho_V^{1-1}], \end{aligned} \tag{A6}$$

with $\rho_V^{\lambda\lambda'}$ normalized according to $\mathrm{Tr}\rho_V = 1$. If the production of V is parity conserving, we have the relations

$$\rho_V^{-\lambda-\lambda'} = (-1)^{\lambda+\lambda'}\rho_V^{\lambda\lambda'}. \tag{A7}$$

Thus, the parity-nonconserving terms are

$$P_V \equiv \rho_V^{11} - \rho_V^{-1-1},$$
$$\mathrm{Im}(\rho_V^{10} - \rho_V^{0-1}),$$
$$\mathrm{Im}\rho_V^{1-1}.$$

P_V is the longitudinal polarization of V. As can be seen from Eq. (A6), $W(\theta,\phi)$ is independent of P_V and, from our previous discussion, the other two parity-nonconserving terms are zero since $\rho_V^{\lambda\lambda'}$ is diagonal. Therefore, the only measurable interference effect in the case of the vector mesons is the forward-backward asymmetry. This can be calculated from Eqs. (A5) and (2) and is given, using Eq. (4), by

$$A_V(s,x) = 4B(s)b_e\,\frac{Q_Q b_Q D_{V/Q}(x) - Q_q b_q D_{V/\bar{q}}(x)}{Q_Q^2 D_{V/Q}(x) + Q_q^2 D_{V/\bar{q}}(x)}, \tag{A8}$$

where Q and $\bar{q}$ are the quarks composing V and

$$D_{V/Q}(x) \equiv D_{Q+}^{V+}(x) + D_{Q+}^{V0}(x) + D_{Q+}^{V-}(x),$$

with a similar definition for $D_{V/\bar{q}}(x)$. Thus, if the charges Q_q and weak-neutral couplings b_q are considered known, a measurement of $A_V(s)$ effectively determines the ratio

$$D(x) \equiv \frac{D_{V/\bar{q}}(x)}{D_{V/Q}(x)}.$$

If we assume SU(3) invariance for the vector mesons composed of the light quarks u, d, s, we have $D(x)=1$ and Eq. (A8) yields

$$A_V(s) = 4B(s)b_e\,\frac{Q_Q b_Q - Q_q b_q}{Q_Q^2 + Q_q^2}.$$

For the mesons composed of a heavy quark $Q = c, b, \ldots,$ and a light antiquark $\bar{q} = \bar{u}, \bar{d}$, we can assume $D(x) \to 0$ on the basis of the argument given in Ref. 2. In this case Eq. (A8) yields

$$A_V(s) = A_Q(s) = \frac{4B(s)b_e b_Q}{Q_Q}.$$

In both cases the asymmetries are equal to the asymmetries of the corresponding pseudoscalar mesons[2] and we obtain, for example,

$$A_{\rho^-}(s) = A_{\pi^-}(s),$$
$$A_{K^{*-}}(s) = A_{K^-}(s),$$
$$A_{D^{*-}}(s) = A_{D^-}(s),$$

etc.

APPENDIX B

In this appendix we wish to distinguish between assumptions regarding the fragmentation probabilities appropriate to Cases I and II:

$$D_{u\lambda_u}^{B\lambda_B}(x) = D_{d\lambda_d}^{B\lambda_B}(x) = D_{s\lambda_s}^{B\lambda_B}(x)\,, \qquad \text{(B1)}$$

$$D_{c\lambda_c}^{B\lambda_B}(x) \gg D_{u\lambda_u}^{B\lambda_B}(x)\,,\ D_{d\lambda_d}^{B\lambda_B}(x)\,,\ D_{s\lambda_s}^{B\lambda_B}(x)\,, \qquad \text{(B2)}$$

and assumptions relating to the same fragmentation probabilities made in the literature in the context of other problems,[9] e.g.,

$$D_{q\lambda_q}^{B\lambda_B}(x) = \delta_{\lambda_q\lambda_B} D_{B/q}(x);\quad q = u,\ d,\ s,\ c, \ldots\,, \qquad \text{(B3)}$$

or

$$D_{q\lambda_q}^{B\lambda_B}(x=1) = \delta_{\lambda_q\lambda_B} D_{B/q}(x=1);\ q = u,\ d,\ s,\ c, \ldots\,. \qquad \text{(B4)}$$

The assumptions in Eqs. (B3) and (B4) describe the dependence on x of the fragmentation probability of a given quark (and thus distinguish between the behavior of a "fast" quark and a "slow" quark) while the assumptions in Eqs. (B1) and (B2) relate the values, at a given x, of the fragmentation probabilities for different quarks. If now, in addition to Eqs. (B1) and (B2), we also assume, for example, the validity of Eq. (B3), our Eqs. (15) and (18) become independent of x corresponding to the replacement in these equations of $C_B(x)$ and $C_{B/Q}(x)$ by 1.

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[2]J. F. Nieves, Phys. Rev. D 19, 2591 (1979).

[3]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (*Nobel Symposium No. 8*), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).

[4]B. Ya. Zel'dovich and M. V. Terent'ev, Yad. Fiz. 7, 1033 (1968) [Sov. J. Nucl. Phys. 7, 650 (1968)]. See also V. K. Cung, Ph.D. thesis, University of Pennsylvania, 1974 (unpublished).

[5]One should, strictly speaking, write relations between density matrices rather than cross sections. Equation (7) is justified in Appendix A.

[6]See, for example, R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969), p. 512.

[7]However, there is an important difference between the $\nu_\mu \to \nu_\mu + B$ process and the $e^- + e^+ \to B$ process. Thus, since the s, c, ..., quarks in the target nucleon are nonvalence and so relatively sparse, their contribution (at least in the high-energy, large-momentum-transfer kinematic region) to the $\nu_\mu \to \nu_\mu + B$ process is small even when $B = \Lambda,\ \Sigma,\ \Lambda_c,\ \Sigma_c, \ldots,$. The $\nu_\mu \to \nu_\mu + B$ cross section is therefore independent of the weak-neutral-current couplings and fragmentation probabilities of these s, c, ..., quarks and yields a B whose longitudinal polarization arises only from the longitudinal polarization of the recoiling u (d) quark in $\nu_\mu + u\ (d) \to \nu_\mu + u\ (d)$.

[8]This has been discussed in detail for $B = \Lambda$ by E. Fischbach and G. Look, Phys. Rev. D 16, 2199 (1977).

[9]See, for example, Hai-yang Cheng and Ephraim Fischbach, Phys. Rev. D 19, 2123 (1979).