PHYSICAL REVIEW D VOLUME 20, NUMBER 11 1 DECEMBER 1979

Existence of the West β correction

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(Received 18 December 1978)

We study the smearing effects, due to the nucleon's Fermi motion inside the deuteron, in a covariant way by using the Brodsky parametrization method and neglecting the spin of the particles involved. We find that West's β correction is absent in the high-energy limit for hadronic scattering, but it is still there for leptonic scattering in the deep-inelastic limit. The form of the correction for the leptonic case is, however, different in our calculations from that given by West. We show explicitly that our approach is consistent and avoid some of the approximations made in the earlier contribution to this problem.

I. INTRODUCTION

The deuteron is used as a neutron target for two reasons. Neutron beams are difficult to work with, and free-neutron targets do not exist. There has in fact been considerable progress in the past few years in the use of high-energy neutron beams, but only neutron-proton and neutron-nucleus reactions can be studied in this way. Reactions involving a neutron and any non-nuclear particles, such as pions and electrons, are best studied using deuteron targets, at least until the time that colliding-secondary-beam techniques are developed. Since the deuteron binding energy is small (~ 2.2 MeV) it is tempting, especially in the asymptotic region, to write the deuteron cross section as the sum of the free-nucleon cross sections (apart from the shadow correction, etc.) $\sigma_d = \sigma_p + \sigma_n$. Intuitively one does not expect the 2.2-MeV binding energy to be significant in a region where the energy scale is many GeV.

However, there is another important correction, the smearing correction due to the fact that the bound nucleons undergo Fermi motion (notice in electron scattering off deuteron that this correction is more important than the shadow correction, which is usually neglected). The fact that the target nucleons are moving affects the cross sections in two distinct ways:

1. The total center-of-mass energy seen by the constituent nucleons is shifted by a "Doppler effect."

2. The flux of incident particles in the rest frame of the moving nucleon is different from that in which the cross section of free nucleons is measured. If the free-nucleon cross sections are strongly energy-dependent, the first effect could be expected to be large. If they are slowly varying, as they are at high energies, one would expect this effect to be negligible. West[1,2] had pointed out that this was not true. He showed that the constraint that real particles be produced in the final state leads to $\sigma_d = (\sigma_p + \sigma_n)(1-\beta)$ if σ_p, σ_n are constant (see Sec. II). This kind of correction is called West's "β correction." He also found that the flux factor depletes σ_d relative to $\sigma_p + \sigma_n$.

Recently Frankfurt and Strikman[3] and Landshoff and Polkinghorne[4] claimed to have shown that West's β correction is in fact absent. If this claim is correct, then the neutron-data extraction using West's procedure needs to be examined again, especially for the deep-inelastic neutron structure function at the Bjorken scaling variable ω close to 1 where σ_p and σ_n vary most rapidly. The information from this region is of very considerable theoretical importance, in that the ratio σ_n/σ_p in deep-elastic electron or muon scattering gives information about the relative shapes of the momentum distributions of u and d quarks in the nucleon.

However, West disagrees with their conclusion.[5] Thus we see there is a controversy here.

The aim of this paper is to give a solution for this controversy. The plan of the paper is as follows. In Sec. II we discuss the source of the controversy and state explicitly the basic assumptions of our formalism. This discussion is important for the refinement, if needed in the future. The normalization of the rela stic deuteron wave function is discussed in Sec. III where we use the common procedure, the relation of the elastic electromagnetic form factor at $q^2 = 0$ to the total charge. In Sec. IV we consider hadronic scattering where the incident particle has a small but finite mass. We find that when the nucleon sections σ_p and σ_n are constant, the West β correction vanishes. In this result we agree with Frankfurt and Strikman[3] and Landshoff and Polkinghorne,[4] who pointed out that the correction is absent. We find, however, that for leptonic scattering (the case of the electron is discussed in Sec. V and that of the neutrino in Appendix D) West's β correction is still there.

In the main part of the paper we show explicitly that our approach is consistent and we have no problem with subasymptotic effects.[6] The problems concerning gauge invariance are the subject of Appendix A. In Appendices B and C, we rederive the result of Sec. III by using the light-cone approach which clarifies the incorrect presentation given in Ref. 3 and TOPTH$_\infty$ (time-ordered perturbation theory in the infinite-momentum frame), respectively.

II. THE CONTROVERSY OF THE WEST β CORRECTION

In this section we would like to review the source of the controversy and state explicitly the basic assumptions which had been used implicitly by the previous authors (Ref. 1, 3, and 4) and also in this present paper. We shall consider hadronic scattering and limit ourselves to the case where the incident particle has a small but finite mass, such as pion-deuteron scattering.

Our basic assumptions are as follows:

1. The deuteron is considered as a (p,n) bound state. We do not consider the isobar admixture, the six-quarks degree of freedom, and the meson exchange current contribution in the deuteron.

2. We use "off-shell kinematics"–"on-shell dynamics" formalism. Stated plainly, we use the on-shell amplitude but off-shell kinematics. This is not a bad approximation if the interacting nucleon is not far off the mass shell, which may be true for a weakly binding system like the deuteron.

3. To illustrate the physical point without obscuring the issue with algebra, we will consider the spin-averaged case and hence will neglect the spin of the particles in the formulation of the model.

4. We work in the (incoherent) impulse approximation (IA). This approximation excludes shadowing corrections. We also neglect in this paper other effects which might impair the validity of the IA. For example, the use of the IA at small q^2's might be questionable, and hence the usual practice of extrapolating to $q^2=0$ for the form factor might not be realistic. We know, however, of no operationally useful approximation that could take into account the deviations from the composite picture implied in the IA, and hence we will join our predecessors in using the IA.

Even doing so, one might think of including processes where the "spectator" nucleon, though dynamically still a spectator, appears in an excited state of the nucleon. We neglect such a possibility in this paper, because including such excited states in the model for the spectator would then also require, for consistency, the inclusion of such excited states in the internal lines of the diagrams we use. To do so would be another, rather ambitious calculation of questionable reliability.

The point we want to study is the behavior of the alleged corrections in the usual IA, which is what West used. It is not our purpose to claim that this model will necessarily give the actual precise *quantitative* magnitude of any corrections one has to make when reducing deuteron data. Such additional contributions to the corrections, due to the deviations from the IA, should be calculated in the future by a string of successive calculations as our knowledge of each of these deviations solidifies.

As pointed out by Bodek,[7] the relevant quantity to smear is the invariant matrix element $|\mathfrak{M}|^2$. The cross section is defined as the invariant matrix element times the invariant flux (the final-state phase space is included in the definition of the matrix element). Smearing the invariant matrix elements (see Fig. 1) we get

$$|\mathfrak{M}_d(s)|^2 = \sum_{i=p,n} \int \frac{d^4k}{(2\pi)^4} \frac{\phi^2(p^2)}{(p^2-M^2+i\epsilon)^2} |\mathfrak{M}_i(s')|^2 \times 2\pi\delta(k^2-M^2)\theta(s'-M^2) , \quad (2.1)$$

where $\phi(p^2)$ is the *truncated* n-p-d vertex function (truncated simply means that one of the nucleons, here the spectator, is on shell). We incorporate the θ function explicitly in Eq. (2.1). This is due to the threshold condition on $|\mathfrak{M}_i(s')|^2$, i.e., $|\mathfrak{M}_i(s')|^2=0$ when $s'<M^2$ which was first suggested by West[1]; we parametrize the $|\mathfrak{M}|^2$'s in terms of the relevant total center-of-mass energies $s=(\mathcal{P}+q)^2$ and $s'=(p+q)^2$.

In the laboratory (lab) system or deuteron rest frame,

$$\begin{aligned} p^0 &\equiv E = M_d - k^0 = M_d - (M^2+\vec{k}^2)^{1/2} , \\ \vec{p}+\vec{k} &= 0 . \end{aligned} \quad (2.2)$$

By defining

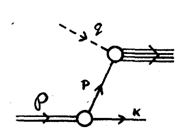

FIG. 1. Impulse-approximation Feynman graph. The broken line represents the incident particle of four-momentum q; the single solid lines are the nucleons of momenta p (the interacting particle) and k (the spectator). The double line represents the deuteron of four-momentum $\mathcal{P}$.

$$M|f(\vec{k})|^2 \equiv \frac{1}{4(2\pi)^3} \frac{\phi^2(p^2)}{(p^2-M^2+i\epsilon)^2}, \tag{2.3}$$

we can rewrite Eq. (2.1) as

$$|\mathfrak{M}_d(s)|^2 = \sum_{i=p,n} \int \frac{d^3k}{k_0/M} 2|f(\vec{k})|^2 |\mathfrak{M}_i(s')|^2 \times \theta(s'-M^2). \tag{2.4}$$

The controversy starts from Eq. (2.4). West interprets $|f(\vec{k})|^2$ as the probability of finding a nucleon with momentum $\vec{k}$ in the rest system of the deuteron, and he chooses the following normalization:

$$\int \frac{d^3k}{k_0/M} |f(\vec{k})|^2 = 1, \tag{2.5}$$

by identifying $|f(\vec{k})|^2/(k_0/M)$ as the usual nonrelativistic deuteron wave function.

