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Weak mixing angle and grand unified gauge theories

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We discuss the role of $\sin^2\theta_W$ (θ_W = weak mixing angle) in the Weinberg-Salam $SU(2)\times U(1)$ model. A definition of the renormalized value of $\sin^2\theta_W$ is given, the effects of radiative corrections are considered, and the possibility of precisely determining this parameter experimentally is examined. Within the framework of grand unified gauge theories, we derive an expression for the effective quantity $\sin^2\theta_W(M_W)$ which can be compared with experiment. Our results are illustrated for the Georgi-Glashow SU(5) model and their implications regarding the predicted lifetime of the proton τ_p are discussed. For $\sin^2\theta_W(M_W)\simeq 0.21$–$0.20$, values consistent with neutral-current measurements, we estimate that the SU(5) model predicts $\tau_p \simeq 10^{30}$–10^{33} yr, a range that is not very far from the present experimental bound. We also show that by increasing the number of scalar multiplets, the model can accommodate larger values for $\sin^2\theta_W(M_W)$.

I. INTRODUCTION

The Weinberg-Salam $SU(2)\times U(1)$ model[1,2] provides us with an elegant unified description of weak and electromagnetic interactions. It preserves the successful precise predictions of quantum electrodynamics ($g-2$, Lamb shift etc.) and accommodates all known charged-current weak-interaction phenomenology (β, muon, pion decays, etc.). In addition, this model correctly predicted recently established weak-neutral-current effects in neutrino scattering[3] and parity-violating electron-scattering-asymmetry experiments,[4] its greatest triumphs to date.

Essentially the only important free parameter in the Weinberg-Salam model is $\sin^2\theta_W$, where the weak mixing angle θ_W is defined by $\tan\theta_W \equiv g_1/g_2$, the ratio of U(1) and SU(2) coupling constants. Values for $\sin^2\theta_W$ are usually obtained by comparing the results of neutral-current experiments with theoretical predictions (which depend only on $\sin^2\theta_W$). Of course, the validity of the Weinberg-Salam model requires that all experiments yield the same value for $\sin^2\theta_W$. At present the world average[5] stands at $\sin^2\theta_W = 0.23 \pm 0.02$, and most individual measurements are consistent with a value of $\sin^2\theta_W$ in the range 0.20 to 0.30. This agreement between the values of $\sin^2\theta_W$ determined by a variety of diverse neutral-current experiments is regarded as strong evidence in support of the Weinberg-Salam model's correctness; however, before the final verdict is rendered a great deal of further proof is required (e.g., discovery of the W and Z vector bosons, Higgs scalar, etc.). Also, more precise determinations of $\sin^2\theta_W$ by several distinct experiments would be helpful. Anticipating that such measurements will eventually be performed, we will devote the first part of this paper to setting down a precise theoretical definition of the renormalized value of $\sin^2\theta_W$ which can be compared with experiments and to a discussion of the effects of higher-order radiative corrections on this important parameter.

We also have at this time an extremely attractive theory of strong interactions, quantum chromodynamics (QCD),[6] which is based on the unbroken gauge group $SU(3)_c$ associated with color. When taken together with the Weinberg-Salam model, there results an $SU(3)_c\times SU(2)\times U(1)$ gauge theory which describes strong, weak, and electromagnetic interactions. This combined theory is often referred to as the standard model. Throughout this paper we accept the premise that the standard model is correct; however, we allow for the possibility that it may be only part of the entire picture. Indeed, the interesting possibility of embedding the standard model in some simple gauge group G: $G \supset SU(3)_c\times SU(2)\times U(1)$ has been advocated by several groups[7-9]; the resulting theories are called grand unified gauge theories. A nice feature of a simple covering gauge group is that it possesses only one coupling constant rather that three independent ones, hence true unification of the fundamental interactions (excluding gravity). That is, the individual couplings g_3, g_2, and g_1 associated with $SU(3)_c$, SU(2), and U(1) must all be the same, up to some group-theoretic weighting factors. In such a scheme, the observed unequal strengths of strong, weak, and electromagnetic interactions is a consequence of performing present-day experiments at relatively low energies, so that higher-order radiative corrections effectively enhance or diminish the various interaction strengths; they become equal only at superhigh energies above all mass scales in the theory (i.e., at very very short distances).

With regard to the weak mixing angle, an appealing property of grand unified models in that $\sin^2\theta_W^0$ is predicted to be some definite number, rather than an (infinite) adjustable counterterm parameter as it is in the Weinberg-Salam model when considered alone. [The superscript zero indicates that we mean the bare (unrenormalized) parameter that appears in the Lagrangian.] The value of $\sin^2\theta_W^0$ depends only on the grand unification group G and the representation assignments of the particles, for example, $\sin^2\theta_W^0=\frac{3}{8}$ in several models (see Sec. III for a discussion of this point). We note, however, that just as the effective interaction strengths depend on the experimental energy scale examined, so does the effective value of $\sin^2\theta_W$, since it is a measure of the ratio of electromagnetic to weak couplings, $\sin\theta_W=e/g_2$. So, within the framework of grand unified gauge theories, the effective value of $\sin^2\theta_W$ measured in present-day experiments may differ significantly from its bare asymptotic value $\sin^2\theta_W^0$ because of large radiative corrections (finite renormalization effects); a feature initially pointed out and explored by Georgi, Quinn, and Weinberg.[10] In this paper we discuss further the origin of these corrections, derive a simple expression for an effective quantity $\sin^2\theta_W(M_W)$ which can be compared with experiment and explain the possible theoretical sources of uncertainty in our result.

Although our results for the effective value of $\sin^2\theta_W$ are somewhat more general, they are perhaps most relevant for the Georgi-Glashow SU(5) model,[7] the earliest, most economical and still the front runner of grand unified theories. In that case we find that the effective value of $\sin^2\theta_W$ which can be compared with present-day experimental measurements is reduced from $\frac{3}{8}$ to

$$\sin^2\theta_W(M_W)\simeq\tfrac{3}{8}[1-0.015\ \ln(M_S/M_W)]\,, \tag{1.1}$$

where M_W is the mass of the usual charged vector boson $W^\pm\simeq 75$–90 GeV and M_S is the mass scale of the superheavy (fractionally charged) vector bosons in the model, which must be very large $\simeq 10^{14}$–10^{17} GeV.[7,10,11,12] [An estimate of the error in (1.1) along with a definition of $\sin^2\theta_W(M_W)$ are given in the text.]

Within the framework of the SU(5) model, the proton is not absolutely stable; but its lifetime is expected to be very long. The predicted lifetime, τ_p, exhibits a very sensitive dependence on the mass, M_S, of the superheavy bosons that mediate proton decay ($\tau_p\propto M_S{}^4$) or equivalently through (1.1) on the effective value of $\sin^2\theta_W$ measured experimentally. As we shall see, the present experimental limits on the proton lifetime[13,14] imply $M_S\gtrsim 3\times10^{14}$ GeV which implies [from (1.1)] that $\sin^2\theta_W(M_W)\leq 0.212$ in the SU(5) model.

The prediction for the effective value of $\sin^2\theta_W$ in (1.1) gets modified if we enlarge the Higgs-scalar content of the SU(5) model. By adding more scalar multiplets, the prediction for $\sin^2\theta_W(M_W)$ can be increased; we will exhibit this feature when we discuss the SU(5) model.

The remainder of this paper is organized as follows: In Sec. II we review the role of $\sin^2\theta_W$ in the Weinberg-Salam model, examine possible experimental measurements that may be able to determine the value of $\sin^2\theta_W$ very precisely, and discuss the effects of radiative corrections on such determinations. In Sec. III we compute the effective quantity $\sin^2\theta_W(M_W)$ relevant for comparison with neutral-current experiments, assuming the framework of a grand unified theory. As a specific example, our results are illustrated for the Georgi-Glashow SU(5) model[7] in Sec. IV. There we discuss the question of proton stability and relate the predicted proton lifetime to the value of $\sin^2\theta_W(M_W)$. We also show that the SU(5) model's prediction for $\sin^2\theta_W(M_W)$ can be increased by enlarging the Higgs-scalar content of the theory. Finally, we conclude in Sec. V with a discussion of our results and their implications.