In terms of the experimentally measured total cross section, rewrite Eq. (2.4) as[7]

$$\sigma_d(s) = \sum_{i=p,n} \int \frac{d^3k}{k_0/M} 2|f(\vec{k})|^2 \frac{M}{M_d} \left(\frac{\tilde{\nu}^2-q^2}{\nu_d{}^2-q^2}\right)^{1/2} \times \sigma_i(s')\theta(s'-M^2), \tag{2.6}$$

where

$$M_d\nu_d = \mathcal{P}\cdot q,$$

$$M\tilde{\nu} = M\nu + (p^2-M^2)/2; \quad M\nu = p\cdot q.$$

In the *high-energy limit*

$$\sigma_d(\nu_d) \simeq \sum_{i=p,n} \int \frac{d^3k}{k_0/M} |f(\vec{k})|^2 \frac{\nu}{\nu_d} \sigma_i(\nu)\theta(\nu). \tag{2.7}$$

In the *deuteron rest frame* where

$$\mathcal{P}_\mu = (M_d, \vec{0}),$$

$$p_\mu = (M_d - (M^2+\vec{k}^2)^{1/2}, -\vec{k}),$$

we can write

$$\nu - \nu_d = -\frac{\nu_d}{M}(\epsilon + T - |\vec{k}|\cos\theta) \tag{2.8}$$

by using

$$|\vec{q}| \simeq q_0 \equiv \nu_d$$

$$M_d = 2M - \epsilon,$$

where $\epsilon =$ the deuteron binding energy and by defining $T \equiv (M^2+\vec{k}^2)^{1/2} - M =$ relativistic kinetic energy. The angle θ above is the angle between $\vec{k}$ and $\vec{q}$. For slowly varying cross sections we can expand $\sigma(\nu)$ around $\nu = \nu_d$. Then from Eq. (2.7) we obtain

$$\sigma_d(\nu_d) = \sum_{i=p,n} \int \frac{d^3k}{k_0/M} |f(\vec{k})|^2 \theta(\nu) \times \left\{\sigma_i(\nu_d) - \frac{\epsilon + T - |\vec{k}|\cos\theta}{M} \times [\nu\sigma_i(\nu)]'_{\nu=\nu_d} + \cdots\right\}, \tag{2.9}$$

where the prime means $d/d\nu$. If we define

$$\int \frac{d^3k}{k_0/M} |f(\vec{k})|^2 |\vec{k}|\, \theta(\nu)\cos\theta \equiv \gamma,$$

$$\int \frac{d^3k}{k_0/M} |f(\vec{k})|^2 \theta(\nu) \equiv 1-\beta, \tag{2.10}$$

$$\int \frac{d^3k}{k_0/M} |f(\vec{k})|^2 T\theta(\nu) \equiv \langle T\rangle,$$

then we obtain

$$\sigma_d(\nu_d) \simeq [\sigma_p(\nu_d) + \sigma_n(\nu_d)](1-\beta) - \frac{\epsilon + \langle T\rangle - \gamma}{M} \nu_d \sigma_i'(\nu_d) + \cdots. \tag{2.11}$$

If σ_p and σ_n are constant, then

$$\sigma_d(\nu_d) = [\sigma_p(\nu_d) + \sigma_n(\nu_d)](1-\beta). \tag{2.12}$$

This β is known as the West β corrections

The normalization Eq. (2.5) had been criticized by Frankfurt and Strikman[3] since it is inconsistent with the usual normalization by using the relation of the elastic electromagnetic form factor at $q^2=0$ to the total charge. They suggested that the normalization

$$\int \frac{d^3k}{k_0/M} |f(\vec{k})|^2 \frac{2(E+k_z)}{M_d} \theta(E+k_z) = 1 \tag{2.13}$$

should be used instead of Eq. (2.5). We will repeat the light-cone derivation of Eq. (2.13) in Appendix B since the original derivation is not explicit and also we need to do it again in connection with the gauge-invariance problem (see Appendix A).

We believe that the normalization Eq. (2.13) is the correct one. We rederive it by using the

Brodsky parametrization method.[8] This will be discussed in Sec. III. In fact Landshoff and Polkinghorne get a similar result by using the Sudakov variable method, though they need to go to high-energy limit to get it. This implies that West's interpretation and the identification of $|f(\vec{k})|^2$ as the usual nonrelativistic deuteron wave function cannot be correct, even apart from the question of whether the kinematic form factor M/k_0 should or should not be included.

Frankfurt and Strikman showed that the correct normalization Eq. (2.13) will lead to the vanishing of β correction in high-energy hadron scattering. Their argument goes as follows: Rewrite Eq. (2.7) as

$$\sigma_d(\nu_d)=\sum_{i=p,n}\int\frac{d^3k}{k_0/M}\,|f(\vec{k})|^2\,\frac{2(E+k_z)}{M_d}\times\sigma_i(\nu)\theta(E+k_z)\;. \quad (2.14)$$

Combining Eqs. (2.13) and (2.14) leads them to conclude that when the nucleon cross sections σ_p and σ_n are constant, the β correction will vanish[9]: $\sigma_d=\sigma_p+\sigma_n$. Landshoff and Polkinghorne using the Sudakov-variable approach also get the same conclusion. This conclusion will be confirmed again by using the Brodsky parametrization method (see Sec. IV), which we claim has a more sound derivation than the previous ones (see Sec. VI).

III. THE NORMALIZATION OF THE RELATIVISTIC DEUTERON WAVE FUNCTION

We will consider first the normalization of the deuteron (relativistic) wave function (see Fig. 2).

$$\begin{aligned}\tilde{\Gamma}_\mu&=(2\mathcal{P}+q)_\mu F_d(q^2)\\&=\sum_{i=p,n}\int\frac{d^4k}{(2\pi)^4 i}\,\frac{\phi(p^2,k^2)}{p^2-M^2+i\epsilon}\,\frac{\phi((p+q)^2,k^2)}{(p+q)^2-M^2+i\epsilon}\,\frac{1}{k^2-M^2+i\epsilon}\\&\qquad\times\left\{(2p+q)_\mu F_i(q^2)+q_\mu\frac{(p+q)^2-p^2}{q^2}\left[1-F_i(q^2)\right]\right\}.\end{aligned}\quad (3.1)$$

We showed in Appendix A that this form satisfies the gauge-invariance condition.

We will use the Brodsky parametrization method to parametrize the four momenta as follows:

$$\begin{aligned}\mathcal{P}_\mu&=\left(P+\frac{M_d^2}{4P},\;\vec{0}_\perp,\;P-\frac{M_d^2}{4P}\right),\quad q_\mu=\left(\frac{\mathcal{P}\cdot q}{2P},\;\vec{q}_\perp,\;-\frac{\mathcal{P}\cdot q}{2P}\right),\\p_\mu&=\left(xP+\frac{P^2+\vec{k}_\perp^{\,2}}{4xP},\;\vec{k}_\perp,\;xP-\frac{P^2+\vec{k}_\perp^{\,2}}{4xP}\right),\quad k_\mu=\left((1-x)P+\frac{k^2+\vec{k}_\perp^{\,2}}{4(1-x)P},\;-\vec{k}_\perp,\;(1-x)P-\frac{k^2+\vec{k}_\perp^{\,2}}{4(1-x)P}\right).\end{aligned}\quad (3.2)$$

Here $P=\frac{1}{2}(\mathcal{P}_0+\mathcal{P}_3)$ is an arbitrary parameter and notice that all invariants are independent of P. In the deuteron rest frame $P=\frac{1}{2}M_d$.

The great merit of the Brodsky parametrization is the simple factorization of the k^2 integration:

$$\int\frac{d^4k}{(2\pi)^4 i}=\int d^2\vec{k}_\perp\int_{-\infty}^{\infty}\frac{dx}{2(2\pi)^3|1-x|}\int_{-\infty}^{\infty}\frac{dk^2}{2\pi i}\;. \quad (3.3)$$

Since $F_d(q^2)=(\tilde{\Gamma}_0+\tilde{\Gamma}_3)/2P$, by using Eq. (3.2) and Eq. (3.3) we get (set $q=0$ and notice that $\mathcal{P}\cdot q=\frac{1}{2}\vec{q}_\perp^{\,2}$ for elastic scattering)

$$1=\int d^2\vec{k}_\perp\int_{-\infty}^{\infty}\frac{x\,dx}{2(2\pi)^3|1-x|}\int_{-\infty}^{\infty}\frac{dk^2}{2\pi i}\,\frac{\phi^2(p^2,k^2)}{k^2-M^2+i\epsilon}\,\frac{1}{\left(xM_d^2-M^2-\dfrac{xk^2+\vec{k}_\perp^{\,2}}{1-x}+i\epsilon\right)^2}\;. \quad (3.4)$$

Neglecting the singularity of ϕ in the k^2 plane, if any,[10] the first denominator represents a simple pole in the lower k^2 half-plane. The second denominator leads to one double pole in the upper or lower k^2 half-plane depending on whether $(1-x)/x$ is positive or negative. Thus x is restricted to the interval $0<x<1$ (see a similar discussion in Appendix B). Notice that $x=0$ or 1 gives zero con-

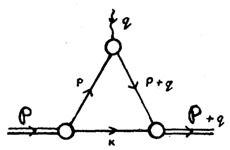

FIG. 2. The deuteron electromagnetic form factor in the impulse approximation.

tribution in Eq. (3.4).