II. $\sin^2\theta_W$ AND THE WEINBERG-SALAM MODEL

We begin this section with a brief review of what is often called the sequential Weinberg-Salam model[1,2] with a Glashow-Iliopoulos-Maiani mechanism.[15] The Weinberg-Salam model of weak and electromagnetic interactions is based on the local gauge group SU(2)$\times$U(1). The group generators are T_a ($a=1,2,3$) [weak isospin generators of SU(2)] and Y [weak hypercharge generator of U(1)]. The electric charge is given by the linear combination

$$Q=T_3+Y/2 \quad \text{(electric charge)}\,. \tag{2.1}$$

Associated with the local symmetry are four gauge fields, an isotriplet W_μ^a, and an isoscalar B_μ. The theory also contains a complex scalar doublet made up of a positively charged and neutral component

$$\phi=\begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix},\quad T=\tfrac{1}{2}\,,\quad Y=1\,. \tag{2.2}$$

Fermions are incorporated into the model through the following multiplet assignments of leptons and quarks:

$$\text{Leptons:}\ \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L,\ \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L,\quad T=\tfrac{1}{2}\,,\quad Y=-1\,, \tag{2.3a}$$

$$e_R\,,\ \mu_R\,,\ \tau_R\,,\quad T=0\,,\quad Y=-2\,;$$

$$\text{Quarks: } \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L, \quad T=\tfrac{1}{2}, \quad Y=\tfrac{1}{3},$$
$$u_R, c_R, t_R, \quad T=0, \quad Y=\tfrac{4}{3}, \tag{2.3b}$$
$$d_R, s_R, b_R, \quad T=0, \quad Y=-\tfrac{2}{3},$$

where R and L subscripts denote right- and left-handed components of the fermion fields

$$\psi_{R,L} = \frac{1\pm\gamma_5}{2}\psi. \tag{2.3c}$$

Although experimental evidence in support of the top quark t has not yet been found, it is included in (2.3b) so that the GIM mechanism[15] will be fully operative, i.e., no flavor-changing neutral currents. If further leptons and quarks exist, they can be easily incorporated into the model as sequential additions to (2.3), i.e., left-handed components in doublets right-handed components in singlets. The primes in (2.3b) allow for quark mixing and CP-violating phases through the Kobayashi-Maskawa matrix[16]

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix}_L = \begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}_L, \quad c_i \equiv \cos\theta_i, \quad s_i \equiv \sin\theta_i, \tag{2.4}$$

with d, s, and b the quark-mass eigenstates. We have suppressed quark color indices in the above discussion. Each quark comes in three colors; hence there are really three times the number of quark fields as listed in (2.3b).

The SU(2)× U(1) gauge-invariant Lagrangian density which describes the gauge fields and their interactions with scalars and fermions is given by[2,17]

$$\mathcal{L} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} + D^\mu\phi^\dagger D_\mu\phi + i\sum_f \bar{\psi}_f\gamma^\mu D_\mu\psi_f, \tag{2.5a}$$

$$\mu,\nu = 0,1,2,3; \quad a=1,2,3 \text{ [SU(2) index]}$$

where

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_{2_0}\epsilon^{abc}W^b_\mu W^c_\nu, \tag{2.5b}$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{2.5c}$$
$$D_\mu = \partial_\mu + ig_{2_0}T\tau^a W^a_\mu + ig_{1_0}\tfrac{1}{2}YB_\mu \quad \text{(covariant derivative)}. \tag{2.5d}$$

[Subscript zero denotes bare (unrenormalized) quantities.] We have used ψ_f to denote any fermion multiplet (singlet or doublet); the summation in (2.5a) is over all fermion multiplets in (2.3). Note the appearance of two independent bare coupling constants in (2.5) g_{2_0} and g_{1_0} which are associated with the SU(2) and U(1) groups, respectively; most of the discussion of this paper is centered around these couplings and their relative magnitude.

By means of the Higgs mechanism,[18] the scalar doublet acquires a vacuum expectation value

$$\langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_c \end{pmatrix}, \tag{2.6}$$

and the symmetry of the theory thereby breaks down to a local U(1) invariance generated by the electric charge in (2.1), i.e., the remaining U(1) gauge invariance is that of electromagnetism. In this way, three intermediate vector bosons acquire mass, while one (identified as the photon) remains massless. The resulting mass eigenstates and their corresponding bare masses are[2]

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu), \quad M^0_W = g_{2_0}v_0/2, \tag{2.7a}$$

$$Z_\mu = W^3_\mu\cos\theta^0_W - B_\mu\sin\theta^0_W, \quad M^0_Z = \tfrac{1}{2}v_0(g_{1_0}{}^2 + g_{2_0}{}^2)^{1/2}, \tag{2.7b}$$

$$A_\mu = B_\mu\cos\theta^0_W + W^3_\mu\sin\theta^0_W, \quad M^0_\gamma = 0, \tag{2.7c}$$

where θ^0_W is the bare (unrenormalized) mixing angle defined by

$$\theta^0_W = \tan^{-1}(g_{1_0}/g_{2_0}) \tag{2.8}$$

[Notice that we use a subscript (or superscript) zero to denote bare (unrenormalized) parameters of the model; this is done when we feel that it is important to distinguish them from the renormalized parameters. All of the fields, including those in (2.7), and parameters used up to now are actually unrenormalized quantities; they give rise to finite renormalized parts plus counterterms necessary to render the theory free of ultraviolet divergences.]

Because only an isodoublet of scalars is employed to break the symmetry, the masses of the charged vector bosons $W^{\pm}$ and the neutral Z satisfy the natural relationship [see (2.7)]

$$M_W^0/M_Z^0=\cos\theta_W^0\,; \tag{2.9a}$$

hence, the physical masses (which will be experimentally measured) must satisfy

$$M_W/M_Z=\cos\theta_W[1+\text{finite } O(g^2)\text{ corrections}]\,, \tag{2.9b}$$

where by $O(g^2)$ we mean $O(g_1^2)$ and $O(g_2^2)$ corrections that arise from one-loop Feynman diagrams.[19–21] The finite corrections in (2.9b) depend on the precise definition of θ_W (renormalized) used; we will return to this point later on.

A further consequence of the Higgs mechanism is that three of the four scalar fields in (2.2) become the longitudinal components of the massive vector bosons, while one remains as a neutral physical scalar field with arbitrary mass M_ϕ. For simplicity, we usually assume in this paper $M_\phi \simeq M_W$. All of the fermions (except neutrinos) also acquire mass because of their interactions with the scalar doublet. (We have not exhibited those Yukawa-type interaction terms.) The fermion masses and the mixing angles θ_i in (2.4) are arbitrary; the values assigned to them are phenomenologically determined.[2]

By identifying the coupling of the photon to the electron with the bare electric charge e_0, one finds the relationships

$$e_0=\frac{g_{1_0}g_{2_0}}{(g_{1_0}{}^2+g_{2_0}{}^2)^{1/2}}=g_{1_0}\cos\theta_W^0=g_{2_0}\sin\theta_W^0\,. \tag{2.10}$$

The renormalized value of the electric charge e depends on the renormalization conditions chosen. It is conventional (although not always most convenient) to define the renormalized charge at zero momentum transfer as in Thomson scattering at $q^2=0$; we will refer to that value as $e(0)$. Very precise experimental measurements then yield (here we use the Josephson-effect determination of α)

$$\frac{e^2(0)}{4\pi}=\alpha(0)\simeq(137.035\,987)^{-1}$$
$$\text{(fine-structure constant)}\,. \tag{2.11}$$

The effective value of α relevant for other q^2 ranges will be estimated later on in this section and its implications discussed.

Another phenomenological constraint on the parameters of the theory comes from the muon decay rate, which implies (to lowest order)

$$\frac{G_F}{\sqrt{2}}\simeq\frac{g_2{}^2}{8M_W{}^2}\simeq\frac{e^2}{8M_W{}^2\sin^2\theta_W}\,, \tag{2.12}$$

where G_F is the Fermi constant $\simeq 1.1664\times10^{-5}$ GeV^{-2}. The relationships in (2.12) are valid only to lowest order in perturbation theory because g_2, e, and θ_W (the renormalized values) can change by finite $O(\alpha)$ corrections depending on how they are precisely defined (the renormalization prescription). [Note that from here on we use $O(\alpha)$ and $O(g^2)$ interchangeably to denote the same order-of-magnitude corrections.]

Because of the constraints in (2.10) and (2.12), the three originally independent parameters g_1, g_2, and M_W are related by the phenomenologically determined quantities α and G_F. Hence, there remains only one important free parameter in the theory, and it is conveniently taken to be $\sin^2\theta_W$. Neutral-current experiments can be compared with the predictions of the Weinberg-Salam model, which depend only on $\sin^2\theta_W$, and in this way $\sin^2\theta_W$ is determined (at least to lowest order in α). In order for the Weinberg-Salam model to be correct, all experimental measurements must yield the same value for $\sin^2\theta_W$. The present world average[5] is $\sin^2\theta_W=0.23\pm0.02$ with most individual measurements in the 0.2 to 0.3 range.

As experimental measurements of $\sin^2\theta_W$ become more precise, the effects of radiative corrections will have to be considered. In principle, this can be done exactly (to order α) by computing the full one-loop radiative corrections to the neutral-current processes investigated; of course, in comparing different processes, a single definition of the renormalized value of $\sin^2\theta_W$ should be adhered to. We will now set down two distinct working definitions of $\sin^2\theta_W$ and argue that one of them is better for direct comparison with experiment, if the $O(\alpha)$ radiative corrections are not accounted for separately.

To start with, we employ the following definition for the renormalized value of $\sin^2\theta_W$ [see (2.10)]:

$$\sin^2\theta_W(0)\equiv e^2(0)/g_{2_R}{}^2$$
$$\text{(renormalized value)}\,, \tag{2.13}$$

where $e(0)$ is the very precisely determined value of the renormalized electric charge given in (2.11) and g_{2_R} is a finite renormalized coupling which we will discuss in a moment. The zero in $\sin^2\theta_W(0)$ indicates that the electric charge employed in (2.13) is renormalized at $q^2=0$. In the Weinberg-Salam sequential model as described earlier in this section, the finite renormalized coupling g_{2_R} is related to the bare SU(2) coupling g_{2_0} by the following expression:

$$g_{2_R}=g_{2_0}\left[1-\frac{g_{2_0}{}^2}{16\pi^2}\left(\frac{19}{6}\frac{1}{n-4}+C\right)+O(g_{2_0}{}^4)\right], \tag{2.14}$$

where n is the dimension of space-time (dimensional regularization assumed).[22] The infinite renormalization in (2.14) is required to render the theory finite, while C represents (at this point) arbitrary finite $O(1)$ terms which we take to be independent of all fermion masses.[23] This is to be contrasted with the relationship between $e(0)$ and the bare electric charge[24]

$$e(0)=e_0\left\{1-\frac{e_0^2}{16\pi^2}\left[-\frac{11}{3}\frac{1}{n-4}+\sum_f\frac{4}{3}Q_f^2\ln(M_W/m_f)-\frac{1}{3}\right]+O(e_0^4)\right\}, \tag{2.15}$$

where the sum is over all charged fermions (left- and right-handed components are not counted separately, Q_f = electric charge) with quarks counted three times because of their color. [The mass scale appropriate for light quarks u, d, and s in (2.15) will be discussed later.] Unlike (2.14), the expression in (2.15) contains sizable logarithms which depend on the fermion masses (mass singularities). They can be removed by defining the renormalized electric charge at $q^2 \simeq -M_W^2$ instead of zero (we shall do so later); however, then one loses the precise determination given in (2.11) and can only estimate the magnitude of the new charge $e(M_W)$.

The finite corrections designated by C in (2.14) are still somewhat arbitrary. We could for simplicity choose $C=0$; however, instead we will adopt a specification of C that allows g_{2_R} to be precisely extracted from experiment. Our definition of g_{2_R} involves equating the experimental value for the muon's total decay rate with the theoretical prediction[25,26]

$$\Gamma(\mu\to e\nu\bar{\nu}(\gamma))=\frac{g_{2_R}{}^4 m_\mu{}^5}{3\times 2^{11}\pi^3 M_W{}^4}\left(1-\frac{8m_e{}^2}{m_\mu{}^2}\right)=4.551\,384\times10^5\ \sec^{-1}, \tag{2.16}$$

where (γ) means that we allow photons in the final state, and terms of $O(m_e{}^4/m_\mu{}^4)$ and $O(m_\mu{}^2/M_W{}^2)$ have been neglected. What we have done is absorbed the radiative corrections to muon decay into the definition of g_{2_R} through (2.16). By explicitly computing all $O(\alpha)$ corrections to muon decay in the Weinberg-Salam model and comparing the result with (2.16), using (2.14) as the definition of g_{2R}, one can determine the value of C in (2.14). C determined in this manner is guaranteed to be free of $\ln(M_W/m_f)$ contributions by the Kinoshita-Sirlin theorem[27] for the muon's total decay rate; hence C should not be very large. The actual value of C can be obtained from previous calculations[28,29]; it will depend on M_ϕ/M_W and the other parameters of the theory. Throughout this paper we assume that C defined in the forementioned manner is small [i.e., truly of $O(1)$], so we neglect it. Of course, before g_{2_R} can be accurately determined from (2.16), we need a precise measurement of M_W and that will require its discovery. In any case, using (2.11), (2.13), and (2.16), together we find

$$\sin\theta_W(0)=\frac{37.320\ \text{GeV}}{M_W}, \tag{2.17}$$

so if and when M_W is measured, $\sin\theta_W(0)$ can be very precisely determined using (2.17). This is the reason for our choice of $\sin^2\theta_W(0)$ as an interesting renormalized quantity. [An alternative possibility is to define g_{2_R} using the W-boson decay-rate formula.[30] The value of C determined by this procedure will differ from that outlined above; however, in both cases C is free from $\ln(M_W/m_f)$ terms and should therefore not be very large.[31]]

The question we now address is: What is the relationship between the renormalized quantity $\sin^2\theta_W(0)$ defined in (2.13) and the quoted value for $\sin^2\theta_W$ as determined by neutral-current experiments? To lowest order they are the same; however, they will differ by terms of order α [or more precisely in this case by terms of order $\alpha\ln(M_W/m_f)$]. An exact determination of their $O(\alpha)$ differences would require a computation of the radiative corrections to the neutral-current processes considered and knowledge of the contributions to C in (2.14). Fortunately, we can bypass such laborious calculations if we wish to estimate only what should be the dominant differences, i.e., terms of the form $\alpha\ln(M_W/m_f)$, since they can be obtained very simply by using renormalization-group techniques.[32] [Actually, we will include all corrections of the form $\alpha^n\ln^n(M_W/m_f)$, $n=1,2,\ldots$, the leading logarithmic corrections.]

The effective coupling strength g for any interaction changes as we vary the mass scale considered (or the renormalization point), with the coupling's orbit governed by its β function $\beta(g)$, such that

$$\mu\frac{\partial}{\partial\mu}g=\beta(g)=b_0g^3+b_1g^5+\cdots . \tag{2.18}$$

By employing a $g(\mu)$ determined from (2.18), when μ is the mass scale relevant for a particular process, one can include the effects of logarthmic radiative corrections. That is, by using $g(\mu)$ as the expansion parameter when μ is the relevant mass scale for the process considered, the resulting perturbative expansion will be free from logarithmic corrections, whereas if expressed in terms of $g(\mu')$, $\mu'\neq\mu$, terms of the form $g^2(\mu')\ln(\mu'/\mu)$ will appear in the expansion; hence use of $g(\mu)$ is usually better for comparison with experiment.

Defining the effective quantity

$$\sin^2\theta_W(\mu) = e^2(\mu)/g_2^{\,2}(\mu)\,, \tag{2.19}$$

we can ask, "What is the mass scale, μ, relevant for comparing $\sin^2\theta_W(\mu)$ with experimental determinations of $\sin^2\theta_W$?" For the quantity $\theta_W \simeq \cos^{-1}(M_W/M_Z)$ given in (2.9b), the answer is clearly $\mu \simeq M_Z \simeq M_W$ (we do not distinguish M_Z and M_W as distinct mass scales). In the case of neutral-current experiments at $|q^2| \ll M_W^{\,2}$, one has to examine the higher-order radiative corrections to ascertain the answer, $\mu \simeq |q|$ or $\mu \simeq M_W$ may be the better scale. We note that the renormalized coupling g_{2_R} in (2.14) and (2.16) is defined at $\mu \simeq M_W$, i.e., $g_{2_R} \simeq g_2(M_W)$, because the W boson mass determines the scale in muon decay, even though $q^2 \simeq m_\mu^{\,2}$. For that reason, charge renormalization in (2.15) includes $\ln(M_W/m_f)$ terms, while (2.14) has none. [The quantity $\sin^2\theta_W(0)$ in (2.13) does *not* equal $\sin^2\theta_W(\mu=0)$ in (2.19) because we used g_{2_R} $(=g_2(M_W))$ in the defining Eq. (2.13).]

Throughout the remainder of this paper, we shall employ $\sin^2\theta_W(M_W) = e^2(M_W)/g_2^{\,2}(M_W)$ as the effective quantity to be compared with experiment; however, the reader should keep in mind that some low-energy neutral-current experiments may be determining a somewhat different quantity such as $\sin^2\theta_W(|q|) = e^2(|q|)/g_2^{\,2}(|q|)$. Because of this possibility, we will always assume a theoretical uncertainty of ± 0.01 when comparing $\sin^2\theta_W(M_W)$ with experimental measurements of $\sin^2\theta_W$.