Thus we get the result[11]

$$1=\int d^2\vec{k}_\perp \int_0^1 \frac{dx}{2(2\pi)^3 x(1-x)} \frac{\phi^2(x,\vec{k}_\perp)}{[M_d{}^2-M^2(x,\vec{k}_\perp)]^2} \tag{3.5}$$

where

$$M^2(x,\vec{k}_\perp)=\frac{M^2+\vec{k}_\perp{}^2}{x(1-x)}$$

is the invariant mass of the two-nucleon system $(p+k)^2$, where

$$p^2=xM_d{}^2-\frac{xM^2+\vec{k}_\perp{}^2}{x(1-x)} . \tag{3.6}$$

Notice that the result Eq. (3.5) is independent of the parameter P as it should be. It can be shown that this result exactly agrees with the form given by time-ordered perturbation theory in the infinite-momentum frame (see Appendix C).

Defining

$$G(x,\vec{k}_\perp)=\frac{1}{2(2\pi)^3}\frac{1}{x(1-x)}\frac{\phi^2(x,\vec{k}_\perp)}{[M_d{}^2-M^2(x,\vec{k}_\perp)]^2} , \tag{3.7}$$

we can rewrite Eq. (3.5) as[12]

$$\int d^2\vec{k}_\perp \int_0^1 dx\, G(x,\vec{k}_\perp)=1 , \tag{3.8}$$

which implies that we can interpret $G(x,\vec{k}_\perp)$ as the probability of finding the nucleon to have $x=(p_0+p_3)/(\mathcal{P}_0+\mathcal{P}_3)$ along the direction of the deuteron and transverse momentum $\vec{k}_\perp$ which is perpendicular to $\vec{\mathcal{P}}$. Defining

$$G(x)=\int d^2\vec{k}_\perp G(x,\vec{k}_\perp) , \tag{3.9}$$

Eq. (3.8) can also be written as

$$\int_0^1 dx\, G(x)=1 . \tag{3.10}$$

In the above discussion we have implicitly used the isospin symmetry, i.e., $G_{p/d}(x)=G_{n/d}(x)\equiv G(x)$. Since the interacting nucleon (spectator) is off (on) the mass shell, neither ϕ nor G are necessarily symmetric around $x=\frac{1}{2}$. However, for the interacting nucleon which is not far off the mass shell it is not a bad approximation to assume ϕ or G to have such property,[23] i.e.,

$$G_{p/d}(x)\simeq G_{n/d}(1-x) . \tag{3.11}$$

This property implies that Eq. (3.10) is equivalent to

$$\int_0^1 [xG_{p/d}(x)+(1-x)G_{n/d}(1-x)]\,dx=1 , \tag{3.12}$$

which means the sum of the fractional momenta of the proton and the neutron is the total (fraction) momentum of the deuteron.

IV. HADRONIC SCATTERING

In this section we will consider hadronic scattering and limit ourselves to the case where the incident particle has a small but finite mass, such as pion-deuteron scattering. See Fig. 1 and the equation (2.1). We will use again the Brodsky parametrization [see Eq. (3.2) with the *on-shell*

$$q_\mu=\left(P'+\frac{m^2}{4P'},\ \vec{0}_\perp,\ -P'+\frac{m^2}{4P'}\right),\quad q^2=m^2 . \tag{4.1}$$

Notice that $\mathcal{P}_\mu$ and q_μ have been defined in a general set of frames along the interaction axis. A specific frame in this set is selected by relating P and P'. For example, the deuteron rest frame is defined by the conditions

$$P=\tfrac{1}{2}M_d \text{ and } \nu_d=M_d\left(P'+\frac{m^2}{4P'}\right), \tag{4.2}$$

where ν_d is the incident particle energy (modulo M_d).

The invariant matrix elements in the *deuteron rest frame* can be written now as[13]

$$|\mathfrak{M}_d(s)|^2=\sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1 \frac{dx}{x}G(x,\vec{k}_\perp)|\mathfrak{M}_i(s')|^2 \times\theta(s'-M^2) , \tag{4.3}$$

where

$$\begin{aligned} s&=(\mathcal{P}+q)^2=M_d{}^2+m^2+2\nu_d ,\\ s'&=(p+q)^2=p^2+m^2+2\nu ,\\ \nu&\equiv p\cdot q=x\nu_d+\frac{m^2}{4\nu_d}\left(M_d{}^2-\frac{M^2+\vec{k}_\perp{}^2}{1-x}\right),\\ p^2&=xM_d{}^2-\frac{xM^2+\vec{k}_\perp{}^2}{1-x} . \end{aligned} \tag{4.4}$$

Notice, for our convenience, that our definition of ν and ν_d here is different from the one in Sec. II.

In terms of experimentally measured total cross section

$$\sigma_d(s)=\sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}\,G(x,\vec{k}_\perp)\times\left(\frac{\tilde{\nu}^2-m^2M^2}{\nu_d^{\,2}-m^2M_d^{\,2}}\right)^{1/2}\times\sigma_i(s')\theta(s'-M^2)\,, \tag{4.5}$$

where

$$\tilde{\nu}=\nu+\tfrac{1}{2}x\left(M_d^{\,2}-\frac{M^2+\vec{k}_\perp^{\,2}}{x(1-x)}\right).$$

In the high-energy limit, $\nu_d\to\infty$, $\nu\simeq x\nu_d$, then

$$\sigma_d(\nu_d)=\sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1 dx\,G(x,\vec{k}_\perp)\sigma_i(\nu)\theta(\nu). \tag{4.6}$$

Since ν is always positive, i.e., $\theta(\nu)=1$,

$$\sigma_d(\nu_d)=\sum_{i=p,n}\int_0^1 dx\,G(x)\sigma_i(x\nu_d)\,. \tag{4.7}$$

Equation (4.7) leads us to the conclusion:

1. The threshold condition is always satisfied in the high-energy scattering, i.e., it plays no role at all. This is what we expect intuitively.
2. The physical quantity to be smeared by the Fermi motion is the cross section itself.
3. If $\sigma_{p,n}$ are constant, then $\sigma_d=\sigma_p+\sigma_n$.

Thus *West's β correction is absent for hadronic processes*. All of these conclusions are the opposite to what West obtained.[1,2]

V. DEEP-INELASTIC ELECTRON SCATTERING

We now consider the case where the "projectile" is a virtual (off-shell) photon of four-momentum q. The $|\mathfrak{M}|^2$ now becomes the forward virtual Compton amplitude $W_{\mu\nu}$. For the case of electroproduction at small q^2, or indeed for photoproduction there is little change as compared to the previous analysis. In this section we consider the case of large (negative) q^2.

$$W^{ed}_{\mu\nu}(q^2,\nu_d)=\sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}\,G(x,\vec{k}_\perp)\times W^{ei}_{\mu\nu}(q^2,\nu)\,, \tag{5.1}$$

where (see Appendix A)

$$\begin{aligned}
&\nu=x\nu_d-\vec{q}_\perp\cdot\vec{k}_\perp\,,\\
&W^{ei}_{\mu\nu}=\tilde{g}_{\mu\nu}W^{ei}_1(q^2,\nu)+\tilde{p}_\mu\tilde{p}_\nu W^{ei}_2(q^2,\nu)M_d^{\,-2},\\
&\tilde{g}_{\mu\nu}=g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\,,\\
&\tilde{p}_\mu=p_\mu-\frac{\nu}{q^2}\,q_\mu\,,
\end{aligned} \tag{5.2}$$

and similarly for $W^{ed}_{\mu\nu}$ by changing $p_\mu\to\mathcal{P}_\mu$ and $\nu\to\nu_d$. This form satisfies the gauge-invariance condition.

To project out the W^{ed}_1 and W^{ed}_2 from $W^{ed}_{\mu\nu}$ we can use the projection operator $P^{\mu\nu}$ with the property

$$P^{\mu\nu}_i\,W^{ed}_{\mu\nu}=W^{ed}_i\,,\quad i=1,2\,. \tag{5.3}$$

It is not difficult to verify that

$$\begin{aligned}
&P^{\mu\nu}_1=-\frac{1}{2}\,g^{\mu\nu}+\frac{1}{2}\,\frac{q^2\mathcal{P}^\mu\mathcal{P}^\nu}{q^2M_d^{\,2}-\nu_d^{\,2}}\,,\\
&P^{\mu\nu}_2=\frac{1}{2}\,\frac{q^2M_d^{\,2}}{q^2M_d^{\,2}-\nu_d^{\,2}}\left(-g^{\mu\nu}+\frac{3q^2\mathcal{P}^\mu\mathcal{P}^\nu}{q^2M_d^{\,2}-\nu_d^{\,2}}\right)
\end{aligned} \tag{5.4}$$

will satisfy Eq. (5.3).

Applying Eq. (5.3) to Eq. (5.1) we get[14]

$$W^{ed}_1(q^2,\nu_d)=\sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}\,G(x,\vec{k}_\perp)[W^{ei}_1(q^2,\nu)+AW^{ei}_2(q^2,\nu)]\,, \tag{5.5}$$

$$W^{ed}_2(q^2,\nu_d)=\sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}\,G(x,\vec{k}_\perp)B\,W^{ei}_2(q^2,\nu)\,, \tag{5.6}$$

where

$$A=[2M_d{}^2(q^2M_d{}^2-\nu_d{}^2)]^{-1}[-\nu_d{}^2\vec{k}_\perp{}^2+\nu_d(\vec{q}_\perp\cdot\vec{k}_\perp)C_1+q^2C_2+M_d{}^2(\vec{q}_\perp\cdot\vec{k}_\perp)^2],$$

$$B=\tfrac{1}{2}(q^2M_d{}^2-\nu_d{}^2)^{-2}\{2x^2\nu_d{}^4-4x\nu_d{}^3(\vec{q}_\perp\cdot\vec{k}_\perp)+\nu_d{}^2[q^2C_3+2(\vec{q}_\perp\cdot\vec{k}_\perp)^2]+\nu_d q^2(\vec{q}_\perp\cdot\vec{k}_\perp)C_4+q^4C_5+q^2M_d{}^2(\vec{q}_\perp\cdot\vec{k}_\perp)^2\},$$

$$C_1=(1-x)M_d{}^2-\frac{M^2+\vec{k}_\perp{}^2}{1-x},$$

$$C_2=\frac{k_\perp{}^4}{4(1-x)^2}+\frac{1}{2}\vec{k}_\perp{}^2\left[M_d{}^2+\frac{M^2}{(1-x)^2}\right]+\left[\frac{M_d{}^2(1-x)^2-M^2}{2(1-x)}\right]^2, \tag{5.7}$$

$$C_3=\frac{1}{1-x}[2xM^2-2x(1-x^2)M_d{}^2+(3x-1)\vec{k}_\perp{}^2],$$

$$C_4=\frac{1}{1-x}\{-3(M^2+\vec{k}_\perp{}^2)+[4-(1+x)^2]M_d{}^2\},$$

$$C_5=\frac{3\vec{k}_\perp{}^4}{4(1-x)^2}+\frac{\vec{k}_\perp{}^2}{2(1-x)^2}[3M^2+(3x^2-2x-1)M_d{}^2]$$
$$+\frac{1}{4(1-x)^2}[3M^4+2M_d{}^2M^2(x^2+2x-3)+M_d{}^4(3x^4-4x^3+2x^2-4x+3)].$$

In the deep-inelastic limit

$$\nu_d\to\infty,\quad q^2\to-\infty,\quad \omega_d=\frac{2\nu_d}{-q^2}\text{ fixed}, \tag{5.8}$$

then

$$W_1^{ed}(q^2;\omega_d)=\sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}G(x,\vec{k}_\perp)\left(W_1^{ei}(q^2,\omega)+\frac{\vec{k}_\perp{}^2}{2M_d{}^2}W_2^{ei}(q^2,\omega)\right), \tag{5.9}$$

$$W_2^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1 dx\,xG(x,\vec{k}_\perp)W_2^{ei}(q^2,\omega), \tag{5.10}$$

where

$$\omega=\frac{2\nu}{-q^2}=\frac{2x\nu_d}{-q^2}=x\omega_d=\frac{x}{x_d}. \tag{5.11}$$

Defining

$$W_1^{ed}=F_1^{ed},\quad \nu_dM_d{}^{-2}W_2^{ed}=F_2^{ed},$$
$$W_1^{ei}=F_1^{ei},\quad \nu M_d{}^{-2}W_2^{ei}=F_2^{ei}, \tag{5.12}$$

we can rewrite Eqs. (5.9)–(5.10) as[15]

$$F_1^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int_0^1\frac{dx}{x}G(x)F_1^{ei}(q^2,\omega)\theta(\omega-1), \tag{5.13}$$

$$F_2^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int_0^1 dxG(x)F_2^{ei}(q^2,\omega)\theta(\omega-1). \tag{5.14}$$

We write the θ function explicitly as a threshold condition, i.e., $F=0$ if $\omega<1$. Notice in our discussion it is not necessary to assume Bjorken scaling, so that we retain the possibility of a q^2 dependence in the structure function.

Our result for F_2^d, Eq. (5.14), agrees with the result obtained in Ref. 4, though it does not mention F_1^d.

In terms of x_d:

$$F_1^{ed}(q^2,x_d)=\sum_{i=p,n}\int_{x_d}^1\frac{dy}{y}G(y)F_1^{ei}\left(q^2,\frac{x_d}{y}\right), \tag{5.15}$$

$$F_2^{ed}(q^2,x_d)=\sum_{i=p,n}\int_{x_d}^1 dyG(y)F_2^{ei}\left(q^2,\frac{x_d}{y}\right), \tag{5.16}$$

a form familiar to the believers of the parton model.[16]

A simple direct consequence of Eqs. (5.13)–(5.14) and of the normalization condition Eq. (3.10) are the *wave-function-independent sum rules*:

$$\int_1^\infty\frac{d\omega_d}{\omega_d{}^2}F_1^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int_1^\infty\frac{d\omega}{\omega^2}F_1^{ei}(q^2,\omega), \tag{5.17}$$

$$\int_1^\infty\frac{d\omega_d}{\omega_d}F_2^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int_1^\infty\frac{d\omega}{\omega}F_2^{ei}(q^2,\omega). \tag{5.18}$$

Notice that these two sum rules are consistent with the Callan-Gross relation:

$$F_1^{ed}(q^2,\omega_d)=\omega_d F_2^{ed}(q^2,\omega_d)\,,$$
$$F_1^{ei}(q^2,\omega)=\omega F_2^{ei}(q^2,\omega)\,. \tag{5.19}$$

The higher moments can be easily obtained, especially

$$\int_1^\infty \frac{d\omega_D}{\omega_D^{\,3}}\,F_1^{ed}(q^2,\omega_D)=\langle x\rangle \sum_{i=p,n}\int_1^\infty \frac{d\omega}{\omega^3}\,F_1^{ei}(q^2,\omega)\,, \tag{5.20}$$

$$\int_1^\infty \frac{d\omega_D}{\omega_D^{\,2}}\,F_2^{ed}(q^2,\omega_d)=\langle x\rangle \sum_{i=p,n}\int_1^\infty \frac{d\omega}{\omega^2}\,F_2^{ei}(q^2,\omega)\,, \tag{5.21}$$

where

$$\langle x\rangle=\int_0^1 x\,G(x)dx\,. \tag{5.22}$$

In general, we expect $\langle x\rangle \simeq 1/N$, where N is the number of constituents in the deuteron. For our two-nucleon model $\langle x\rangle\simeq\frac{1}{2}$.

Going back to Eqs. (5.13)–(5.14) we see that for large ω, where $F_2^{ei}(q^2,\omega)$ is approximately constant (fixed q^2), the West β correction is still present, but the form of this West β correction is different according to us than what was given by West.[1,2] Thus the result is different from hadronic scattering. The reason is that in electron scattering the incoming "projectile" is a *virtual* (off-shell) *photon* with a *large* (negative) q^2, and thus the threshold condition is

$$\theta(s'-M^2)\to\theta(\omega-1)=\theta(x\omega_d-1)\,,$$

since[15]

$$s'-M^2=\frac{-q^2}{1-x}\left[-x^2\left(\omega_d+\frac{M_d^{\,2}}{-q^2}\right)+x\left(1+\omega_d+\frac{M_d^{\,2}}{-q^2}\right)-\frac{M^2+\vec{k}_\perp^{\,2}}{-q^2}-1\right]$$
$$\underset{-q^2\gg M_d^2}{\simeq}\frac{-q^2}{1-x}\,[-\omega_d x^2+(1+\omega_d)x-1]\,.$$

In hadronic scattering the incoming "projectile" is on shell with a *small* (positive) q^2 ($=m^2$, the mass of the incident particle), and so the threshold condition in this case is

$$\theta(s'-M^2)\to\theta(\nu)=\theta(x\nu_d)=1\,,$$

since[15]

$$s'-M^2=\frac{M_d^{\,2}+2\nu_d}{1-x}\left[-x^2+\left(1-\frac{m^2}{M_d^{\,2}+2\nu_d}\right)x-\frac{M^2+\vec{k}_\perp^{\,2}-m^2}{M_d^{\,2}+2\nu_d}\right]$$
$$\simeq(M_d^{\,2}+2\nu_d)x\,.$$

A similar calculation with similar results can be carried out also for the neutrino case. It is given in Appendix D.

VI. CONCLUSION AND DISCUSSION

We have considered the smearing effect due to the Fermi motion of the constituent nucleons in deuteron. The analysis has been done covariantly. The proper relativistic treatment of the deuteron as a (p,n) bound state implies that we need to consider also the $N\bar{N}$ pairs (due to the vacuum fluctuation) in the deuteron. As in the parton model of the nucleon one can use the infinite-momentum frame to deal with $N\bar{N}$ pairs. For the deuteron, however, this is unnecessary since its binding energy is small, and furthermore we do not know the infinite-momentum wave function. We prefer to work in the deuteron rest frame, where we can identify the wave function as of the Bethe-Salpeter type.