The difference between $\sin^2\theta_W(M_W)$ and $\sin^2\theta_W(0)$ in (2.13) is determined simply by the β function for the electric charge, since as we previously mentioned, g_{2_R} as given in (2.14) and (2.16) is already defined at $\mu \simeq M_W$; hence

$$\sin^2\theta_W(M_W) = \frac{e^2(M_W)}{g_2^{\,2}(M_W)} = \frac{e^2(M_W)}{g_{2_R}^{\,2}}\,, \tag{2.20}$$

with the same g_{2_R} as in (2.13). To estimate $e^2(M_W)$, we will keep only the first term in its β function, [In this way only the leading logarithms, $\alpha^n \ln^n(M_W/m_f)$, $n = 1, 2, \ldots$ are accounted for.]

$$\mu\frac{\partial}{\partial\mu}e = \beta(e) = \frac{1}{12\pi^2}\sum_f Q_f^{\,2} e^3 + O(e^5)\,, \tag{2.21}$$

where the sum is over all fermions in (2.3) with quarks counted three times because of their color (left- and right-handed components are not treated separately). The charged vector bosons do not influence (by very much) the evolution of $e(\mu)$ in the region $0 < \mu < M_W$, hence, they do not contribute to (2.21). Furthermore, a fermion should be included in (2.20) only when $\mu > m_f$. Taking this into account, we find from (2.21)

$$\frac{4\pi}{e^2(M_W)} = \alpha^{-1}(M_W) \simeq \alpha^{-1}(0) - \frac{2}{3\pi}\sum_f Q_f^{\,2}\ln(M_W/m_f)\,. \tag{2.22}$$

The expression in (2.22) is to be evaluated in the following way. For light quarks, u, d, and s their current quark masses should not be used; instead some strong-interaction mass scale $\simeq 0.5$ to 1.0 GeV is more appropriate. Using 1 GeV, the known lepton masses and $m_c \simeq m_b/3 \simeq m_t/9 \simeq 1.5$ GeV along with $M_W \simeq 85$ GeV, we find

$$\alpha^{-1}(M_W) \simeq \alpha^{-1}(0) - \frac{80}{3\pi} \simeq 128.5\,. \tag{2.23}$$

This value is not very sensitive to the actual mass parameters employed and probably represents a good estimate of $\alpha^{-1}(M_W)$; we will use the numerical value in (2.23) throughout this paper and assume that other $O(\alpha)$ corrections that we have neglected do not significantly alter $\sin^2\theta_W$.

From the estimate in (2.23), we find that the value of $\sin^2\theta_W(M_W)$ in (2.20) is related to $\sin^2\theta_W(0)$ by

$$\sin^2\theta_W(M_W) \simeq 1.066\,\sin^2\theta_W(0)\,, \tag{2.24}$$

i.e., it is about 6% or 7% larger. In terms of $\sin\theta_W(M_W)$, (2.17) becomes

$$\sin\theta_W(M_W) \simeq \frac{38.53\ \text{GeV}}{M_W} \tag{2.25a}$$

or

$$M_W \simeq \frac{38.53\ \text{GeV}}{\sin\theta_W(M_W)}\,. \tag{2.25b}$$

Furthermore, from the relationship in (2.9b) we have

$$M_Z \simeq \frac{77.06\ \text{GeV}}{\sin 2\theta_W(M_W)}\,, \tag{2.26}$$

which should not be very sensitive to neglected radiative corrections as long as $M_\phi \simeq M_W$.[33] The results in (2.25b) and (2.26) tell us that M_W and M_Z are about 3% ($\simeq 3$ GeV) heavier than what one usually estimates using (2.17), since $\sin\theta_W(M_W)$ is closer to the experimentally measured quantity than $\sin\theta_W(0)$. If the predictions for M_W and M_Z in (2.25b) and (2.26) are borne out experimentally, it would be a triumph for the Weinberg-Salam model.

Before closing this section, we would like to discuss the prospects for precisely determining $\sin^2\theta_W$. Clearly, the most accurate determination will result from a measurement of M_W used in conjunction with (2.17) or (2.25a). A separate determination of $\sin^2\theta_W$ will become available when both M_W and M_Z are measured; then assuming that there are only scalar isodoublets in the model, one finds from (2.9b)[33,34]

$$\sin^2\theta_W(M_W) = (1 - M_W{}^2/M_Z{}^2)[1 + O(\alpha)]\,, \qquad (2.27)$$

where the $O(\alpha)$ corrections should not be very significant since they do not include $\alpha \ln(M_W/m_f)$ contributions [that is why we used $\sin^2\theta_W(M_W)$ and not $\sin^2\theta_W(0)$ in (2.27), they are related by (2.24)]. In the case of neutral-current processes, neutrino scattering experiments and parity-violating asymmetry measurements in deep-inelastic electron scattering have already yielded values for $\sin^2\theta_W$ with about a 10% experimental uncertainty, and they should do better as more data are analyzed.[4,5] Unfortunately, these types of measurements carry an underlying theoretical uncertainty because they involve strong-interaction effects and hence require theoretical approximations. Some experiments that may eventually yield precise values for $\sin^2\theta_W$ with very little theoretical uncertainty are (1) searches for parity-violating neutral-current effects in hydrogen,[35] (2) ν_μ-e scattering, for which the full $O(\alpha)$ radiative corrections have been calculated,[36] and (3) parity-violating asymmetries in polarized electron scattering on an unpolarized electron target.[37] If and when any experiment does become sophisticated enough to yield a very precise determination of $\sin^2\theta_W$, all of the $O(\alpha)$ radiative corrections will have to be examined, not just those leading logarithms that our analysis includes.[32]

The main conclusion that we will take from this section is that $\sin^2\theta_W(M_W)$ as defined in (2.20) is the more relevant quantity for comparison with experimental neutral-current determinations of $\sin^2\theta_W$, even though $\sin^2\theta_W(0)$ will be the more precisely determined quantity after M_W is measured [via our result in (2.17)]. We should, however, emphasize that even $\sin^2\theta_W(M_W)$ may differ somewhat from any particular experimental determination of $\sin^2\theta_W$ because of the effect of radiative corrections that we have not accounted for by our prescription; this might especially be the case for very low-energy experiments.[32] All things considered, one should expect $\sin^2\theta_W(M_W)$ to agree with most experimental measurements up to an uncertainty of about ± 0.01.

III. $\sin^2\theta_W$ AND GRAND UNIFIED MODELS

As we mentioned in the Introduction, embedding the standard $SU(3)_c \times SU(2) \times U(1)$ model in a grand unified theory based on a simple gauge group G [such as SU(5)] leads to some very interesting consequences.[7-12] A property of these schemes that we find particularly appealing is their elevation of $\sin^2\theta_W^0$ from an arbitrary counterterm parameter to a predicted number. This has to be the case, since in such theories there is only a single gauge coupling constant g_{G_0}; so g_{3_0}, g_{2_0}, and g_{1_0} must be multiples of g_{G_0} and $\tan\theta_W^0 = g_{1_0}/g_{2_0}$ must be a pure number determined by the group-theoretic structure of the model considered. To further illustrate this feature, let us explicitly exhibit the relationship [obtained from (2.14) and (2.15)] between the renormalized quantity $\sin^2\theta_W(0)$ defined in (2.13) and $\sin^2\theta_W^0$ in the Weinberg-Salam model when considered alone (i.e., before embedding it in G),[38]

$$\sin^2\theta_W(0) = \sin^2\theta_W^0\left\{1 + \frac{e_0{}^2}{8\pi^2}\left[\left(\frac{11}{3} + \frac{19}{6\sin^2\theta_W^0}\right)\frac{1}{n-4} + \text{finite terms}\right] + O(e_0^4)\right\}, \qquad (3.1)$$

where the finite terms are those discussed in the previous section. [Equation (3.1) includes contributions from the $SU(2) \times U(1)$ gauge bosons, one Higgs doublet, and the fermions listed in (2.3).] The ultraviolet divergence in (3.1) points out $\sin^2\theta_W^0$'s role as an infinite counterterm parameter [remember $\sin^2\theta_W(0)$ is finite]. In grand unified theories, $\sin^2\theta_W^0$ is a pure number, so the divergence in (3.1) must be cancelled by other ultraviolet-divergent contributions coming from new particles introduced by enlarging the theory. There will remain, however, sizable finite calculable radiative corrections which can cause $\sin^2\theta_W(0)$ [and the experimentally relevant $\sin^2\theta_W(M_W)$ defined in (2.20)] to differ significantly from $\sin^2\theta_W^0$. The existence of these large finite renormalization effects, along with an estimate of their size, was pointed out in the work of Georgi, Quinn, and Weinberg.[10] In this section we re-evaluate those corrections and discuss possible sources of uncertainty in our estimates; the results presented should be viewed as modest refinements of the earlier calculations.[10-12]

To begin our analysis, we need only assume that the grand unified theory (based on the simple group G) contains the Weinberg-Salam $SU(2) \times U(1)$ model as a subtheory such that the electric charge generator is still given by $Q = T_3 + Y/2$ as in (2.1). [Other assumptions will be given as they are needed. Note also that in our approach the QCD

$SU(3)_c$ subgroup plays no essential role.] Then the predicted value of $\sin^2\theta_W^0$ can be found if we know the values of Q and T_3 for the members of any irreducible representation R belonging to G; it is given by[39]

$$\sin^2\theta_W^0 = \frac{\sum_{i\in R} T_{3i}^{\ 2}}{\sum_{i\in R} Q_i^{\ 2}}$$

$$\text{(sum over all members of } R). \qquad (3.2)$$

Of course the same value of $\sin^2\theta_W^0$ must be obtained for any R in the model, whether it contains fermions, scalars, or gauge bosons; likewise, if we sum over several R's in the numerator and denominator of (3.2) the same $\sin^2\theta_W^0$ will be found. To illustrate how (3.2) works, consider the possibility that all leptons and quarks come in sequential doublets and singlets as described in (2.3). Then using the values of T_3 and Q in (2.3) gives (left- and right-handed components included separately)

$$\sin^2\theta_W^0 = \frac{12(\frac{1}{2})^2+12(-\frac{1}{2})^2}{6(-1)^2+18(\frac{2}{3})^2+18(-\frac{1}{3})^2} = \frac{3}{8}\,. \qquad (3.3)$$

This situation is realized in, for example, the Georgi-Glashow SU(5) model.[7] Note that the gauge bosons or Higgs scalars of the Weinberg-Salam model alone do *not* give $\sin^2\theta_W^0$ equal to $\frac{3}{8}$ in (3.2); this is because there must exist other gauge fields and scalars which complete their SU(5) multiplets such that (3.3) is realized. One thing that we learn from (3.3) is that any grand-unified model with $\sin^2\theta_W^0 \neq \frac{3}{8}$ requires the existence of somewhat exotic fermions [i.e., fermions outside the sequential scheme of (2.3)].

What is the difference between $\sin^2\theta_W^0$ as given in (3.2) and the effective value of $\sin^2\theta_W$ measured in present-day neutral-current experiments? The answer is that they differ by finite calculable renormalization effects. As pointed out in the previous section, neutral-current experiments approximately measure the effective value of $\sin^2\theta_W(\mu) = e^2(\mu)/g_2^{\ 2}(\mu)$ at $\mu \simeq M_W$, whereas $\sin^2\theta_W^0$ is the asymptotic limit approached as $\mu\to\infty$. We now calculate the relationship between $\sin^2\theta_W(M_W)$ and $\sin^2\theta_W^0$ using the renormalization-group approach of Georgi, Quinn, and Weinberg.[10]

We assume that the gauge group G is spontaneously broken down to the standard $SU(3)_c\times SU(2)\times U(1)$ model such that all vector bosons and scalar fields which are *not* part of pure QCD or the Weinberg-Salam model acquire about the same superheavy mass $M_S \gg M_W$. In the case of fermions, we assume that all fermions that belong to sequential multiplets as in (2.3) have fairly light masses $\lesssim M_W$; but we leave open the possibility that there are exotic fermions with masses $\simeq M_S$ which are outside of the sequential scheme (they could give rise to $\sin^2\theta_W^0 \neq \frac{3}{8}$). Because there exist two very different boson mass scales, M_W and M_S, the effective couplings $g_3(\mu)$, $g_2(\mu)$, and $g_1(\mu)$ evolve quite differently as μ varies from M_S to $\simeq 0$, even though $g_3(\mu)=g_2(\mu)=g_1(\mu)\cot\theta_W^0$ as $\mu\to\infty$. Their behavior below $\mu\simeq M_S$ is governed by their respective effective β functions, which do *not* include contributions from superheavy particles (they decouple) and hence can differ appreciably. For any gauge coupling constant g_i in the theory, we have[40]

$$\mu\frac{\partial}{\partial\mu}\,g_i = \beta(g_i) = (b_B+b_H+b_f)\,g_i^{\ 3} + O(g_i^{\ 5})\,, \qquad (3.4)$$

where we have divided up the contributions from gauge bosons, Higgs scalars, and fermions into b_B, b_H, and b_f, respectively. In our analysis, we keep only the first term in the β function, this means that we include all leading logarithm corrections, i.e., those of the form $[g_i^{\ 2}\ln(\mu_1/\mu_2)]^n$, $n=1,2,\ldots$. We also use the fact that a single asymptotic coupling g_G is approached as $\mu\to\infty$.

To determine the value of $\sin^2\theta_W(M_W)$, we consider the evolution of $g_2(\mu)$ and $e(\mu)$ separately and find from their $SU(2)\times U(1)$ β functions [using (3.4)]

$$g_2^{\ -2}(M_W) \simeq g_G^{\ -2} + 2(b_B+b_H+b_f)\ln(M_S/M_W)\,, \qquad (3.5a)$$

where

$$b_B = -\frac{1}{16\pi^2}\,\frac{22}{3}\,, \qquad (3.5b)$$

$$b_H = \frac{1}{16\pi^2}\,\frac{1}{6}\,N_H\,, \qquad (3.5c)$$

$$b_f = \frac{1}{16\pi^2}\,\frac{2}{3}\,N_f\,, \qquad (3.5d)$$

and

$$e^{-2}(M_W) \simeq \frac{1}{\sin^2\theta_W^0}\,g_G^{\ -2} + 2(b_B'+b_H'+b_f')\ln(M_S/M_W)\,, \qquad (3.6a)$$

where

$$b_B' = b_B = -\frac{1}{16\pi^2}\,\frac{22}{3}\,, \qquad (3.6b)$$

$$b_H' = 2b_H = \frac{1}{16\pi^2}\,\frac{1}{3}\,N_H\,, \qquad (3.6c)$$

$$b_f' = \frac{8}{3}\,b_f = \frac{1}{16\pi^2}\,\frac{16}{9}\,N_f\,. \qquad (3.6d)$$

We have allowed for N_H complex Higgs doublets (all physical scalars resulting from these are assumed to have masses $\approx M_W$) and N_f sequential fermion flavors in the above expressions ($N_H=1$, $N_f=6$ in Sec. II). The quantities b and b' are related in the following way:

$$b' = \frac{\sum_i Q_i^2}{\sum_i T_{3i}^2}\, b\,, \tag{3.7}$$

where the sum is over all vector bosons, scalars, or fermions (left and right components counted separately) with masses $\lesssim M_W$ (i.e., no superheavy particles included). If there are no superheavy fermions, but there are light exotic ones which do not fall into the sequential scheme of (2.3) so $\sin^2\theta_W^0 \neq \frac{3}{8}$, then (3.5d) and (3.6d) are both changed; but they still satisfy

$$b_f' = \frac{1}{\sin^2\theta_W^0}\, b_f\,. \tag{3.8}$$

We will also consider this possibility in our following analysis.

Combining (3.5) and (3.6) we find

$$g_2^{-2}(M_W) = \sin^2\theta_W^0 e^{-2}(M_W) + 2[b_B \cos^2\theta_W^0 + b_H(1-2\sin^2\theta_W^0) + b_f - b_f' \sin^2\theta_W^0]\ln(M_S/M_W)\,. \tag{3.9}$$

[Notice that fermions drop out of (3.9) if (3.8) is satisfied.] From this expression we obtain [using (3.5) and (3.6)]

$$\sin^2\theta_W(M_W) = \frac{e^2(M_W)}{g_2^2(M_W)} = \sin^2\theta_W^0\left\{1 - \frac{\alpha(M_W)}{2\pi}\left[\tfrac{22}{3}\cot^2\theta_W^0 - \tfrac{1}{6}N_H\left(\frac{1}{\sin^2\theta_W^0} - 2\right)\right]\ln\frac{M_S}{M_W}\right\},$$
$$\alpha(M_W) = \frac{e^2(M_W)}{4\pi} \simeq (128.5)^{-1} \quad \text{[see (2.23)]} \tag{3.10a}$$

when (3.8) is satisfied (no superheavy fermions), while

$$\sin^2\theta_W(M_W) = \sin^2\theta_W^0\left\{1 - \frac{\alpha(M_W)}{2\pi}\left[\tfrac{22}{3}\cot^2\theta_W^0 - \tfrac{1}{6}N_H\left(\frac{1}{\sin^2\theta_W^0} - 2\right) - \tfrac{2}{3}N_f\left(\frac{1}{\sin^2\theta_W^0} - \frac{8}{3}\right)\right]\ln\frac{M_S}{M_W}\right\}, \tag{3.10b}$$

if $\sin^2\theta_W^0 \neq \frac{3}{8}$ and there are superheavy exotic fermions. We will not consider the possibility in (3.10b) any further in this paper.