The Brodsky parametrization method allows us to work in the deuteron rest frame. The result we get agrees with the infinite-momentum-frame calculation. Furthermore, it can deal with the problem in a general way and does not need to go to the limiting case, the high-energy limit.

We get the correct normalization condition for the relativistic deuteron wave function. This result leads us to the vanishing of the West β correction for high-energy hadronic scattering. This absence is not surprising since in the high energy limit the threshold condition can always be satisfied, i.e., it has no role at all. Thus, we confirm the previous claims that the West β correction is absent in this case. However, we believe that our approach is more general than the previous ones since it avoids making several approximations used in earlier papers.[6,12,14] Furthermore, we find some inconsistency[9] in Ref. 3.

Similarly, also in Ref. 4 we do not see any reason why the ξ variable (which is equivalent to our x (variable) should have values between 0 and 1.[12] Note that the nucleon as the constituent of the deuteron should not be treated as the parton-quark (which is supposed to be the constituent of the nucleon), especially if we work in the deuteron rest frame.

In electron scattering we have found that the West β correction exists, though not quite in the same form as given by West.[1,2] In fact, we get a result that can be regarded as the complement to Ref. 4. The results of the latter can also be obtained easily by our approach. In the deep-inelastic limit, but not necessarily in the Bjorken scaling limit, our result agrees, though not quite exactly, with the third paper in Ref. 3. The variable α there is not the same as our variable[22] x.

Our general expressions for the W_1^{ed} and W_2^{ed} [see Eqs. (5.5)–(5.6)] are more complicated than the ones obtained by West.[1,2] This complication is the price we pay for taking the $N\bar{N}$ pairs in the deuteron properly into account. We believe that our expressions are the correct ones. This feeling is strengthened by the fact that our results leads to a form which has a simple parton interpretation (in the infinite-momentum frame). The difference between our results and West's is reflected also in the deep-inelastic form of the structure function. Eqs. (5.13)–(5.14) in the deuteron rest frame can be written as[12]

$$F_1^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int\frac{d^3k}{k_0/M}\,2|f(\vec{k})|^2\theta(E+k_z)\times F_1^{ei}(q^2,\omega)\theta(\omega-1)\,,\qquad(6.1)$$

$$F_2^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int\frac{d^3k}{k_0/M}\,|f(\vec{k})|^2\,\frac{2(E+k_z)}{M_d}\times\theta(E+k_z)F_2^{ei}(q^2,\omega)\theta(\omega-1)\,.\qquad(6.2)$$

If we define

$$|\psi(\vec{k})|^2\equiv\frac{1}{k_0/M}\,|f(\vec{k})|^2\,\frac{2(E+k_z)}{M_d}\,\theta(E+k_z)\,,\qquad(6.3)$$

which satisfies

$$\int d^3k|\psi(\vec{k})|^2=1\,,\qquad(6.4)$$

then

$$\omega_d^{-1}F_1^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int d^3k|\psi(\vec{k})|^2\omega^{-1}\times F_1^{ei}(q^2,\omega)\theta(\omega-1)\,,\qquad(6.5)$$

$$F_2^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int d^3k|\psi(\vec{k})|^2\times F_2^{ei}(q^2,\omega)\theta(\omega-1)\,.\qquad(6.6)$$

In comparison, the equations obtained by West are[23]

$$F_1^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int d^3k|\psi_W(\vec{k})|^2\times F_1^{ei}(q^2,\omega)\theta(\omega-1)\,,\qquad(6.7)$$

$$\omega_d F_2^{ed}(q^2,\omega_d)=\sum_{i=p,n}\int d^3k|\psi_W(\vec{k})|^2\times\omega F_2^{ei}(q^2,\omega)\theta(\omega-1)\,.\qquad(6.8)$$

Similarly, our result for the hadronic total cross section in the high-energy limit differs from West's. Eq. (4.7) in the deuteron rest frame can be written as

$$\sigma_d(\nu_d)=\sum_{i=p,n}\int d^3k|\psi(\vec{k})|^2\sigma_i(\nu)\,.\qquad(6.9)$$

In comparison, the equation obtained by West is

$$\nu_d\sigma_d(\nu_d)=\sum_{i=p,n}\int d^3k|\psi_W(\vec{k})|^2\nu\sigma_i(\nu)\theta(\nu)\,.\qquad(6.10)$$

Although, as we mentioned, the present paper avoided some of the approximations made in earlier contributions to this problem, one assumption was made here also, namely that the conclusions concerning the existence of West's β correction are independent of the spin structure of the reaction, and hence in our proof we used a spin-averaged formalism. Whether this assumption is valid or not will be investigated later.

ACKNOWLEDGMENT

We are indebted to Professor P. V. Landshoff, Professor L. L. Frankfurt, and Professor M. I. Strikman for illuminating comments made during our correspondence concerning the West β correction. This research was supported in part by the U. S. Department of Energy under Contract No. EY-76-06-2230.

APPENDIX A

In this appendix we will discuss the gauge-invariance problem for the diagram shown in Figs. 1 and 2. Consider Fig. 2:

$$\tilde{\Gamma}_\mu=(2\mathcal{P}+q)_\mu F_d(q^2)=\sum_{i=p,n}\int\frac{d^4k}{(2\pi)^4i}\,\frac{\phi(k^2,p^2)}{p^2-M^2+i\epsilon}\,\Gamma_\mu^i(p+q,p)\times\frac{\phi(k^2,(p+q^2))}{(p+q)^2-M^2+i\epsilon}\,\frac{1}{k^2-M^2+i\epsilon}\,,\qquad(\mathrm{A1})$$

where the ϕ is the n-p-d vertex function and the $\Gamma_\mu(p+q,p)$ is the γ-N-N vertex function which satisfies the Ward-Takahashi identity

$$q^\mu\Gamma_\mu(p+q,p)=\Delta_F'(p+q)^{-1}-\Delta_F'(p)^{-1}\simeq(p+q)^2-p^2=-2q\cdot k\,,\qquad(\mathrm{A2})$$

by using $\mathcal{P}_\mu=(p+k)_\mu$ and $q^2+2\mathcal{P}\cdot q=0$. Notice that in Eqs. (A1)–(A2) we approximate the full dressed propagator Δ_F' by the free propagator Δ_F. This is consistent with the assumption(2) discussed in Sec. II.

The gauge-invariance condition will be satisfied if the scalar

$$I(q^2, \mathcal{P}\cdot\mathcal{P}') = q^\mu \tilde{\Gamma}_\mu = 0\,,$$

where $\mathcal{P}'_\mu = (\mathcal{P}+q)_\mu$. Using Eq. (A2) we obtain

$$I = \int \frac{d^4k}{(2\pi)^4 i}\,\phi(k^2, \mathcal{P}\cdot k)\phi(k^2, \mathcal{P}'\cdot k)\,\frac{1}{k^2 - M^2 + i\epsilon} \times\left[\frac{1}{(\mathcal{P}-k)^2 - M^2 + i\epsilon} - \frac{1}{(\mathcal{P}'-k)^2 - M^2 + i\epsilon}\right]. \tag{A3}$$

Notice that the scalar I is invariant under the interchange $\mathcal{P}\leftrightarrow\mathcal{P}'$; however, the right-hand side of Eq. (A3) changes sign. This implies I must be zero, and hence the gauge-invariance condition is satisfied.

The general form of $\Gamma_\mu(p+q, p)$ is given by

$$\Gamma_\mu(p+q,p) = (2p+q)_\mu F(p^2, q^2, p\cdot q) + q_\mu G(p^2, q^2, p\cdot q)\,. \tag{A4}$$

F and G here are not independent, but are related by Eq. (A2); thus

$$\Gamma_\mu(p+q,p) = (2p+q)_\mu F + q_\mu\,\frac{(p+q)^2 - p^2}{q^2}\,(1-F)\,. \tag{A5}$$

Notice that the second term in Eq. (A5) will be zero on the mass shell limit, and thus we get the usual form.

Now consider Fig. 1 where q is the four-momentum of the virtual photon. We will use the identity[17]

$$q^\mu T_{\mu\nu}(p,p;q,q) = \Gamma_\nu(p+q,p) - \Gamma_\nu(p,p-q)\,, \tag{A6}$$

where the graphical representation is shown in

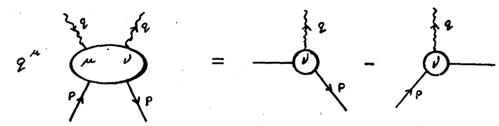

FIG. 3. Graphical representation of the identity Eq. (A6).

Fig. 3. In Eq. (A6) $T_{\mu\nu}$ is the virtual forward Compton scattering amplitude. Recall that $W_{\mu\nu}$ is the imaginary part of $T_{\mu\nu}$. Since we use the off-shell kinematic and on-shell dynamic formalism, i.e., F is a *real* function of q^2 only in Eq. (A5), the imaginary part of right-hand side of Eq. (A6) is equal to zero. Thus,

$$q^\mu W_{\mu\nu} = 0 \tag{A7}$$

for an interacting nucleon which is off shell. Hence, we can use the usual form for the $W_{\mu\nu}$, given by Eq. (5.2).

APPENDIX B

Charge conservation is used for the normalization of the wave function (see Fig. 2). Consider Eq. (A1) and Eq. (A5). By using the light-cone parameterization

$$k^\pm = k^0 \pm k^3\,;\qquad q^+ = q^0 + q^3 = 0\,;\qquad d^4k = \tfrac{1}{2}\,dk^+dk^-d^2\vec{k}_\perp\,, \tag{B1}$$

we find, in the *deuteron rest frame* [set $q=0$, $F_d = (\tilde{\Gamma}_0 + \tilde{\Gamma}_3)/2M_d$]

$$1 = \frac{1}{2(2\pi)^3 M_d}\sum_{i=p,n}\int d^2\vec{k}_\perp \int_{-\infty}^{\infty} dk^+(M_d - k^+)\int_{-\infty}^{\infty}\frac{dk^-}{2\pi i}\,\phi^2(k^+,k^-,\vec{k}_\perp)(k^+k^- - \vec{k}_\perp^{\,2} - M^2 + i\epsilon)^{-1} \times [M_d^{\,2} - M_d(k^+ + k^-) + k^+k^- - \vec{k}_\perp^{\,2} - M^2 + i\epsilon]^{-2}\,. \tag{B2}$$

In the formula above we used the fact that $F_d(0) = F_p(0) = 1$ and $F_n(0) = 0$.

Neglecting the singularity of ϕ in the K^- plane, if any,[10] the denominators lead to poles in the upper or lower K^- half plane (fixed K^+) depending on whether $K^+ < 0$ or $K^+ > M_d$. In this case we can close the integration contour in the lower or upper half plane, pushing a semicircle to infinity and we end up with zero. Also, from Eq. (B2) we see that $K^+ = 0$ or M_d gives zero contribution. Thus K^+ is restricted to the interval $0 < K^+ < M_d$. If we close the contour in this case, we can do it in the upper or lower half plane picking up one double pole or one simple pole, respectively. The two expressions, of course, have to be equal. The result is[11]

$$1 = \frac{M}{M_d}\int d^2\vec{k}_\perp \int_0^{M_d} dk^+\,\frac{M_d - k^+}{k^+}\,2\,|f(k^+, \vec{k}_\perp)|^2\,, \tag{B3}$$

where f is defined by Eq. (2.3). Using the relation

$$k_3 = \frac{1}{2k^+}(k^{+2} - \vec{k}_\perp^{\,2} - M^2)\,, \tag{B4}$$

we can rewrite Eq. (B3) as

$$\int d^2\vec{k}_\perp \int_a^\infty dk_3 \frac{1}{k_0/M} |f(\vec{k})|^2 \frac{2(E+k_3)}{M_d} = 1\,, \tag{B5}$$

where

$$a = \frac{1}{2M_d}(M^2 + \vec{k}_\perp{}^2 - M_d{}^2)\,.$$

Notice that Eq. (B5) can also be written as

$$\int \frac{d^3k}{k_0/M} |f(\vec{k})|^2 \frac{2(E+k_3)}{M_d} \theta(E+k_3) = 1\,. \tag{B6}$$

In the derivation of Eq. (B6) we do not make any approximation, apart from the singularity of ϕ in the K^- plane which we neglected. Thus with this appendix we clarify the incorrect presentation given in the Ref. 3.

APPENDIX C

In this appendix we will rederive Eq. (3.5) by using TOPTH$_\infty$ (time-ordered pertubation theory in the infinite-momentum frame).[8,18] We will assume that we can use the Weinberg rule (Feynman's rule in the infinite-momentum frame) for spinless particles here, though it is derived for the ϕ^3 interaction (we just change the g, the coupling constant, to ϕ, the vertex function).

In the infinite-momentum frame only the graphs shown in Fig. 4 will contribute to the Feynman graph Fig. 2. By choosing the reference frame

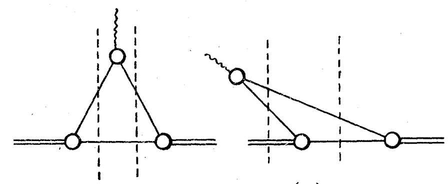

FIG. 4. The time-ordered contributions to the Feynman graph of Fig. 2 in the infinite-momentum frame. By convention, time flows from left to right.

$$\begin{aligned} \mathcal{P}_\mu &= \left(P + \frac{M_d{}^2}{2P}, \vec{0}_\perp, P\right), \\ q_\mu &= \left(\frac{\vec{q}_\perp{}^2}{2P}, \vec{q}_\perp, 0\right), \end{aligned} \tag{C1}$$

with

$$(\mathcal{P}+q)^2 = M_d{}^2 = M_d{}^2 + 2\mathcal{P}\cdot q + q^2 + O(1/P^2)\,,$$
$$q^2 = -\vec{q}_\perp{}^2 + O(1/P^2)\,,$$

only the single time ordering of Fig. 4(a) needs to be explicitly considered at $P\to\infty$.

Using the Weinberg rule and the parametrization

$$\begin{aligned} p_\mu &= \left(xP + \frac{M^2+\vec{k}_\perp{}^2}{2xP}, \vec{k}_\perp, xP\right), \\ k_\mu &= \left((1-x)P + \frac{M^2+\vec{k}_\perp{}^2}{2(1-x)P}, -\vec{k}_\perp, (1-x)P\right), \\ p'_\mu &= \left(xP + \frac{M^2+(\vec{k}_\perp+\vec{q}_\perp)^2}{2xP}, \vec{k}_\perp+\vec{q}_\perp, xP\right), \end{aligned} \tag{C2}$$

where $0<x<1$, we obtain[19]

$$(2\mathcal{P}+q)_\mu F_d(q^2) = \int d^2\vec{k}_\perp \int_0^1 \frac{dx}{2(2\pi)^3x^2(1-x)} \frac{\phi(x,\vec{k}_\perp)}{M_d{}^2 - M^2(x,\vec{k}_\perp)+i\epsilon} \sum_{i=p,n} (p+p')_\mu F^i(q^2) \frac{\phi(x,\vec{k}_\perp+(1-x)\vec{q}_\perp)}{M_d{}^2 - M^2(x,\vec{k}_\perp+(1-x)\vec{q}_\perp)+i\epsilon}\,, \tag{C3}$$

where

$$M^2(x,\vec{k}_\perp) = \frac{M^2+\vec{k}_\perp{}^2}{x(1-x)}\,.$$

By setting $q \doteq 0$ and taking, for example, the third component we find

$$\int d^2\vec{k}_\perp \int_0^1 \frac{dx}{2(2\pi)^3x(1-x)} \frac{\phi^2(x,\vec{k}_\perp)}{[M_d{}^2 - M^2(x,\vec{k}_\perp)]^2} = 1\,, \tag{C4}$$

which exactly agrees with Eq. (3.5). The result is not surprising since if $P\to\infty$ in the Brodsky parametrization, we have exactly the infinite-momentum frame, where x becomes the fractional longitudinal momentum.

After we finished this calculation we learned that a similar calculation has been done by Gunion, Brodsky, and Blankenbecler,[20] though their purpose is different from ours.

APPENDIX D

In this appendix we consider deep-inelastic neutrino scattering off the deuteron, say $\nu_\mu + d \to \mu^- + x$. Consider again Fig. 1 where q now is the four-momentum of the W boson. The general form of the $W^{\nu d}_{\lambda\rho}$ is given by[21]

$$W_{\lambda\rho}^{\nu d}(q^2,\nu_d) = -g_{\lambda\rho}W_1^{\nu d}(q^2,\nu_d) + \mathcal{P}_\mu\mathcal{P}_\nu M_d^{-2}W_2^{\nu d}(q^2,\nu_d) - i\epsilon_{\lambda\rho\beta\gamma}\mathcal{P}^\beta q^\gamma(2M_d^2)^{-1}W_3^{\nu d}(q^2,\nu_d) + q_\lambda q_\rho M_d^{-2}W_4^{\nu d}(q^2,\nu_d) + (\mathcal{P}_\lambda q_\rho + q_\lambda\mathcal{P}_\rho)(2M_d^2)^{-1}W_5^{\nu d}(q^2,\nu_d)\,, \tag{D1}$$

and similarly for $W_{\lambda\rho}^{\nu i}$.