Equation (3.10) represents the major result of this section. Let us examine some of its features: (1) Since our analysis included all leading logarithms, the next-order logarithmic corrections to (3.10) are $O(\alpha^2\ln(M_S/M_W))$ *not* $O(\alpha^2\ln^2(M_S/M_W))$ and hence are about the same magnitude as neglected nonlogarithmic $O(\alpha)$ corrections. (2) For $\sin^2\theta_W^0 < \frac{1}{2}$, adding more isodoublet scalars, which give rise to physical scalar particles with masses $\simeq M_W$, tends to reduce the radiative corrections in (3.10). (3) $\alpha(M_W)$, which is somewhat larger that the usual fine-structure constant, appears in our expression. This last point implies a degree of uncertainty in comparing (3.10) with experiment, since direct examination of the renormalization of $\sin^2\theta_W$ in grand unified models[37] seems to suggest that $\alpha(|q|)$, where q^2 is the momentum transfer for the process considered, may appear in the coefficient of $\ln(M_S/M_W)$. Of course a precise determination of this coefficient requires a full two-loop calculation which we have not done. This is in keeping with our comment at the end of the last section that we anticipate an uncertainty of about ±0.01 in comparing $\sin^2\theta_W(M_W)$ with experiment.

As an alternative derivation of our result, consider the case $\sin^2\theta_W^0 = \frac{3}{8}$ and $N_H=1$, so that (3.10a) becomes

$$\sin^2\theta_W(M_W) = \frac{3}{8}\left(1 - \frac{\alpha(M_W)}{2\pi}\,\frac{109}{9}\ln(M_S/M_W)\right). \tag{3.11}$$

This result could have been simply obtained using (3.1). For $\sin^2\theta_W^0 = \frac{3}{8}$, the coefficient of the divergence in (3.1) becomes $109\alpha/(18\pi)$, the same as the coefficient of the logarithm in (3.11) (but with opposite sign). In the dimensional regularization prescription a $\ln M$ always accompanies a divergence; so light particles ($\simeq M_W$) and heavy particles ($\simeq M_S$) renormalize $\sin^2\theta_W^0$ in the following way:

$$\sin^2\theta_W = \frac{3}{8}\left[1 + \frac{109\alpha}{18\pi}\left(\frac{1}{n-4} + \ln M_W\right) - \frac{109\alpha}{18\pi}\left(\frac{1}{n-4} + \ln M_S\right) + O(\alpha)\right], \tag{3.12}$$

where the second term in the square brackets comes from light particles and the third term from superheavy particles. The divergences from light and heavy particles cancel, and we are left with the result in (3.11); this is a check on our previous calculation. [Note that (3.12) does not specify whether $\alpha(|q|)$ or $\alpha(M_W)$ is the better expansion parameter. That depends on the experimental situation and its corresponding two-loop corrections.]

Accepting (3.10) as a reliable estimate of the finite renormalization of $\sin^2\theta_W^0$ in grand unified gauge theories (with an uncertainty of about ± 0.01), we now go on to apply this result to the SU(5) Georgi-Glashow model.[7]

IV. SU(5) GEORGI-GLASHOW MODEL

The SU(5) model of Georgi and Glashow[7] is the earliest, most economical and perhaps most carefully studied example of a grand unified gauge theory. It exhibits many nice features[7-12] which we feel give it a degree of credibility, and so it seems worthwhile to study some of its properties in more detail than has been done previously. In that regard, we will apply our analysis of $\sin^2\theta_W$ from the previous sections to this model and comment on the relationship between the experimentally observed value of $\sin^2\theta_W$ and the predicted lifetime of the proton in this theory.

The SU(5) model is minimal in that it requires only the fermions in (2.3). They make up three generations (six flavors), each of which is composed of a $\underline{5}+\underline{10}$ representation of SU(5). Merely knowing the fermion content of this model implies from (3.2) and (3.3)

$$\sin^2\theta_W^0 = \tfrac{3}{8} \quad [\text{SU(5) model}]\,, \tag{4.1}$$

a property already noted.

There are 24 gauge bosons in the SU(5) model. Twelve of these are rather exotic in that they carry color and have electric charges $\pm\frac{1}{3}$ and $\pm\frac{4}{3}$. They mediate phenomena such as proton decay; however, the predicted rate for this (as yet unobserved) process is extremely small because the masses of these fractionally charged bosons are extremely heavy. We assume that all such bosons have about the same superheavy mass $M_S \gg M_W$; they obtain that mass as a result of the spontaneous breaking of the gauge symmetry from SU(5) to $\text{SU(3)}_c\times\text{SU(2)}\times\text{U(1)}$ via a real 24-plet of Higgs scalars. The second step in the symmetry breakdown is due to N_H complex 5-plets of Higgs scalars (usually $N_H=1$; however, we leave it arbitrary) which leave only an $\text{SU(3)}_c\times\text{U(1)}$ local gauge symmetry unbroken.[41] In this way, $W^\pm$ and Z acquire masses M_W and M_Z, while the eight gluons of QCD and the photon of QED remain massless. The masses of the physical scalar particles in this model are rather arbitrary. We shall assume that all physical scalars which originate from the 24-plet and all fractionally charged colored scalars coming from the 5-plets have superheavy masses $\simeq M_S$. The remaining scalars which come from the SU(2) isodoublet parts of the 5-plets (these are just like ordinary Weinberg-Salam doublets) are taken to have masses $\simeq M_W$.

Because there are two very different mass scales M_S and M_W in this model, the value of $\sin^2\theta_W(M_W)$ relevant for present-day experiments will be renormalized away from its zeroth-order value of $\frac{3}{8}$ owing to radiative corrections. From our previous result in (3.10) and the assumptions regarding vector and scalar boson mass scales given above, we find

$$\sin^2\theta_W(M_W) \simeq \frac{3}{8}\left[1-\frac{\alpha(M_W)}{2\pi}\left(\frac{110-N_H}{9}\right)\ln(M_S/M_W)\right]. \tag{4.2}$$

This compact expression represents a refinement of the Georgi-Quinn-Weinberg calculation[10,42] in the following ways: (1) The coefficient of the correction in (4.2) depends on $\alpha(M_W)$, which is about 6.6% larger than the usual fine-structure constant. [However, as previously mentioned, for processes at $|q^2|\ll M_W{}^2$, $\sin^2\theta_W$ with $\alpha(M_W)$ replaced by $\alpha(|q|)$ may be more appropriate for comparison with some experiments[43]]. (2) Our result demonstrates the dependence of the correction on the number of SU(2) scalar isodoublets N_H. Notice that the effect of these scalars can be substantial if N_H is fairly large. (3) The logarithm in (4.2) depends on the mass ratio M_S/M_W. We assert on the basis of explicit one-loop calculations that this quantity, rather than something like $10M_S/M_W$, is the relevant argument of the logarithm.[43] A source of some uncertainty in our calculation is the assumption that all superheavy particles have exactly the same mass M_S and all ordinary bosons have mass M_W. Variations from this idealized situation would alter our result somewhat. (We could correct for this effect if we knew all particles' masses.) Let us also note that if all physical scalars actually have mass M_S, that is if we promote the isodoublet scalars to superheavy status, then the coefficient of the logarithm in (4.2) increases from $(110-N_H)/9$ to $\frac{439}{36}$. However, a word of caution, in that case there may be additional large corrections of $O(\alpha\ln(M_S/M_W))$ which have to be separately included in phenomenological analyses of neutral-current processes.[33]