To project out the W's from $W_{\lambda\rho}$ we need the projection operator $P^{\lambda\rho}$:

$$P_i^{\lambda\rho}W_{\lambda\rho}^{\nu d} = W_i^{\nu d}, \quad i=1,\ldots,5\,. \tag{D2}$$

We find

$$\begin{aligned}
P_1^{\lambda\rho} &= [2(q^2M_d^2-\nu_d^2)]^{-1}[-(q^2M_d^2-\nu_d^2)g^{\lambda\rho} + q^2\mathcal{P}^\lambda\mathcal{P}^\rho + M_d^2q^\lambda q^\rho - \nu_d(\mathcal{P}^\lambda q^\rho + q^\lambda\mathcal{P}^\rho)]\,,\\
P_2^{\lambda\rho} &= M_d^2 2^{-1}(\nu_d^2-q^2M_d^2)^{-2}[q^2(\nu_d^2-q^2M_d^2)g^{\lambda\rho} + 3q^4\mathcal{P}^\lambda\mathcal{P}^\rho + (2\nu_d^2+q^2M_d^2)q^\lambda q^\rho - 3q^2\nu_d(\mathcal{P}^\lambda q^\rho + q^\lambda\mathcal{P}^\rho)]\,,\\
P_3^{\lambda\rho} &= -iM_d^2(q^2M_d^2-\nu_d^2)^{-1}\epsilon^{\lambda\rho\beta\gamma}\mathcal{P}_\beta q_\gamma\,,\\
P_4^{\lambda\rho} &= M_d^2 2^{-1}(\nu_d^2-q^2M_d^2)^{-2}[M_d^2(\nu_d^2-q^2M_d^2)g^{\lambda\rho} + (2\nu_d^2+q^2M_d^2)\mathcal{P}^\lambda\mathcal{P}^\rho + 3M_d^4q^\lambda q^\rho - 3M_d^2\nu_d(\mathcal{P}^\lambda q^\rho + q^\lambda\mathcal{P}^\rho)]\,,\\
P_5^{\lambda\rho} &= M_d^2(\nu_d^2-q^2M_d^2)^{-2}[-\nu_d(\nu_d^2-q^2M_d^2)g^{\lambda\rho} - 3q^2\nu_d\mathcal{P}^\lambda\mathcal{P}^\rho - 3\nu_dM_d^2q^\lambda q^\rho + (2\nu_d^2+q^2M_d^2)(\mathcal{P}^\lambda q^\rho + q^\lambda\mathcal{P}^\rho)]\,.
\end{aligned} \tag{D3}$$

Applying these projection operators to the equation

$$W_{\lambda\rho}^{\nu d}(q^2,\nu_d) = \sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}G(x,\vec{k}_\perp)W_{\lambda\rho}^{\nu i}(q^2,\nu)\,, \tag{D4}$$

we obtain

$$W_1^{\nu d}(q^2,\nu_d) = \sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}G(x,\vec{k}_\perp)[W_1^{\nu i} + AW_2^{\nu i}]\,, \tag{D5}$$

$$W_2^{\nu d}(q^2,\nu_d) = \sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}G(x,\vec{k}_\perp)BW_2^{\nu i}\,, \tag{D6}$$

$$W_3^{\nu d}(q^2,\nu_d) = \sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}G(x,\vec{k}_\perp)CW_3^{\nu i}\,, \tag{D7}$$

$$W_4^{\nu d}(q^2,\nu_d) = \sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}G(x,\vec{k}_\perp)[DW_2^{\nu i} + W_4^{\nu i} + EW_5^{\nu i}]\,, \tag{D8}$$

$$W_5^{\nu d}(q^2,\nu_d) = \sum_{i=p,n}\int d^2\vec{k}_\perp\int_0^1\frac{dx}{x}G(x,\vec{k}_\perp)[FW_2^{\nu i} + GW_5^{\nu i}]\,, \tag{D9}$$

where A and B are given by Eq. (5.7),

$$\begin{aligned}
C &= (q^2M_d^2-\nu_d^2)^{-1}[-x\nu_d^2 + \nu_d(\vec{q}_\perp\cdot\vec{k}_\perp) + C_1]\,,\\
C_1 &= q^2[2(1-x)]^{-1}[(1-x^2)M_d^2 - M^2 - \vec{k}_\perp^2]\,,
\end{aligned} \tag{D10}$$

$$\begin{aligned}
D &= [2(\nu_d^2-q^2M_d^2)^2]^{-1}[\nu_d^2D_1 + \nu_d(\vec{q}_\perp\cdot\vec{k}_\perp)D_2 + q^2D_3]\,,\\
D_1 &= [2(1-x)]^{-2}\{\vec{k}_\perp^4 + 2\vec{k}_\perp^2[M^2 - 2(1-x)^2M_d^2] + 4x(1-x)M_d^2[M^2-(1-x)M_d^2] + [(1-x)^2M_d^2 - M^2]^2\}\,,\\
D_2 &= 3(1-x)^{-1}M_d^2[M_d^2(1-x)^2 - M^2 - \vec{k}_\perp^2]\,,\\
D_3 &= M_d^2[4(1-x)^2]^{-1}[M_d^2(1-x)^2 - M^2]^2\,,
\end{aligned} \tag{D11}$$

$$\begin{aligned}
E &= [2(\nu_d^2-q^2M_d^2)^2]^{-1}[\nu_d^3E_1 - M_d^2\nu_d^2(\vec{q}_\perp\cdot\vec{k}_\perp) + \nu_dq^2E_2 - 2q^2M_d^4(\vec{q}_\perp\cdot\vec{k}_\perp)]\,,\\
E_1 &= (1+2x)M_d^2 - (1-x)^{-1}(M^2+\vec{k}_\perp^2)\,,\\
E_2 &= (1-x)^{-1}M_d^2[M^2+\vec{k}_\perp^2-(1-x)^2M_d^2]\,,
\end{aligned} \tag{D12}$$

$$
\begin{aligned}
F &= (\nu_d^2 - q^2 M_d^2)^{-2}[\nu_d^3 F_1 + 2\nu_d^2(\vec{q}_\perp\cdot\vec{k}_\perp)F_2 + \nu_d q^2 F_3 - q^2(\vec{q}_\perp\cdot\vec{k}_\perp)F_4]\,,\\
F_1 &= (1-x)^{-1}[x(1-x)^2 M_d^2 - xM^2 + (1-2x)\vec{k}_\perp^{\,2}]\,,\\
F_2 &= (2x-1)M_d^2 + (1-x)^{-1}(M^2+\vec{k}_\perp^{\,2})\,,\\
F_3 &= -3M_d^2 q^{-2}(\vec{q}_\perp\cdot\vec{k}_\perp)^2 + x(x+2)M_d^4 - M_d^2(1-x)^{-1}[2xM^2 + (1+x)\vec{k}_\perp^{\,2}]\\
&\quad -3[4(1-x)^2]^{-1}\{\vec{k}_\perp^{\,4} + \vec{k}_\perp^{\,2}[2M^2 - 2(1-x^2)M_d^2] + [(1-x^2)M_d^2 - M^2]^2\}\,,\\
F_4 &= M_d^2(1-x)^{-1}[(1-x^2)M_d^2 - M^2 - \vec{k}_\perp^{\,2}]\,,
\end{aligned}
\tag{D13}
$$

$$
\begin{aligned}
G &= (\nu_d^2 - q^2 M_d^2)^{-2}[x\nu_d^4 - (\vec{q}_\perp\cdot\vec{k}_\perp)\nu_d^3 + q^2\nu_d^2 G_1 + \nu_d q^2(\vec{q}_\perp\cdot\vec{k}_\perp)M_d^2 + q^4 G_2]\,,\\
G_1 &= -\tfrac{1}{2}M_d^2(1+3x) + \tfrac{1}{2}(1-x)^{-1}[M^2+\vec{k}_\perp^{\,2}]\,,\\
G_2 &= M_d^2[2(1-x)]^{-1}[(1-x^2)M_d^2 - M^2 - \vec{k}_\perp^{\,2}]\,.
\end{aligned}
\tag{D14}
$$

In the deep-inelastic limit,

$$
\begin{aligned}
W_1^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int d^2\vec{k}_\perp \int_0^1 \frac{dx}{x} G(x,\vec{k}_\perp)\left(W_1^{\nu i} + \frac{\vec{k}_\perp^{\,2}}{2M_d^2}W_2^{\nu i}\right),\\
W_2^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int d^2\vec{k}_\perp \int_0^1 dx\, xG(x,\vec{k}_\perp)W_2^{\nu i}\,,\\
W_3^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int d^2\vec{k}_\perp \int_0^1 dx G(x,\vec{k}_\perp)W_3^{\nu i}\,,\\
W_4^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int d^2\vec{k}_\perp \int_0^1 \frac{dx}{x} G(x,\vec{k}_\perp)W_4^{\nu i}\,,\\
W_5^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int d^2\vec{k}_\perp \int_0^1 dx G(x,\vec{k}_\perp)W_5^{\nu i}\,.
\end{aligned}
\tag{D15}
$$

Defining

$$
\begin{aligned}
&W_1^{\nu d} = F_1^{\nu d},\ \ \nu_d M_d^{-2}W_2^{\nu d} = F_2^{\nu d},\ \ \nu_d M_d^{-2}W_3^{\nu d} = F_3^{\nu d}\,,\\
&\nu_d M_d^{-2}W_4^{\nu d} = F_4^{d},\ \ \nu_d M_d^{-2}W_5^{\nu d} = F_5^{\nu d}\,,
\end{aligned}
\tag{D16}
$$

and similarly for W^i by changing $\nu_d \to \nu$, we can rewrite Eq. (D15) as[15]

$$
\begin{aligned}
F_{1,3,5}^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int_0^1 \frac{dx}{x} G(x) F_{1,3,5}^{\nu i}(q^2,\omega)\theta(\omega-1)\,,\\
F_2^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int_0^1 dx G(x) F_2^{\nu i}(q^2,\omega)\theta(\omega-1)\,,\\
F_4^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int_0^1 \frac{dx}{x^2} G(x) F_4^{\nu i}(q^2,\omega)\theta(\omega-1)\,.
\end{aligned}
\tag{D17}
$$