Let us consider some of the implications of

(4.2). For definiteness, take $N_H=1$ and use the estimate $\alpha(M_W)\simeq(128.5)^{-1}$ found in (2.23); in that case (4.2) becomes

$$\sin^2\theta_W(M_W)\simeq\tfrac{3}{8}[1-0.015\ln(M_S/M_W)]\,. \qquad (4.3)$$

This is the result quoted in our introduction. [We estimate an uncertainty of about ± 0.01 in comparing (4.3) with present-day experimental results.] An interesting interpretation of (4.3) is that an experimental determination of $\sin^2\theta_W(M_W)$ actually determines the value of M_S through the expression in (4.3) [using (2.25b)]. Neutral-current experiments are observing superheavy boson effects.[37]

Before proceeding with our analysis of $\sin^2\theta_W(M_W)$ and its implications, let us mention its connection with the predicted lifetime of the proton. Since the superheavy bosons in the SU(5) model violate baryon-number conservation, they can mediate decays such as $p^+\to e^+ +\pi^0$, $p^+\to\nu_e+\pi^+$, etc. It was estimated in Refs. 12 and 14 that taking into account all possible decay modes and including the effects of enhancement factors, the predicted lifetime of the proton, τ_p, in the SU(5) model has the following dependence on M_S (Refs. 12, 14):

$$\tau_p\simeq 2\times 10^{-29}(M_S \text{ in GeV})^4 \text{ yr}\,. \qquad (4.4)$$

Although we guess that the uncertainty in this estimate is at least a factor of $10^{\pm 1}$, we shall accept its validity for the subsequent analysis.

Now that we have the formulas in (4.3) and (4.4) along with the relationship $M_W\simeq[38.53/\sin\theta_W(M_W)]$ GeV in (2.25b), we can determine the values of M_W, M_S, and τ_p that correspond to any value of $\sin^2\theta_W(M_W)$. [Remember, we contend that $\sin^2\theta_W(M_W)$ is a good approximation to the quantity $\sin^2\theta_W$ measured experimentally, uncertainty $\simeq\pm 0.01$.] To illustrate this point, we have listed in Table I the values of M_W, M_S, and τ_p for $\sin^2\theta_W(M_W)$ in the range 0.170 to 0.240, using increments of 0.005. Notice that M_S decreases by about 50% for each increase in $\sin^2\theta_W(M_W)$ by 0.005, a very sensitive dependence. If we were to accept the present average[5] of $\sin^2\theta_W=0.23\pm 0.02$, it would suggest that $M_S\leqslant 5\times 10^{14}$ GeV and the proton lifetime is not so very long $\tau_p\lesssim 10^{30}$ yr. However, allowing for the previously mentioned uncertainty of ± 0.01 between $\sin^2\theta_W(M_W)$ and experimental values for $\sin^2\theta_W$, these become $M_S\lesssim 3\times 10^{15}$ GeV and $\tau_p\lesssim 10^{33}$ yr. The theoretical uncertainty in $\sin^2\theta_W$ becomes a factor-of-10^3 uncertainty in τ_p. Past experimental searches for proton decay have yielded the approximate bound[13,14,44]

$$\tau_p\gtrsim 2\times 10^{29} \text{ yr}\,. \qquad (4.5)$$

Using (4.4), this translates into a bound on the superheavy mass scale

$$M_S\gtrsim 3\times 10^{14} \text{ GeV}\,, \qquad (4.6)$$

which in turn implies, through (4.3), the following bound on $\sin^2\theta_W(M_W)$:

$$\sin^2\theta_W(M_W)\lesssim 0.212\,. \qquad (4.7)$$

So if we accept (4.7) and the world average for $\sin^2\theta_W$, we should expect in the SU(5) model the following consequences: $\sin^2\theta_W(M_W)\simeq 0.21-0.20$, $M_W\simeq 84-86$ GeV, $M_S\simeq(5-30)\times 10^{14}$ GeV, and $\tau_p\simeq 10^{30}-10^{33}$ yr. This prediction for τ_p is temptingly close to the experimental bound in (4.5).

Recently, new proposals to search for proton decay have been put forward.[45] Such experiments should be capable of observing this decay if $\tau_p\lesssim 2\times 10^{34}$ yr. Within the framework of SU(5), our formulas in (4.3) and (4.4) predict the following outcomes: If $\sin^2\theta_W(M_W)$ lies in the range 0.196 to 0.212, they should be able to detect proton decay, since

$$2\times 10^{29}\lesssim\tau_p\lesssim 2\times 10^{34} \text{ yr} \qquad (4.8a)$$

for

$$0.212\gtrsim\sin^2\theta_W(M_W)\gtrsim 0.196\,. \qquad (4.8b)$$

On the other hand, if $\sin^2\theta_W(M_W)$ is less than 0.196 (which corresponds to $M_S>5.6\times 10^{15}$ GeV), the proton's lifetime is predicted to be too long for these experiments to detect its decay.

Perhaps at this point we should make some correspondence between our findings and the results of early SU(5)-model calculations.[10-12] First of all, we note that our values for M_S are smaller

TABLE I. Values for M_W (W-boson mass), M_S (superheavy mass scale), and τ_p (proton lifetime) corresponding to $\sin^2\theta_W(M_W)$ in the range 0.170 to 0.240 predicted by the SU(5) Georgi-Glashow model (Ref. 7). The values quoted were found using Eqs. (2.25b), (4.3), and (4.4).

$\sin^2\theta_W(M_W)$	M_W (GeV)	M_S (GeV)	τ_p (yr)
0.170	93.4	6.3×10^{17}	3×10^{42}
0.175	92.1	2.5×10^{17}	8×10^{40}
0.180	90.8	1.0×10^{17}	2×10^{39}
0.185	89.6	4.2×10^{16}	6×10^{37}
0.190	88.4	1.7×10^{16}	2×10^{36}
0.195	87.3	6.9×10^{15}	5×10^{34}
0.200	86.2	2.8×10^{15}	1×10^{33}
0.205	85.1	1.1×10^{15}	3×10^{31}
0.210	84.1	4.6×10^{14}	9×10^{29}
0.215	83.1	1.9×10^{14}	3×10^{28}
0.220	82.1	7.6×10^{13}	7×10^{26}
0.225	81.2	3.1×10^{13}	2×10^{25}
0.230	80.3	1.3×10^{13}	6×10^{23}
0.235	79.5	5.1×10^{12}	1×10^{22}
0.240	78.7	2.1×10^{12}	4×10^{20}

than the results of those early calculations[11,12] because of our refinements in (4.2), particularly, our use of $\alpha(M_W)$. This also translates into smaller values for τ_p [for $\sin^2\theta_W(M_W)$ fixed]. Numerically, this difference means that for $\sin^2\theta_W(M_W)=0.200$, our estimate for M_S is about $\frac{1}{6}$ of the earlier value and τ_p is $(\frac{1}{6})^4 \simeq 8\times10^{-4}$ times the earlier predictions. Essentially, the same conclusion (regarding the reduction of M_S and τ_p) was reached by Ross[46]; but his analysis was totally different from ours. We must note, however, that allowing for the possible uncertainty of ±0.01 between our $\sin^2\theta_W(M_W)$ and the experimental quantity could bring our values for M_S and τ_p closer to the earlier results.[11,12] In any case, the SU(5) prediction that $10^{30} \lesssim \tau_p \lesssim 10^{33}$ yr for $0.21 \gtrsim \sin^2\theta_W(M_W) \gtrsim 0.20$ should provide some impetus for proton-decay searches.

A distinction between our analysis and that of earlier calculations[10-12] is that we have ignored the strong-interaction (QCD) sector of the SU(5) model (we compared weak and electromagnetic couplings), while earlier work concentrated on comparing strong and electromagnetic couplings. Their approach provides a rather severe constraint on M_S; whereas, we leave M_S as a free parameter determined by $\sin^2\theta_W(M_W)$ as input. The results obtained by those early calculations[12,47] imply that M_S lies in the range 10^{15}–10^{16} GeV, a finding consistent with the proton stability bound in (4.6). If we borrow these independently determined bounds on M_S, then we find from (2.25b), (4.3), and (4.4)

$$0.206 \gtrsim \sin^2\theta_W(M_W) \gtrsim 0.193\,, \tag{4.9a}$$

$$85 \lesssim M_W \lesssim 88 \text{ GeV}\,, \tag{4.9b}$$

$$2\times10^{31} \lesssim \tau_p \lesssim 2\times10^{35} \text{ yr}\,, \tag{4.9c}$$

for $10^{15} \lesssim M_S \lesssim 10^{16}$ GeV. These are very tight constraints. They are consistent with our earlier analysis using proton stability and the world average $\sin^2\theta_W \simeq 0.23\pm0.02$ as input, which implied $0.21 \gtrsim \sin^2\theta_W(M_W) \gtrsim 0.20$. We should also note that new preliminary results from SLAC[48] give a value $\sin^2\theta_W(M_W)=0.224\pm0.012$. This finding suggests that $\sin^2\theta_W(M_W)$ may be rather close to the bound 0.212 in (4.7) and consequently τ_p may be very near the experimental bound $\tau_p \simeq 2\times10^{29}$ yr. This new SLAC result should provide additional motivation for proton decay searches.

Before closing this section, we will comment on the consequences of enlarging the Higgs sector of the theory such that $N_H>1$ in (4.2). In that case the radiative corrections to $\sin^2\theta_W(M_W)$ are reduced. Retaining the constraint[12,47] $10^{15} \lesssim M_S \lesssim 10^{16}$ GeV, we find that the predicted range of values for $\sin^2\theta_W(M_W)$ in the SU(5) model increases

TABLE II. The ranges of values for $\sin^2\theta_W(M_W)$ and M_W predicted in the SU(5) model for M_S in the domain 10^{15}–10^{16} GeV, as a function of N_H, the number of isodoublet Higgs scalars. The numbers quoted were obtained using Eqs. (4.2) and (2.25b). For all cases, τ_p is predicted to be between 2×10^{31} and 2×10^{35} yr.

N_H	$\sin^2\theta_W(M_W)$	M_W (GeV)
1	0.206–0.193	84.9–87.7
2	0.207–0.195	84.7–87.3
5	0.212–0.200	83.7–86.2
10	0.220–0.208	82.1–84.5
15	0.228–0.217	80.7–82.7
20	0.236–0.225	79.3–81.2
25	0.244–0.234	78.0–79.7
30	0.251–0.242	76.9–78.3

as we increase N_H. The dependence of $\sin^2\theta_W(M_W)$ and consequently M_W on N_H is illustrated in Table II for a variety of N_H values. We see that although for $N_H=1$, $\sin^2\theta_W(M_W)$ is predicted to lie in the range 0.193 to 0.206, its allowed values can be increased somewhat by increasing N_H. Of course, since Higgs scalars are often considered the ugliest feature of gauge theories, one might prefer to have as few multiplets as possible; that is why we concentrated most of our analysis on the minimal situation $N_H=1$. The main point of Table II is to illustrate the fact that the SU(5) model can be altered so as to tolerate somewhat larger values of $\sin^2\theta_W(M_W)$ than the range 0.193 to 0.206 previously allotted to it.

V. DISCUSSION

We would like to conclude this paper with a discussion of our results and their implications. In so doing we will summarize the main conclusions of the previous sections.

We found that in the Weinberg-Salam model the renormalized quantity $\sin^2\theta_W(0)$ defined in (2.13) can be very precisely determined through the relationship $\sin\theta_W(0)=37.320/M_W$. All that is required is an accurate measurement of the W boson's mass M_W. We also noted that $\sin^2\theta_W(0)$ is probably somewhat smaller than the effective value of $\sin^2\theta_W$ presently measured in neutral-current experiments. In that regard, we argued that $\sin^2\theta_W(M_W)$ as defined in (2.20) is a better quantity to compare with experiment, and estimated that $\sin^2\theta_W(M_W)\simeq1.066\sin^2\theta_W(0)$. The difference between these is due to radiative corrections of the form $\alpha\ln(M_W/m_f)$. As a result of this analysis, we estimated that M_W and M_Z are probably about 3 GeV heavier than the values usually assigned to them. The Weinberg-Salam model predicts $M_W \simeq 38.53/\sin\theta_W(M_W)$ GeV and $M_Z \simeq 77.06/\sin 2\theta_W(M_W)$ GeV. So, an experimental value of $\sin^2\theta_W(M_W)$ in the world-average range[5] 0.21–0.25 implies $M_W \simeq 77-84$ GeV and $M_Z \simeq 89-95$ GeV.

In Sec. III we presented a rather general analysis of the (finite) renormalization of $\sin^2\theta_W$ in grand unified gauge theories. Our main result, the compact expression for the corrections to $\sin^2\theta_W^0$ in (3.10), is applicable to a fairly large class of grand unified models. Its main drawback is that it assumes there are only two distinct mass scales M_S, the superheavy mass, and M_W, the ordinary vector-boson mass, in the theory. If we somehow knew the actual values of all masses in a given theory, we could easily correct this deficiency.

When our general results were applied to the SU(5) model of Georgi and Glashow,[7] we found that $\sin^2\theta_W$, which has the zeroth-order value $\sin^2\theta_W^0 = \frac{3}{8}$, was renormalized to a much smaller value, a consequence originally pointed out by Georgi, Quinn, and Weinberg.[10] For the case of a single Higgs SU(2) isodoublet ($N_H = 1$), we found that the value of M_S corresponding to $\sin^2\theta_W(M_W) = 0.20$ was about $\frac{1}{6}$ of the value obtained by earlier estimates.[12] This finding agrees with results obtained by Ross,[44] through a totally different type of analysis. We also estimated that our $\sin^2\theta_W(M_W)$ could differ from the value of $\sin^2\theta_W$ currently measured experimentally by about ± 0.01. Our results when applied to the question of the proton's lifetime in the SU(5) model lead to the following sensitive relationship between τ_p and $\sin^2\theta_W(M_W)$:

$$\tau_p \simeq 1\times 10^{33}[0.200/\sin^2\theta_W(M_W)]^2 \times \exp\{-711[\sin^2\theta_W(M_W) - 0.200]\}\ \mathrm{yr}. \qquad (5.1)$$

This prediction implies that τ_p will lie in the experimentally observable domain[45] $\tau_p \lesssim 2\times 10^{34}$ yr, if $\sin^2\theta_W(M_W)$ is greater than 0.196 and that it will satisfy the present bound $\tau_p \gtrsim 2\times 10^{29}$ yr if $\sin^2\theta_W(M_W) \lesssim 0.212$. The present world average,[5] $\sin^2\theta_W = 0.23 \pm 0.02$, along with the new SLAC results[48] suggest that $\sin^2\theta_W(M_W)$ may be in the range 0.21–0.20 and correspondingly $\tau_p \simeq 10^{30}-10^{33}$ yr. This prediction should provide further impetus for proton-decay searches.

Previous analyses of the strong-interaction sector in the SU(5) model indicated that M_S was in the range $10^{15}-10^{16}$ GeV.[12,47] When we borrow this result, we find that $0.193 \lesssim \sin^2\theta_W(M_W) \lesssim 0.206$, a rather severe constraint, but one that agrees well with the limitations found from proton-stability considerations. All things considered, we should expect $\sin^2\theta_W(M_W)$ to lie in the range 0.19 to 0.21 and $\sin^2\theta_W$ (experimental) $\lesssim 0.22$ if the SU(5) model is valid.

One possibility which detracts somewhat from the precise predictability of the SU(5) model is the effect of enlarging the Higgs content of the theory. Although we have no good reason for increasing the number of scalar isodoublets beyond $N_H = 1$ (unless perhaps one wants a calculable Cabibbo angle[49]), we find that for a given value of M_S, increasing N_H can lead to a larger SU(5) prediction for $\sin^2\theta_W(M_W)$ and consequently a smaller M_W. However, as illustrated in Table II, N_H must be large before this effect becomes substantial. Furthermore, even this effect goes away if all physical scalars have superheavy mass M_S rather than some having mass M_W as we have assumed. (Although superheavy masses for some of the scalars may produce other more several problems.[33])

In closing, we would like to re-emphasize the importance of precisely determining $\sin^2\theta_W$ by as many distinct methods as possible. Such measurements will test the Weinberg-Salam model at the level of its radiative corrections and provide a severe constraint for grand unified gauge theories.

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[41]For a detailed discussion of the Higgs mechanism in the SU(5) model and its consequences, see Ref. 12. We note that in this theory there is an isotriplet part of the 24 which gets a vacuum expectation value and thus explicitly breaks the natural mass relationship in (2.9a).

[42]The results of Georgi, Quinn, and Weinberg in Ref. 10 can be manipulated to look like (4.2) with the coefficient of the logarithm being $55\alpha/9\pi$ (see Ref. 37).

[43]In Ref. 37 it was pointed out that the correction to $\sin^2\theta_W^0$ in (4.2) come from charge radii diagrams (e.g., γ-Z mixing diagrams). This interpretation provides us with a simple procedure for explicitly computing the correction in (4.2). A preliminary analysis confirms the correctness of (4.2), but indicates that for comparison with very-low-energy experimental determinations of $\sin^2\theta_W$, $\alpha(|q|)$ may be more appropriate than $\alpha(M_W)$ in this expression.

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