Simple direct consequences of Eq. (D17) and of the normalization condition are the wave-function-independent sum rules for the neutrino scattering:

$$
\begin{aligned}
\int_1^\infty \frac{d\omega_d}{\omega_d^2} F_{L,R}^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int_1^\infty \frac{d\omega}{\omega^2} F_{LR}^{\nu i}(q^2,\omega)\,,\\
\int_1^\infty \frac{d\omega_d}{\omega_d} F_2^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int_1^\infty \frac{d\omega}{\omega} F_2^{\nu i}(q^2,\omega)\,,\\
\int_1^\infty \frac{d\omega_d}{\omega_d^3} F_4^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int_1^\infty \frac{d\omega}{\omega^3} F_4^{\nu i}(q^2,\omega)\,,\\
\int_1^\infty \frac{d\omega_d}{\omega_d^2} F_5^{\nu d}(q^2,\omega_d) &= \sum_{i=p,n}\int_1^\infty \frac{d\omega}{\omega^2} F_5^{\nu i}(q^2,\omega)\,,
\end{aligned}
\tag{D18}
$$

where

$$F_{L,R} \equiv \tfrac{1}{2}(F_1 \mp F_3)\,.$$

The higher-moment sum rules can also be easily obtained.

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[1]G. B. West, Phys. Lett. 37B, 509 (1971); Ann. Phys. (N. Y.) 74, 464 (1972).

[2]W. B. Atwood and G. B. West, Phys. Rev. D 7, 773 (1973).

[3]L. L. Frankfurt and M. I. Strikman, Phys. Lett. 64B, 433 (1976); 65B, 51 (1976); 76B, 333 (1978).

[4]P. V. Landshoff and J. C. Polkinghorne, Phys. Rev. D 18, 153 (1978).

[5]G. B. West, private communication to one of us (M.J.M.).

[6]In private communication, Landshoff told us that there is a possibility for the Sudakov-variable method to handle this problem, which is mentioned in Ref. 4. As an indication of that, see P. V. Landshoff and D. M. Scott, Nucl. Phys. B131, 173 (1978).

[7]A. Bodek, Phys. Rev. D 8, 2331 (1973). The $|\mathfrak{M}|^2$ here can be regarded also as the imaginary part of the forward scattering amplitude.

[8]D. Sivers, S. J. Brodsky, and R. Blankenbecler, Phys. Rep. 23C, 2 (1976); M. G. Schmidt, Phys. Rev. D9, 408 (1974); I. A. Schmidt and R. Blankenbecler, *ibid.* 15, 3321 (1977).

[9]Though the conclusion is correct, there is an inconsistency in Ref. 3. Equation (2.13) is derived by using the light-cone approach. This method leads to a restriction that $0<K^+<M_d$ (see Appendix B) which is reflected by the factor $[(E+K_z)/M_d]\theta(E+K_z)$. On the other hand, Eq. (2.7) is *not* derived by using this method. In fact the same factor above in Eq. (2.14) is due to both the flux and the threshold condition.

[10]This corresponds to the usual static approximation for the bound-state wave function in which the relative time variable is set to zero. See P. M. Fishbane and I. J. Muzinich, Phys. Rev. D 8, 4015 (1973), Appendix A.

[11]In Eq. (3.1) $\phi(p^2, k^2)$ is the *n-p-d* vertex function. After we do the k^2 integration we are left with $\phi(p^2, k^2=M^2)$. We identify this $\phi(p^2, k^2=M^2)$ as the *truncated n-p-d* vertex function.

[12]It is not difficult to show that Eq. (3.8) in the *deuteron rest frame*, where $x=(E-k_z)/M_d$, is equivalent to Eq. (2.13). Furthermore, this equation exactly agrees with the normalization condition Eq. (2.11) in Ref. 4. though there the condition is obtained by going to a high-energy limit. Notice that Ref. 4 does not show that the variable ξ (here x) has a value between 0 and 1. In private communication with us, Landshoff told us that it is possible to show that by using a similar approach they used for the covariant description of the parton model of the nucleon. [See P. V. Landshoff and H. Osborn, in *Electromagnetic Interactions of Hadrons*, edited by A. Donnachie and G. Shaw (Plenum, New York, 1978), Vol. 2.]

[13]The limits on x ensure that k^2 is timelike and p^2 is the same as in the elastic case [Eq. (3.6) with $0<x<1$].

[14]Notice that our result is independent of the parameter P. It is hard to obtain this result from other methods such as the Sudakov-variable method (see Appendix B of Ref. 4).

[15]In Eq. (5.13 we had neglected the term

$$\sum_{i=p,n}\frac{1}{2\nu_d}\int \vec{k}_\perp^2 d^2\vec{k}_\perp \times\int_0^1 \frac{dx}{x^2} G(x,\vec{k}_\perp)F_2^{\nu i}(q^2, xw_d)\theta(xw_d-1),$$

which is justified since $\vec{k}_\perp^2$ cannot be large. In fact, by analyzing the equation

$$p^2 = xM_d^2 + M^2 + (x-1)^{-1}(M^2+\vec{k}_\perp^2), \quad 0<x<1$$

we find that, strictly speaking, $0.26 \lesssim x \lesssim 0.66$ and $0<\vec{k}_\perp^2 \lesssim 0.14$ GeV^2 if the interacting nucleon is near the mass shell. These numbers are obtained by taking $M^2-\mu^2-2\mu M<p^2<M^2$, where μ is the pion mass. If we relax this physical restriction, and take $0<p^2<M^2$, then $0<x\lesssim 0.75$ and $0<\vec{k}_\perp^2 \lesssim 0.49$ GeV^2. Furthermore, we expect that $G(x,\vec{k}_\perp)$ must be such that

$$\lim_{\vec{k}_\perp\to\infty} G(x,\vec{k}_\perp)=0.$$

[16]In the infinite-momentum frame these equations have a simple parton interpretation: The probability to find a parton in the deuteron, carrying a fraction of the deuteron momentum x_d, is equal to the product of the probability to find a nucleon with a fraction of the deuteron momentum y and the probability to find the parton in the nucleon with a fraction of nucleon momentum x_d/y.

[17]T. D. Lee, Phys. Rev. 128, 899 (1962).

[18]S. J. Brodsky, R. Roskies, and R. Suaya, Phys. Rev. D 8, 4574 (1973).

[19]Notice that $\vec{p}\,'=\vec{p}+\vec{q}$ and $(p+q)_0=P_0'+O(1/P)$, thus $p'_\mu \simeq (p+q)_\mu$, which means that energy nonconservation can be neglected here.

[20]J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Rev. D 8, 287 (1973).

[21]C. H. Llewellyn Smith, Phys. Rep. 3C, 262 (1972). We do not include the term

$$i(2M_d^2)^{-1}W_6^{\nu d}(q^2,\nu_d)(\mathcal{P}_\lambda q_\rho - q_\lambda \mathcal{P}_\rho)$$

here since it gives zero contribution to the cross section. For our convenience we are including the W_3 and W_4 terms in the $W_{\lambda\rho}$, though they are usually neglected since their contributions to the cross section are $O(m^2)$, where m is the lepton mass in the final state.

[22]Reference 3 uses the so-called dispersion approach developed by Gribov. The derivation there is not clear to us. The α in the second and third paper of Ref. 3 is different from the α in the first paper of Ref. 3, the latter being exactly the same as our x. The source of the difference is in fact due to the setting of $M_{NN}^2 = (M^2+\vec{k}_\perp^2)/\alpha(1-\alpha)$, the invariant mass of the two-nu-

cleon system, equal to $4(M^2+\vec{k}^2)$. Notice that there is no physical basis for this equality.

[23]From Eqs. (2.3) and (2.4) we see that

$$|\psi(\vec{k})|^2 = |\psi_W(\vec{k})|^2 \frac{2(E+K_g)}{M_d}\theta(E+K_g).$$

Clearly, the identification of $|\psi_W(\vec{k})|^2$ as the nonrelativistic deuteron wave function is not correct since $\int d^3k\, |\psi_W(\vec{k})|^2 \neq 1$. On the other hand, for quantitative purposes, it is not a bad approximation to set $|\psi(\vec{k})|^2$ equal to the usual nonrelativistic deuteron wave function. This identification is not inconsistent with the approximation given by Eq. (3.11). This equation implies that

$$\langle x\rangle \equiv \int_0^1 x\, G(x)dx = \tfrac{1}{2}.$$

Following Sec. II we find

$$\langle x\rangle = \frac{M}{M_d} - \frac{\epsilon + \langle T\rangle}{M_d}.$$

Estimates from various deuteron nonrelativistic wave functions gives values of $\langle T\rangle$ which vary from a few MeV up to 22 MeV (see Ref. 1). Thus

$$\langle x\rangle \simeq M/M_d \approx \tfrac{1}{2}.$$